HIGHWAY RESEARCH REPORT

METHOD FOR REDUCING THE COST OF CORROSION TESTING OF REINFORCING STEEL

69-22

STATE OF CALIFORNIA

BUSINESS AND TRANSPORTATION AGENCY

DEPARTMENT OF PUBLIC WORKS

DIVISION OF HIGHWAYS

MATERIALS AND RESEARCH DEPARTMENT

RESEARCH REPORT

NO. M & R 635116-5

Prepared in Cooperation with the U.S. Department of Transportation, Bureau of Public Roads November, 1969

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STATE OF CALIFORNIA—TRANSPORTATION AGENCY

RONALD REAGAN, Governor

DEPARTMENT OF PUBLIC WORKS
DIVISION OF HIGHWAYS
MATERIALS AND RESEARCH DEPARTMENT
5900 FOLSOM BLVD., SACRAMENTO 95819

November, 1969
Interim Report
M & R No. 635116-5

Mr. John L. Beaton
Materials and Research Engineer
Sacramento, California

Dear Sir:

Submitted herewith is a statistical research report entitled:

METHOD FOR REDUCING THE COST OF

CORROSION TESTING OF REINFORCING STEEL

This report was prepared under contract No. 19-0206 by Harry Benenson of the Sacramento State College School of Engineering. Findings will be utilized in the project "Corrosion Control of Steel in Concrete" to aid in the analysis and interpretation of corrosion control test data.

Very truly yours,

D. L. Spellman

Donald L. Spellman
Assistant Materials and
Research Engineer

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November , 1969

Mr. R. F. Stratfull
Senior Corrosion Engineer
Materials and Research Dept.
California Division of Highways
5900 Folsom Blvd.
Sacramento, California 95819

Dear Sir:

Submitted herewith is a statistical research report entitled:

METHOD FOR REDUCING THE COST OF

CORROSION TESTING OF REINFORCING STEEL

Prepared under contract

By H. Benenson
Lecturer
Sacramento State College
Department of Engineering

Very truly yours,

Harry Benenson.

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REFERENCE: Benenson, H., "Statistical Procedure for Reducing Time and Samples in Laboratory Corrosion Tests of Steel in Concrete", State of California, Department of Public Works, Division of Highways, Materials and Research Department. Research Report October, 1969.

ABSTRACT:

An analytical procedure was developed that will permit a test to be curtailed after a given percentage of the samples have been tested or when the test is stopped after a specified number of days. The analysis that was mathematically derived was applied to eight sets of data supplied by the Materials and Research Department. The analysis provided for maximum likelihood estimates of the mean and standard deviation from a censored normal population as well as estimates of the variance of the mean and standard deviation. These estimates permitted comparison of the maximum error expected from a 100 percent sample to the maximum error expected from reduced sampling of 75, 50 and 25 percent. In addition, the number of specimens required to compensate for the reduced samples was determined.

It was found in applying the method to the eight sets of data that excellent correlation was obtained, especially at levels greater than 50 percent of the sample. A bimodal normal distribution was detected in the data and a correction was developed and may be applied if such a correction is desired. However, typical testing variation has the same magnitude as the correction, making the correction unnecessary. The procedure was applied to one set of the data to illustrate and provide training to the personnel in the Materials and Research Department.

Key Words : Statistical Analysis, statistical sampling, predictions, Sample size.

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The opinions, findings, and conclusions expressed in this publication are those of the author and not necessarily those of the Bureau of Public Roads.

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Table of Contents

	Page
Introduction	1
Summary and Conclusion	2
Technical Discussion	3
Application	
I Procedure for estimating population parameters from reduced samples	4
II Comparison of reduced (censored) samples to 100 % sample	5
III Application of procedure to test results	5
IV Application and interpretation of test data results	7
Analysis of Censored Data for Small Samples from Normal Distribution	9
Mathematical Development	10
Bibliography	16
Figures	
Tables	

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LIST OF FIGURES

1 - Maximum Error as a Function of Sample Size and Percent of Sample Tested

2 - " " " " " " " " " " " " "

3 - Curtailment Factor vs. Percentage Sample Tested

4 - Comparison of 50 and 100 Percent Samples for 5-Sack Moist Data

5 - Correction Curve for Bimodal Normal Distribution

6 - Frequency Distribution of Days to Potential Jump for 6-Sack Moist Sample

7 - Bimodal Normal Distribution for Potential Jump 6-Sack Moist Sample

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LIST OF TABLES

I Values of z as a Function of ψ and ρ

II Variance Factors for Censored Samples

III Test Data for Days to Potential Drop

IV Summary of Data Evaluation of Censored Data

V Determination of λ from γ and h

VI Error Analysis of Potential Jump Test Data

VII Coefficients for Estimating the Mean and Standard Deviation in Censored Samples

VIII Variance and Co-Variance of Censored Data Estimates

IX Computation of Censored Sampling Procedure to Jump Potential Data

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INTRODUCTION

Laboratory corrosion tests of steel in concrete are continually being conducted by the Materials and Research Department for the Division of Highways. The typical test requires a given time to elapse before a significant event occurs. In the test data being evaluated, the event was potential jump. It is necessary that all the specimens be tested until the event for each specimen occurs, therefore, analysis cannot be carried out until the last data point is obtained. The elapsed time can be several hundred days. The problem presented was to develop and derive analytically a procedure that would permit the testing to be terminated when a given percentage of the specimens had been tested or possibly stop the test after a given number of days had elapsed. It was further requested that the data obtained from reduced testing be equated to the 100 percent test results that had been reported to date. That is, it is desired to obtain an estimate of the expected maximum error for the reduced test results and compare these results to the 100 percent sample results. Also, it was desired that the additional number of specimens required be determined in order to provide that the reduced sample tests be equivalent to the 100 percent sample results. These objectives were all satisfactorily completed during the course of the study. An example is included to illustrate the procedure.

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SUMMARY AND CONCLUSIONS

The analysis procedure and mathematical derivation that were requested were completed and are enclosed in this report. Tables necessary to support the analysis and reduce tedious computations are tabulated and presented herewith. The results of the analysis show that for samples of 20 specimens, a 25 percent reduction in testing time can be accomplished by an addition of only four specimens while a 50 percent reduction in testing time will require approximately 25 additional specimens to provide equivalent results to those obtained from 100 percent full-time tests.

When the procedure was applied to the test data, excellent correlation was obtained with the observed error being approximately 20 to 50 percent of the computed maximum 95 percent confidence error level for the 100 percent sample. The test data showed that curtailing testing at 50 percent of normal time is entirely feasible with errors of only one day observed in estimating the mean of the population when samples are under 100-day duration and approximately 5 percent for periods longer than 100 days. Some of the error observed was attributed to a bimodal normal distribution that characterized the data. A correction curve was established. However, for samples with less than 50 percent curtailment, little error is discernible.

Eight sets of data were supplied by the Materials and Research Department in order to verify the procedure. The results of the analysis indicated that approximately 50 percent of the testing can be curtailed, resulting in substantial savings in test time, personnel time, and permitting decisions to be made much faster.

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TECHNICAL DISCUSSION

When testing is halted so that only a portion of the specimens have been tested and measured and the remaining specimens can only be counted, the sample is called a "censored" sample. For the problem presented by the Materials and Research Department, it was desired that the test be conducted for a given number of days, or for that period of time when data have been collected for a fixed percentage of the sample. When this elapsed time is reached, the test is terminated. Thus, the remaining samples can only be counted. To obtain a procedure to permit restricting or censoring the sample, it is necessary to establish the underlying distribution of the data. From this distribution, it is possible to mathematically derive estimates of the parameters that characterize this distribution. For censored samples, it is necessary to include the test curtailment point in the derivation. The underlying distribution was assumed to be normal, an assumption that was borne out in the test results. The estimates of the mean and standard deviation were determined using maximum likelihood techniques. Depending on the method used for derivation, two possible procedures can be used. Both procedures are similar and are included in this report, although only one method is referenced in the illustration. Also, for completeness, a procedure is included for cases where the total sample size is ten or less since the basic procedure assumes sample sizes greater than ten.

It is the object of this report to present a concise procedure, both applied and theoretical, of censored sampling for normal distributions and to apply this procedure to corrosion testing by the Materials and Research Department. Therefore, several tables that appear in the statistical literature that will simplify computations have been included so that individuals with little statistical background can apply the statistical methods. The sequence of the report to follow will include a procedure to apply to test data, a comparison of the theoretical and applied results, and an illustration of the procedure as applied to the test results. A complete summary of the procedure applied to the test results will follow, including the analysis and interpretation. Concluding the report will be the mathematical derivation for censored samples from normal distributions.

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APPLICATION

I. Procedure for Estimating Population Parameters from Reduced Samples

Given a sample of n specimens of which k of the specimens are to be tested to termination. The test will be terminated when either a predetermined time X_k has been reached, or a given percentage of the sample $(\frac{k}{n})$ has been tested. Either of the two criteria must be established prior to testing. The analysis procedure is similar for both criteria. The following procedure will be used to estimate the mean and standard deviation:

1. Compute the mean $(\bar{X})$ and the standard deviation (S) from the censored sample of k values where

$$\bar{x} = \left(\sum_{i=1}^{k} x_i\right)/k \; ; \quad S^2 = \sum_{i=1}^{k} (x_i - \bar{x})^2/k \; ;$$

2. Determine $d = X_k - \bar{X}$ and $d^2 = (X_k - \bar{X})^2$ where X_k is the terminal level of testing (in days).

3. Compute $\psi = S^2/(S^2 + d^2)$ and $p = k/n$ where k of the n specimens were tested to the time of curtailment.

4. Refer to Table I and determine z which is a function of ψ and p.

5. Determine the estimate of the population mean (μ) and population standard deviation from the following relations:

$$\hat{\mu} = \bar{X} + (\sigma^2 - S^2)/d$$

$$\hat{\sigma} = d/z$$

Sigma $(\hat{\sigma})$ must be computed before $\hat{\mu}$. The (^) refers to population parameters estimated from sample data.

6. Plot population estimates on probability paper by locating the $\hat{\mu}$ at the 50 percent probability point and $(\hat{\mu} + \hat{\sigma})$ at the 84 percent probability point and $(\hat{\mu} - \hat{\sigma})$ at the 16 percent probability point. Connect the three points which are on a straight line.

7. 95% confidence limits are $\hat{\mu} \pm 1.96\, k' \hat{\sigma}/\sqrt{n}$ where k' is obtained from Figure 3 and $\hat{\mu}$ and $\hat{\sigma}$ are obtained from Step 5.

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II. Comparison of Reduced (Censored) Samples to 100% Sample

Reference (1) compares the maximum error as a percentage of the mean vs. number of specimens for different values of coefficient of variation. The censored samples are compared on the same basis as shown in Figures 1 and 2. Maximum error can be shown to be equal to $\sigma/(\bar{X}\sqrt{n})$ or $(CV)/\sqrt{n}$ and for 95 percent confidence assuming normal distribution with known standard deviation, the maximum error is $1.96\ (CV)/\sqrt{n}$. Therefore, parallel lines can be drawn on log log scales as shown for the 100 percent samples of Figures 1 and 2. Where reduced sampling is used, the minimum error at the 95 percent confidence level is $1.96\ k(CV)/\sqrt{n}$ where k is the curtailment factor applied to censored sampling. Figure 3 shows the relation of k as a function of the percentage sampled. These data are also tabulated in Table II in the $u_{11}(k)$ column. Three points were selected from Figure 3 and superimposed on Figures 1 and 2. Thus, it can be seen in Figures 1 and 2 that a 75 percent sample represents an almost insignificant increase in the 95 percent confidence maximum error band at sample sizes greater than 20 and a sample 50 percent shows a reasonable and acceptable number of specimens for test. Another useful conclusion can be extracted from Figures 1 and 2 which specifies the number of specimens from reduced testing that must be tested to yield the same maximum error that would be obtained from a 100 percent sample test. These analyses are limited to the 95 percent confidence level. The results are as follows:

Number of Specimens Needed for Given Test Percentage Using 100% Sample as 20 Specimens

Coefficient of Variation	Testing Percentage 100%	75%	50%	25%
60%	20	23	42	340
30%	20	24	50	400

Again the conclusion can be made that based on theoretical analyses, a 75 percent sample would require only an increase of four specimens while a 50 percent sample would require a 30 specimen increase to obtain equivalent testing results.

III. Application of Procedure to Test Results

The procedure was applied to 5-sack moist. data. It was assumed that the test was curtailed at 50% of the 20 samples which is also equivalent to 50 days. Therefore, $n = 20$, $k = 10$, and $X_k = 50$.

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Step 1

$$\bar{X} = (\sum_{i=1}^{k} X_i)/k = 381/10 = 38.1$$

$$S^2 = \sum_{i=1}^{k}(X_k - \bar{X})^2 /k = 368.9/10 = 36.89; \; S = \sqrt{S^2} = 6.07$$

Step 2

$$d = (X_k - \bar{X}) = 50 - 38.1 = 11.9 \quad d^2 = (X_k - \bar{X})^2 = 141.61$$

Step 3 $\psi = S^2/(S^2+d^2) = (36.89)/(36.89+141.61) = .207$

$p = k/n = 10/20 = 0.50$

Step 4

When $p = 0.50$

$\psi = 0.20, z = 0.9262$ Therefore, for $\psi = 0.207$,

$z = 0.9206$

and, $\psi = 0.25 \; z = 0.8865$

Step 5

$$\hat{\sigma} = d/z = 11.90/0.9206 = 12.93 \text{ days}$$

$$\hat{\mu} = \bar{X} + (\hat{\sigma}^2 - S^2)/d = 38.10 + (12.93^2 - 36.89)/11.9 = 49.05 \text{ days}$$

The 100 percent values of μ and σ computed from the 20 specimens are:

$\mu = 48.25$ days, $\sigma = 11.94$ days

Step 6

The plot is shown in Figure 4 along with the 100% values

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50% Sample $\hat{\mu} = 49.05$ at 50% probability

$\hat{\mu} + \hat{\sigma} = 61.98$ at 84% probability

$\hat{\mu} - \sigma = 36.12$ at 16% probability

100% Sample $\mu = 48.25$ at 50% probability

$\mu + \sigma = 60.19$ at 84% probability

$\mu - \sigma = 36.13$ at 16% probability

It can be noted that the difference is less than one day.

Step 7

The 95% confidence interval can be determined from

$\hat{\mu} \pm 1.96 \frac{k'\hat{\sigma}}{\sqrt{n}}$ where k is determined from Table II μ_{11} (k') for $P = 0.50$. The confidence limits are:

$$49.05 \pm 1.96 \times 1.52 \times \frac{12.93}{\sqrt{20}} = 49.05 \pm 8.53$$

or the 95% confidence limits are between 40.52 to 57.58 days.

IV. Application and Interpretation of the Test Data Results

Eight sets of test data were supplied by the Materials and Research Department. The data represent elapsed time when a potential jump occurs. A specimen is measured for voltage potential. At a specific time during the testing program, an abrupt change from an apparently passive potential to an active potential is observed. The time in days when this event occurs, constitutes the data for the analysis. Table III summarizes the specimen in rank order of testing and also the termination points that were evaluated for each sample. As explained in the mathematical section of this report, two methods of analysis are available, so both methods were applied to this data. The results of the analysis are shown in Table IV. An error analysis showing the coefficient of variation, the standard error of $\hat{\mu}$ and the standard error of $\hat{\sigma}$ is shown in Table V.

To interpret the data, it can be seen that when low percentages of samples are tested, large errors occur. However, the errors from the test data are all less than the true average

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and standard deviation. This fact is shown in Figure 5. In fact, the data are sufficiently repeatable so that a correction factor can be applied at percentage levels less than 50 percent. With this correction curve, the 100 percent average could be estimated within two days. After the 50 percent sampling point, no correction is required, although a bias of + 0.5 days can be applied at the 40-50 percent sampling range. The cause for this error can be traced to essentially two normal distributions with distinct and different means and standard deviations. A histogram is presented in Figure 6 to demonstrate this fact. The data are also shown on probability paper in Figure 7 to illustrate the two distributions. Therefore, at low percentage sampling levels, only the first distribution is estimated. However, when 50 percent sampling or greater is used, the influence of bimodal normal distribution is not significantly indicated.

Reference is made to the 7-sack, steam sample. The last sample took 326 days which was 119 days longer than the 19th sample. This type of testing is very unusual and leads one to suspect that this last specimen is one not belonging to the population of the first 19 values. Where the data are plotted on probability paper, this value is usually given little weight when estimating the underlying normal distribution. In this case the reduced sample gave estimates that were better than the estimates from the 100 percent sample since the last value caused a higher mean of 6 days and a higher standard deviation of 17 days when in actuality it should not have been considered. Also, a 60 percent sample in this instance would have saved 176 days of testing, which is obviously a significant saving.

A comparison of the standard error of the mean and standard deviation as seen in Table VI shows that these values are well within the maximum error that might be expected from censored samples. In these cases, the data compares favorably to the theoretical.

Conclusions

The use of censored testing has been shown to be applicable to the test results and especially with a 50 percent sample or greater, could result in substantial savings in dollars, manpower, and time. It is recommended that the procedure be used. As seen in Figure 5, the errors in estimating the mean and standard deviation are within the realm of testing error.

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ANALYSIS OF CENSORED DATA FOR SMALL SAMPLES FROM NORMAL DISTRIBUTION

When samples are ten or less, the techniques described in this report become inefficient and should not be used. A procedure is presented herewith in which linear methods can be used. Consider testing n specimens and the testing halted after k samples have been tested. Thus n-k samples are known to be greater than X_k. The equations for estimating the population mean and standard deviation are :

$$\hat{\mu} = \sum_{i=1}^{k} \alpha_{1i} X_i$$

$$\hat{\sigma} = \sum_{i=1}^{k} \alpha_{2i} X_i$$

The coefficients of α_{1i} and α_{2i} used in estimating μ and σ are obtained from Table VII. To illustrate the procedure, consider the following ordered observations in which the first seven of ten samples is to be used. The data in the illustration represents propellant samples that were subjected to a constant applied strain. The logarithm of this variate is assumed to be normally distributed. It is desired to estimate the mean time to rupture and the standard deviation of rupture time.

	Ordered Observation Yi	$Log_{10}Yi$	Coefficients 1i	2i
1	41	1.613	.02442821	-.32524451
2	44	1.644	.06355637	-.17575100
3	46	1.663	.08178041	-.10578536
4	54	1.732	.09616208	-.05017756
5	55	1.740	.10885704	-.000056384
6	58	1.763	.12073778	.04685799
7	60	1.778	.50447311	.61066429
8	--	---	0	0
9	--	---	0	0
10	--	---	0	0

The linear coefficient of α_{1i} and α_{2i} are read from Table VII for n = 10 and n-k = 3. The estimates of the mean and standard deviation are

$$\log \hat{\mu} = \sum_{i=1}^{7} \log y_i \quad . \alpha_{1i} = 1.746, \hat{\mu} = 55.72 \text{ hours}$$

$$\log \hat{\sigma} = \sum_{i=1}^{7} \log Y_i \quad . \alpha_{2i} = 8.959-10(-1.041), \hat{\sigma} = .091 \text{ hrs.}$$

This analysis is based on references (6), (7), (8).

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MATHEMATICAL DEVELOPMENT

A.

A. C. Cohen's Method

Consider a sample consisting of n specimens of a characteristic X (a random variable) such that $X \leq X_k$ where X_k is a known and fixed terminal point. When X is normally distributed with mean μ and standard deviation σ, its frequency distribution function is

$$(1) \qquad f(X) = \frac{1}{\sigma\sqrt{2\pi}} \quad e^{-(X-\mu)^2/2\pi^2} \quad -\infty \leq X \leq \infty$$

and the likelihood function for a censored sample from this population may be expressed as

$$(2) \qquad P = \frac{1}{[I_o \sigma\sqrt{2\pi}]^n} \quad e^{-\sum_1^n (X_i-\mu)^2/2\sigma^2}$$

Here n is the number of observations for (which $X \leq X_k$) and I is the proportion of the population from which measured observations are possible.

Let ξ designate the terminal point in standard units of the population or

$$\xi = (X_k - \mu)/\sigma$$

and I_o when expressed as a function of ξ becomes

$$(4) \qquad F(\xi) = \int_{-\infty}^{\xi} \varphi(t)\,dt \quad \text{where} \quad \varphi(t) = \frac{1}{\sqrt{2\pi}} \quad e^{-t^2/2}.$$

Taking the natural logarithms of (2) and writing L for ln P, we have

$$(5) \qquad L = -n \ \ln I_o - n \ln \sigma - n \ln \sqrt{2\pi} - \sum_1^n (X_i - m)^2/2\sigma^2$$

Differentiating (5) to derive the maximum likelihood estimates of μ and σ, and equating the resulting equations to zero, thereby obtaining

$$(6) \qquad \frac{\partial L}{\partial \mu} = \frac{-nz}{\sigma} + \frac{1}{\sigma} \sum_1^n \left(\frac{X_i - \mu}{\sigma}\right) = 0$$

$$\frac{\partial L}{\partial \sigma} = \frac{-n\xi z}{\sigma} - \frac{n}{\sigma} + \frac{1}{\sigma} \sum_1^n \left(\frac{X_i - \mu}{\sigma}\right)^2 = 0$$

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Where Z is defined as

(7) $Z(\xi) = \varphi(\xi)/F(\xi)$

and from (6) it follows that

(8) $\sum_{1}^{n} (X_i - \mu)/n = \sigma Z$

$$\sum_{1}^{n} (X_i - \mu)^2/n = \sigma^2 (1 + \xi Z)$$

Using the equation for the definition of moments about the termination point

(9) $\nu_k = \sum_{1}^{n} (X_i - X_k)^k/n$

Then the first and second moments are

$\nu_1 = \sum_{1}^{n} (X_i - X_k)/n$ and from equation (3) $\mu = x_k + \sigma\xi$

or $\nu_1 = \sigma(Z + \xi)$

(10) and $\nu_2 = \sum_{1}^{n} (X_i - X_k)^2/n = \sigma^2 \left[1 - \xi(\xi - Z)\right]$

Eliminating σ from the two equations in (10) gives

(11) $\dfrac{1 - \xi(Z - \xi)}{(Z - \xi)^2} \qquad \dfrac{\nu_2}{\nu_1^2} = 0$

in which ξ is the only independent variable.

If γ is defined as $\left[1 - Y(Y - \xi)\right]/(Y - \xi)^2$ and $Y = \left[h/(1-h)\right] \times \varphi(\xi)/F(\xi)$

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Where $h = (n-k)/n$

Then tables are available where the best estimate of the mean and standard deviation are

$$\hat{\mu} = \bar{X} + \hat{\lambda}(X_k - \bar{X}) = X + \lambda d \quad \text{where } \lambda = f(\gamma, h)$$

$$\sigma^2 = s^2 + \hat{\lambda}(X_k - \bar{X})^2 = s^2 + \lambda d^2 \quad \text{and } \gamma = s^2/d^2, \; h = \frac{n-k}{n}$$

and λ is a function of γ and h. Table V has been reproduced from the literature to permit ready solution. The use of the Tables V and the mathematical development was due primarily to A. C. Cohen, References 2, 3, and 4.

B.

A. K. Gupta's Method

A second similar derivation is due to A. K. Gupta, Ref.5, which yield slightly different equations, but very similar results. For sake of completeness, the second derivation was also used in the analysis.

Let $X_1, X_2 \ldots X_n$ be a random sample of sizes n from a normal population with mean μ and standard deviation σ and let $x_1 x_2 \ldots x_k$ be the censored sample of size k in which x_k is the greatest observation. The n-k censored observation are known to be greater than x_k. The likelihood function in such a sample is

$$(1) \qquad L = \frac{n!}{(k-1)!\,(n-k)!} (\sigma\sqrt{2\pi})^{-k} \exp\left[-\frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2\right] \times$$

$$\left[(\sigma\sqrt{2\pi})^{-1} \int_{x_k}^{\infty} \exp\left\{-\frac{1}{2\sigma^2} (x-\mu)^2\right\} dx\right]^{n-k}$$

The logarithm of the likelihood function can be written as

$$\text{Log } L = C - k \log \sigma - \frac{1}{2\sigma^2} \Sigma (x_i - \mu)^2 + (n-k) \log \phi(\eta)$$

where C is a constant, $\eta = \frac{x_k - \mu}{\sigma}$ and $\Phi(\eta) = \frac{1}{\sqrt{2\pi}} \int_{\eta}^{\infty} e^{-\frac{1}{2}t^2} dt$

Let $A = \frac{\phi(\eta)}{\Phi(\eta)}$ Where $\phi(\eta) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\eta^2}$

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The likelihood equations are therefore

(2) $$\frac{\partial \log L}{\partial \mu} = \frac{-1}{\sigma^2} \Sigma(x_i - \mu) + (n-k) \frac{A}{\sigma} = 0$$

(3) $$\frac{\partial \log L}{\partial \sigma} = \frac{K}{\sigma} + \frac{1}{\sigma^2} \Sigma(x_i - \mu)^2 + (n-k)\eta \frac{A}{\sigma} = 0$$

Substituting the value of A from equation (2) in (3) gives

(4) $$-\sigma^2 + s^2 + (\bar{x} - \mu)^2 + (\bar{x} - \mu)(x_k - \mu) = 0.$$

or $$\mu^* = \bar{x} + (\sigma^{2*} - s^2)/d$$

where $\bar{x} = \frac{1}{k} \sum_{i=1}^{k} x_i$ $\quad s^2 = \frac{1}{k} \sum_{i=1}^{k} (x_i - \bar{x})^2$, $d = (x_k - \bar{x})$

Thus, given an estimate of σ, μ can be easily obtained from equation (4). Estimates are noted by an asterisk throughout the report.

In order to estimate σ, we rewrite the likelihood equations in the form

(5) $$\sigma(\eta + \frac{n-k}{k} A) = d$$

(6) and $$\sigma^2 + \eta d\sigma - (S^2 + d^2) = 0$$

Writing $z = \eta + (\frac{1}{p} - 1)A$ where $p = \frac{k}{n}$ equation (5) gives

(7) $$\sigma = d/z$$

and substituting the value of σ in equation (6) we have

(8) $$\psi = \frac{S^2}{S^2 + d^2} = \frac{1 - \eta z - z^2}{1 + n z}$$

The lefthand side of equation (8) contains only sample quantities and can easily be calculated. It is always positive and varies between zero and unity. For a given p, the righthand side is a function of η. Values of z for different values of ψ are only needed for the estimation problem. Table I gives the values of z for different values of p and ψ. When p = 1.0, $z = \eta$ and

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(9) $$Z^2 = \frac{1-\psi}{\psi}$$

The variances and covariances of the maximum likelihood estimates are approximated by calculating the expected values of the following quantities:

$$\frac{-\partial^2 \log L}{\partial \mu^2} = \frac{n}{\sigma^2}\left[p+\phi(\eta) \quad (A-\eta)\right]$$

(10) $$\frac{-\partial^2 \log L}{\partial \mu \, \partial \sigma} = \frac{2}{\sigma^2} \sum_{i=1}^{k} \left(\frac{x_i-\mu}{\sigma}\right)^2 + (n-k)\frac{A}{\sigma^2}\left[1+\eta(A-\eta)\right]$$

$$\frac{-\partial^2 \log L}{\partial \sigma^2} = \frac{k}{\sigma^2} + \frac{3}{\sigma^2} \sum_{i=1}^{k} \left(\frac{x_i-\mu}{\sigma}\right)^2 + (n-k)\frac{A\eta}{\sigma^2}\left[2+\eta(A-\eta)\right]$$

The exact expected values of these quantities cannot easily be evaluated in small samples, hence, large sample approximations have been obtained in the limit when $n \to \infty$, p being fixed. Taking the limits

$$\lim_{n\to\infty} -\frac{\sigma^2}{n}\frac{\partial^2 \text{Log } L}{\partial \mu^2} = p + \phi(\hat{\eta}) \quad (A-\hat{\eta}) = \nu 11$$

(11) $$\lim_{n\to\infty} -\frac{\sigma^2}{n}\frac{\partial^2 \log L}{\partial \mu \, \partial \sigma} = -\phi(\hat{\eta}) + \hat{\eta}\phi(\hat{\eta}) \quad (A-\hat{\eta}) = \nu 12$$

$$\lim_{n\to\infty} -\frac{\sigma^2}{n}\frac{\partial^2 \log L}{\partial \sigma^2} = 2p - \hat{\eta}\phi(\hat{\eta}) + \hat{\eta}^2\phi(\hat{\eta})(A-\hat{\eta}) = \nu 22$$

(12) where $\int_{-\infty}^{\eta} \phi(t)dt = p$, $E\left(\frac{x_i-\mu}{\sigma}\right)\frac{1}{p}\int_{-\infty}^{\hat{\eta}} \phi(t)t\,dt = -\frac{1}{p}\phi(\hat{\eta})$

$$E\left(\frac{x_i-\mu}{\sigma}\right)^2\frac{1}{p}\int_{-\infty}^{\hat{\eta}} \phi(t)t^2\,dt = 1-\frac{1}{p}\hat{\eta}\phi(\hat{\eta})$$

$$\sigma_{ij} = \left[\nu_{ij}\right]^{-1}$$

$$V(\mu^*) = \frac{\sigma^{2*}}{n}\sigma_{11}, \quad V(\sigma^*) = \frac{\sigma^{2*}}{n}\sigma_{22}$$

The variances and covariances of the maximum likelihood estimates from censored samples in terms of σ^2/n are presented in Table VIII.

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The z in Gupta terminology is equivalent to $(Y-\xi)$ in Cohen's notation.

Table IX presents the computation using both methods as applied to the eight sets of data.

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3. Cohen, A. C., "On the Solution of Estimating Equations for Truncated and Censored Samples from Normal Populations", Biometrika, Vol. 44 (1957), pp 225-236.

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5. Gupta, A. K., "Estimation of Mean and Standard Deviation of a Normal Population from a Censored Sample", Biometrika, Vol. 30 (1952), pp 260-273, Vol. 39 (1952).

6. Sarhan, A. E., and Greenberg, B. G., "Estimation of Location and Scale Parameters by Order Statistics from Singly and Doubly Censored Samples, Part I. The Normal Distribution up to Samples of Size 10", Annals of Mathematical Statistics, Vol. 27 (1956), pp 427-51.

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Figure 1

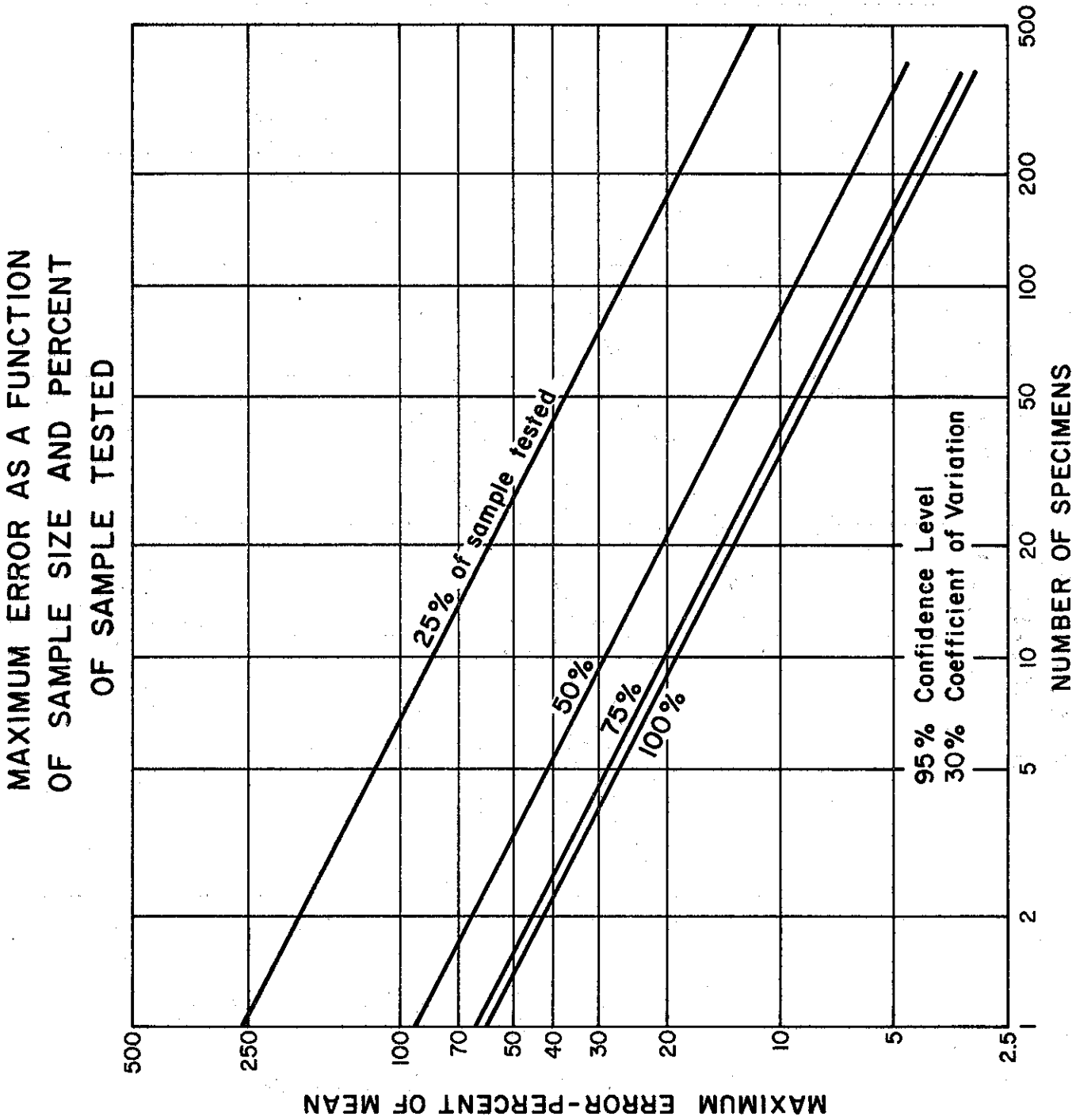

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Figure 2

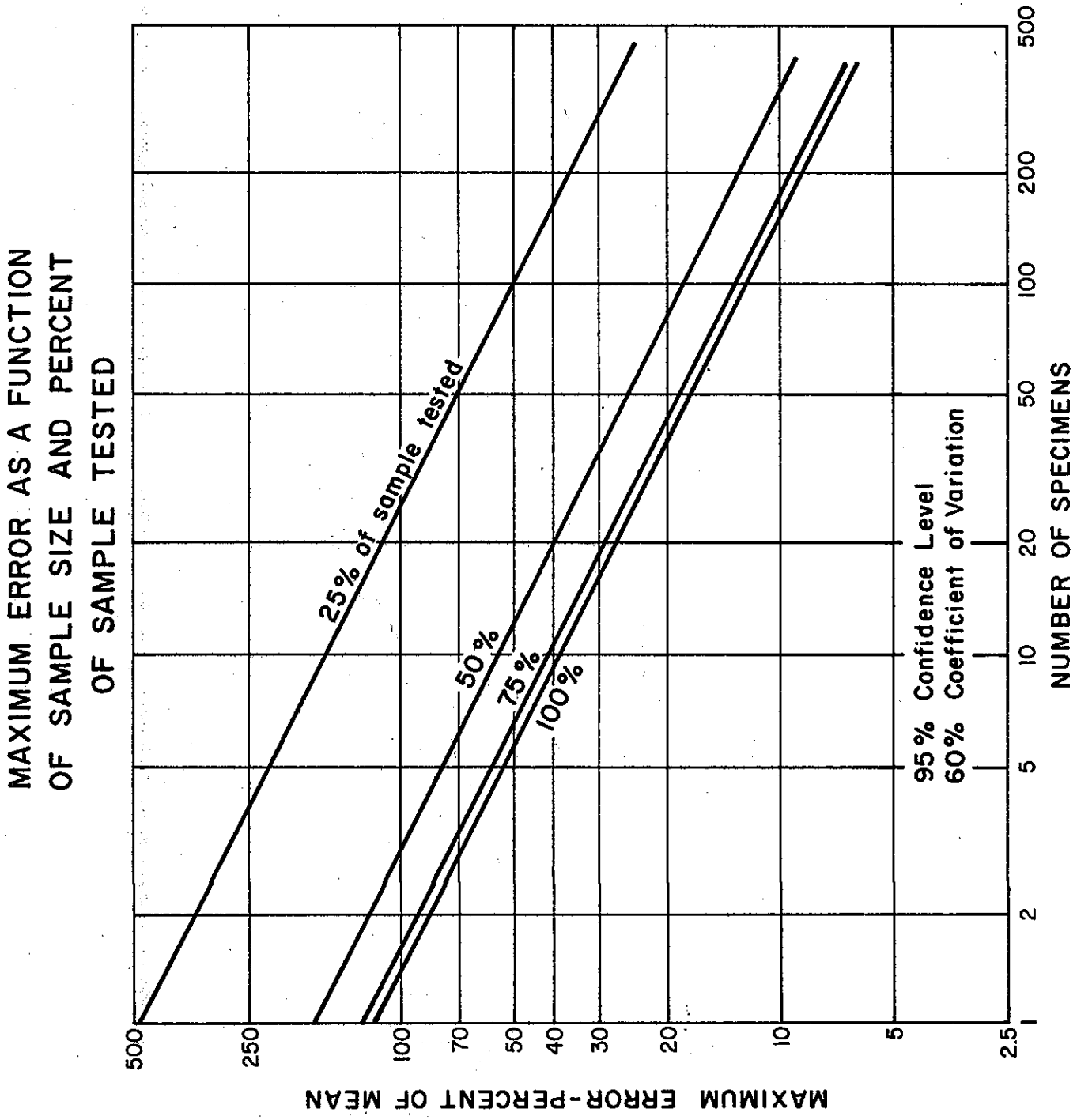

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Figure 3

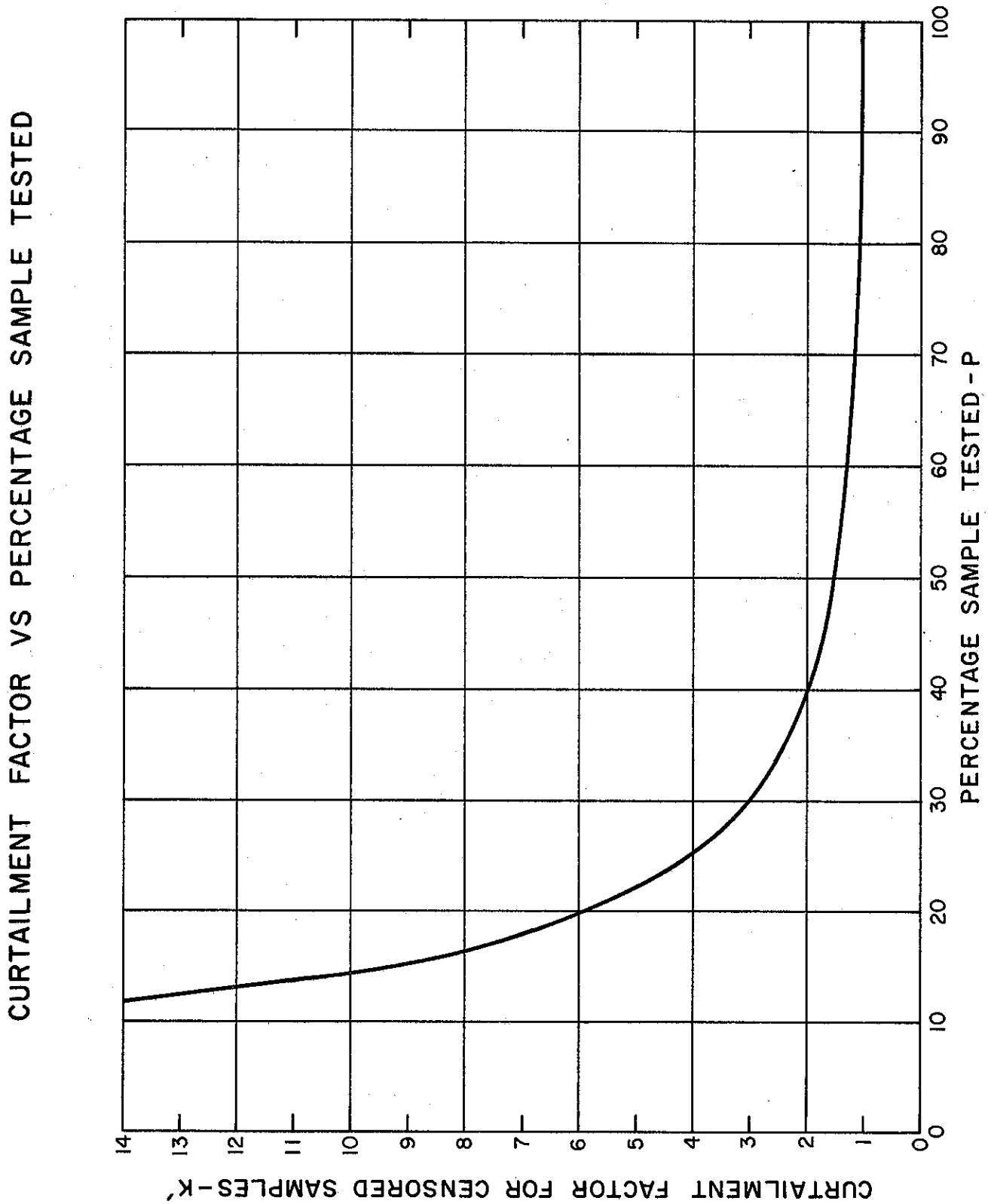

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Figure 4

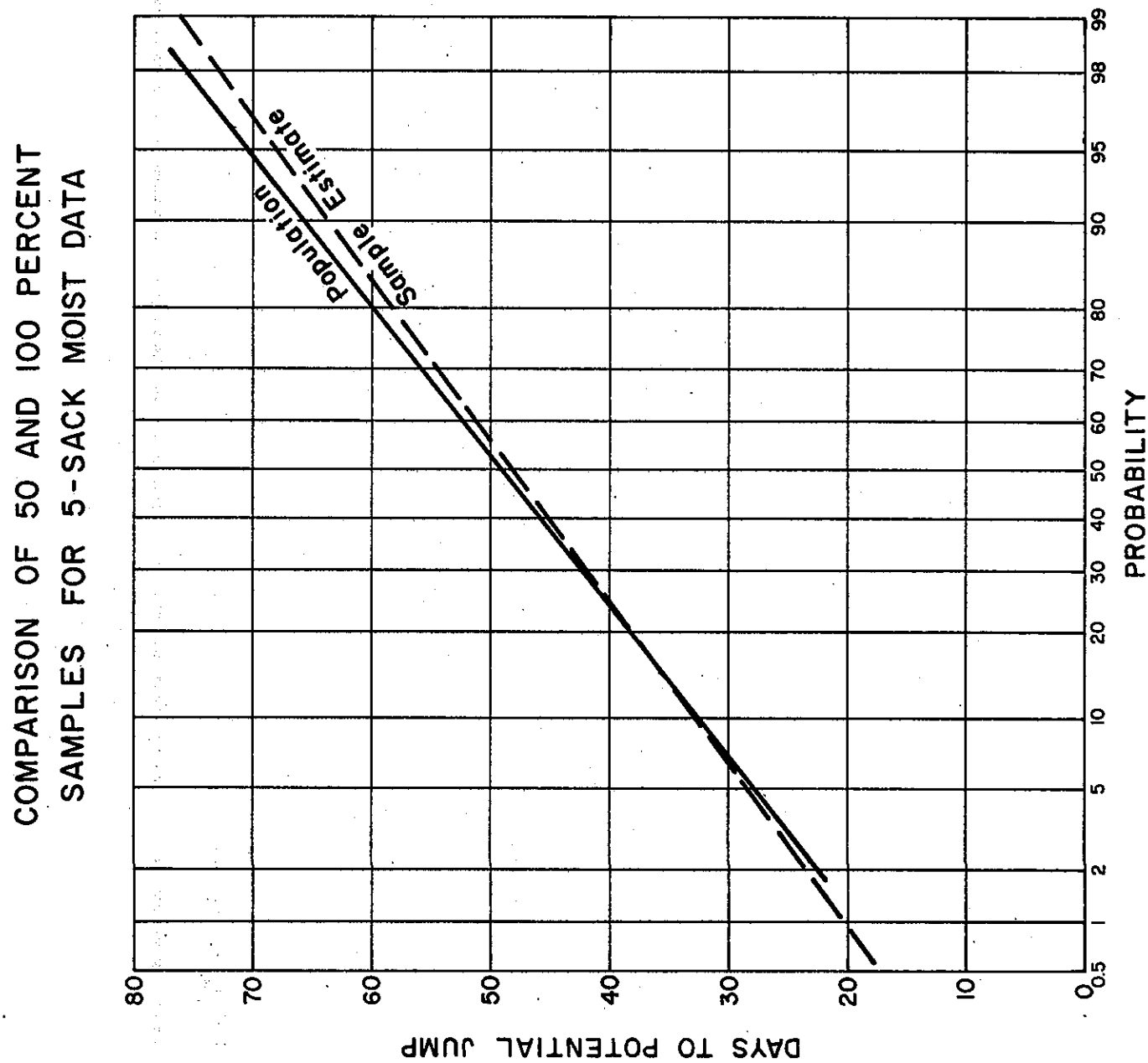

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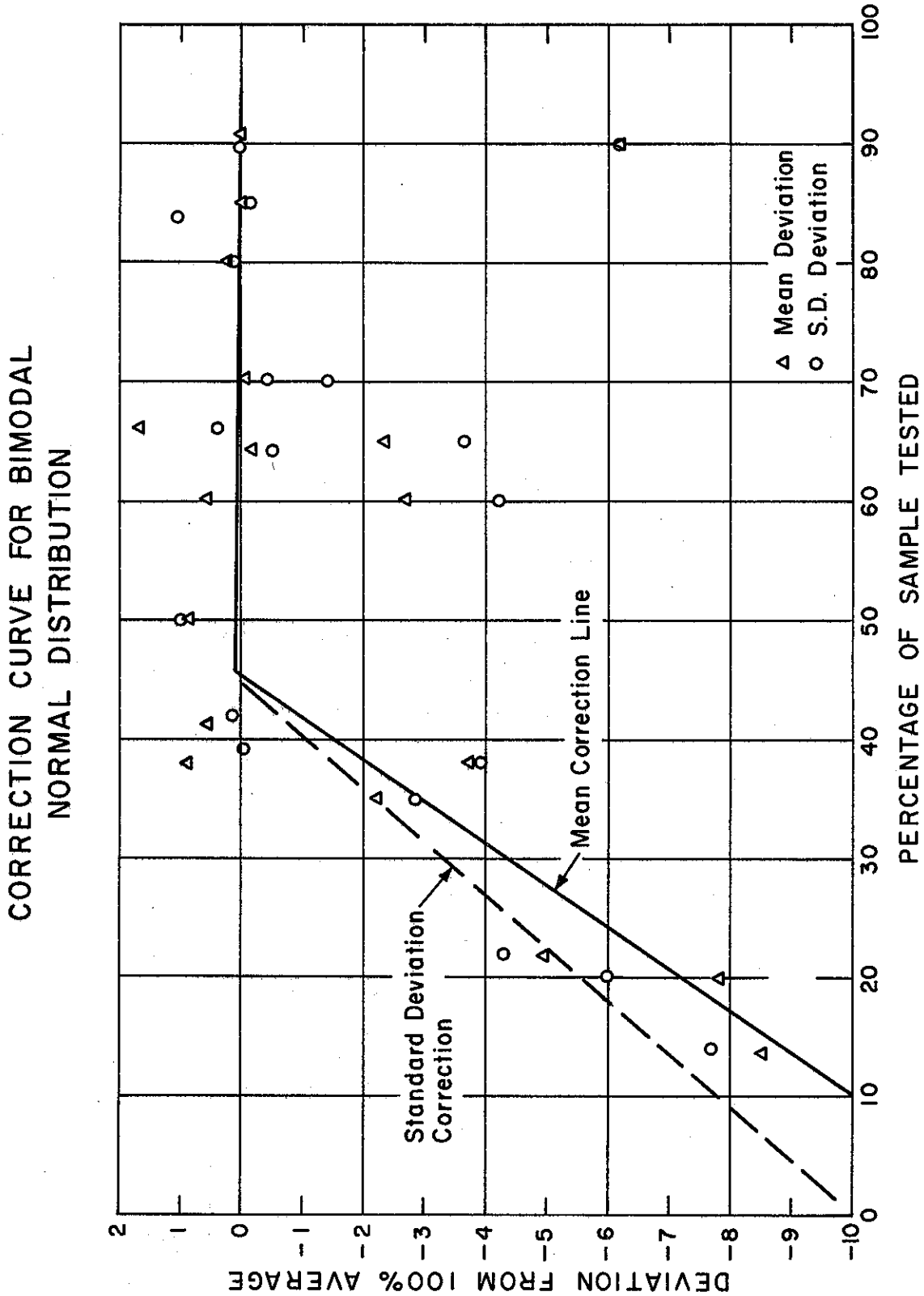
Figure 5



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FREQUENCY DISTRIBUTION OF DAYS TO POTENTIAL JUMP FOR 6-SACK MOIST SAMPLE

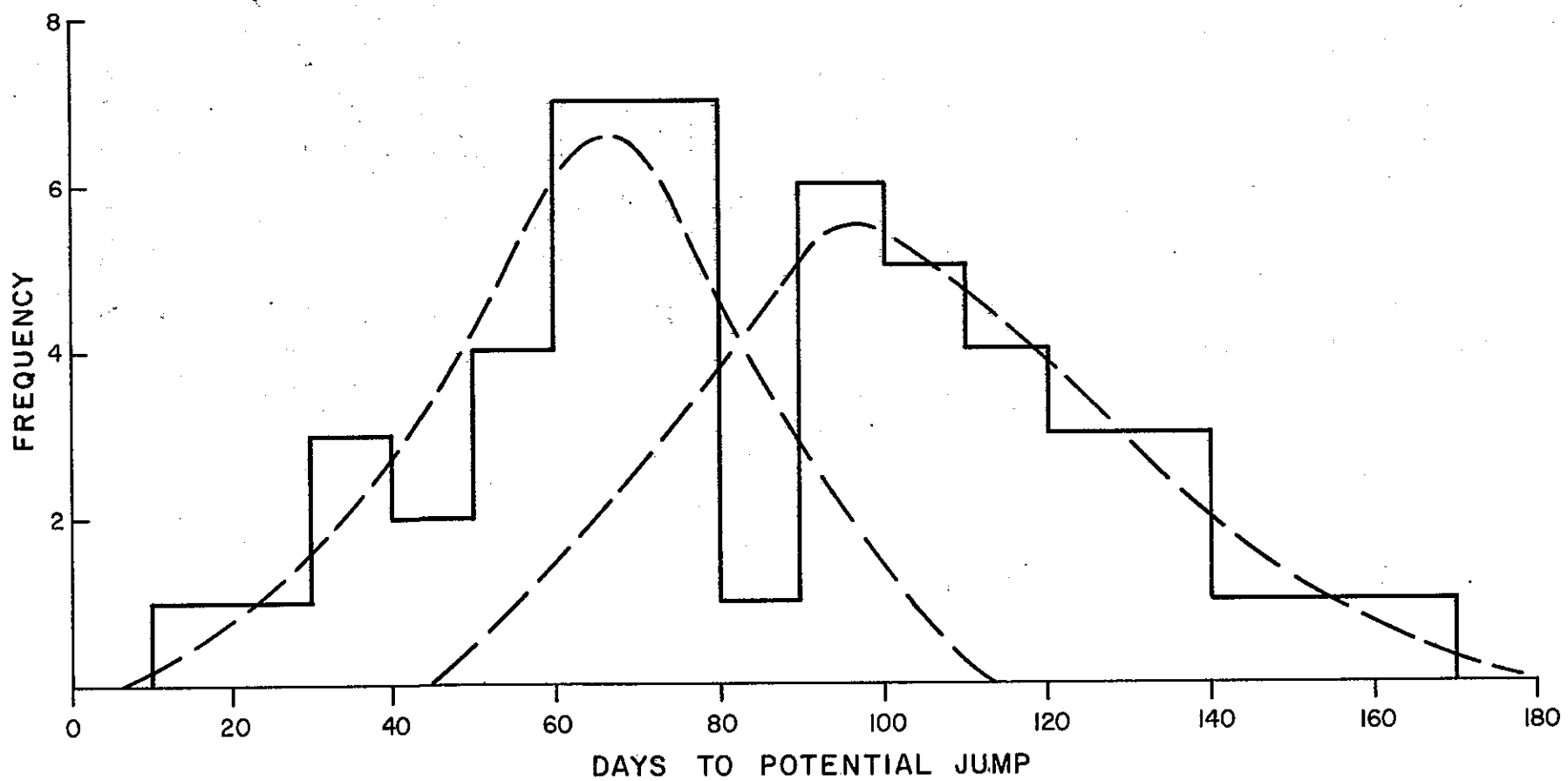

Figure 6

Figure 7

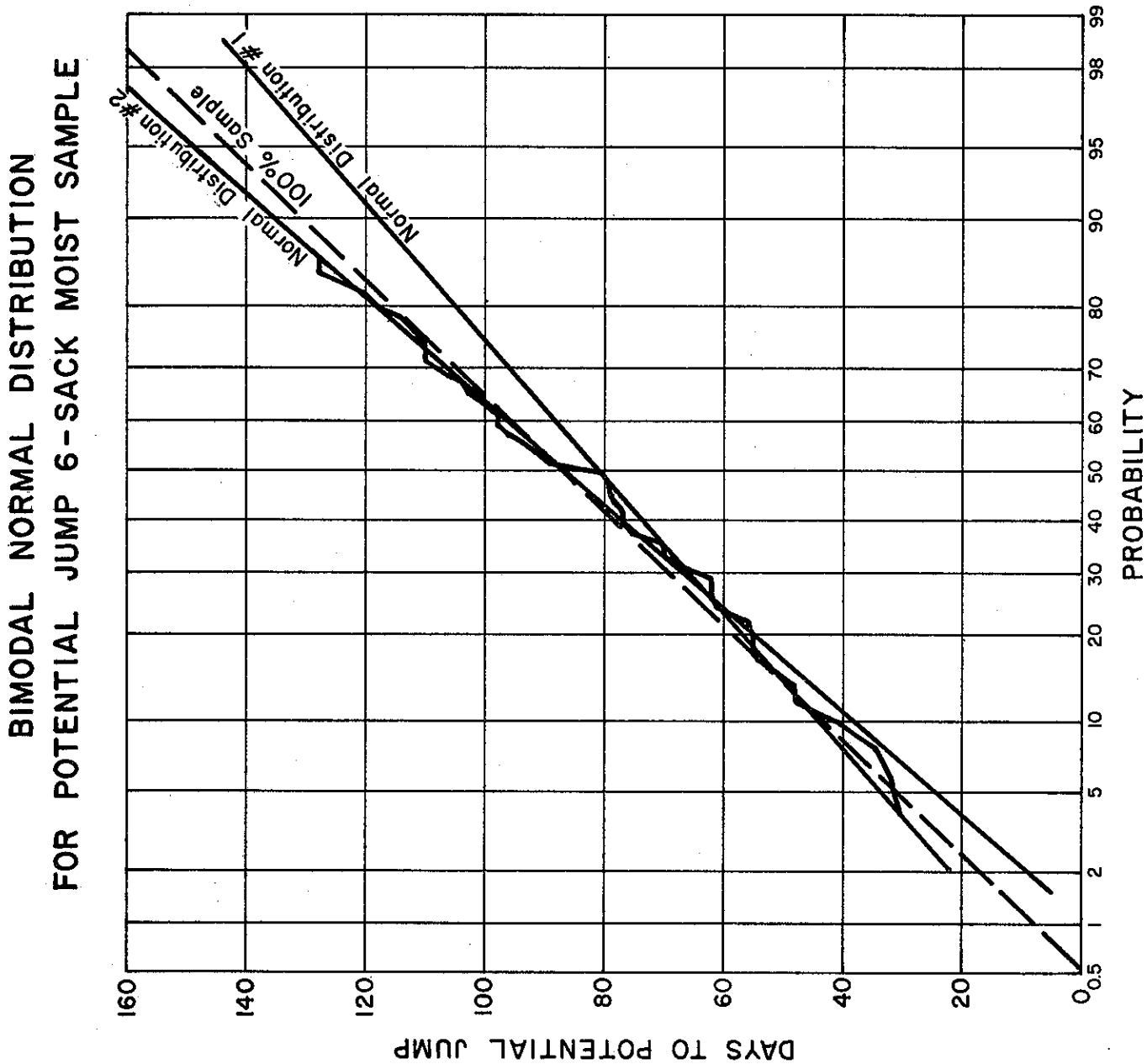

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TABLE I

Values of Z as a Function of ψ and p

$\psi \backslash p$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
.05	0.5449	0.6656	0.7811	0.9056	1.0498	1.2277	1.4646	1.8157	2.4459	4.35890
.10	0.5374	0.6528	0.7616	0.8769	1.0076	1.1645	1.3652	1.6439	2.0854	3.00000
.15	0.5294	0.6395	0.7417	0.8482	0.9665	1.1049	1.2758	1.5010	1.8267	2.38048
.20	0.5210	0.6256	0.7212	0.8193	0.9262	1.0483	1.1944	1.3785	1.6271	2.00000
.25	0.5119	0.6110	0.7002	0.7902	0.8865	0.9941	1.1193	1.2711	1.4653	1.73205
.30	0.5022	0.5957	0.6786	0.7608	0.8473	0.9419	1.0492	1.1752	1.3292	1.52753
.35	0.4918	0.5796	0.6562	0.7310	0.8083	0.8913	0.9832	1.0881	1.2116	1.36277
.40	0.4806	0.5626	0.6329	0.7006	0.7694	0.8419	0.9205	1.0079	1.1076	1.22474
.45	0.4685	0.5445	0.6087	0.6695	0.7303	0.7933	0.8603	0.9330	1.0138	1.05409
.50	0.4552	0.5251	0.5833	0.6375	0.6909	0.7453	0.8020	0.8624	0.9278	1.00000
.55	0.4405	0.5044	0.5565	0.6044	0.6508	0.6973	0.7450	0.7948	0.8476	0.94868
.60	0.4244	0.4819	0.5281	0.5698	0.6097	0.6490	0.6887	0.7295	0.7718	0.81650
.65	0.4063	0.4574	0.4976	0.5335	0.5672	0.6000	0.6326	0.6654	0.6991	0.73380
.70	0.3858	0.4302	0.4647	0.4949	0.5228	0.5496	0.5758	0.6018	0.6280	0.65465
.75	0.3621	0.3998	0.4287	0.4531	0.4757	0.4969	0.5174	0.5375	0.5574	0.57735
.80	0.3342	0.3650	0.3878	0.4072	0.4246	0.4408	0.4562	0.4711	0.4857	0.50000
.85	0.3001	0.3237	0.3408	0.3551	0.3677	0.3793	0.3901	0.4005	0.4104	0.42008
.90	0.2561	0.2722	0.2836	0.2930	0.3011	0.3084	0.3152	0.3215	0.3276	0.33333
.95	0.1921	0.2004	0.2062	0.2107	0.2147	0.2181	0.2212	0.2242	0.2269	0.22942

Reference: Gupta, A. K., "Estimation of the Mean and Standard Deviation of a Normal Population from a Censored Sample", Biometrika, Vol. 39 (1952) pp 260-73

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Table II

Variance Factors for Censored Samples

	$\mu_{11}(k')$	μ_{12}	μ_{22}	ρ	p
-4.0	1.00000	-.000006	.500030	-.000001	100.00
-3.5	1.00001	-.000052	.500208	-.000074	99.98
-3.0	1.00010	-.000335	.501180	-.000473	99.87
-2.5	1.00056	-.001712	.505280	-.002407	99.38
-2.4	1.00078	-.002312	.506935	-.003247	99.18
-2.3	1.00107	-.003099	.509030	-.004341	98.93
-2.2	1.00147	-.004121	.511658	-.005757	98.61
-2.1	1.00200	-.005438	.514926	-.007571	98.21
-2.0	1.00270	-.007123	.518960	-.009875	97.72
-1.9	1.00363	-.009266	.523899	-.012778	97.13
-1.8	1.00485	-.011971	.529899	-.016405	96.41
-1.7	1.00645	-.015368	.537141	-.020901	95.54
-1.6	1.00852	-.019610	.545827	-.026431	94.52
-1.5	1.01120	-.024884	.556187	-.033181	93.31
-1.4	1.01467	-.031410	.568471	-.041358	91.92
-1.3	1.01914	-.039460	.582981	-.051193	90.32
-1.2	1.02488	-.049355	.600046	-.062937	88.49
-1.1	1.03224	-.061491	.620049	-.076861	86.43
-1.0	1.04168	-.076345	.643438	-.093252	84.13
-0.9	1.05376	-.094501	.670724	-.112407	81.59
-0.8	1.06923	-.116674	.702513	-.134620	78.81
-0.7	1.08904	-.143744	.739515	-.160175	75.80
-0.6	1.11442	-.176798	.782574	-.189317	73.57
-0.5	1.14696	-.217183	.832691	-.222233	69.15
-0.5	1.18876	-.266577	.891077	-.259011	65.54
-0.3	1.24252	-.327080	.959181	-.299607	61.79
-0.2	1.31180	-.401326	1.03877	-.343800	57.93
-0.1	1.40127	-.492641	1.13198	-.391156	53.98
0.0	1.51709	-.605233	1.24145	-.441013	50.00
0.1	1.66743	-.744459	1.37042	-.492483	46.02
0.2	1.86310	-.917165	1.52288	-.544498	42.07
0.3	2.11857	-1.13214	1.70381	-.595891	38.21
0.4	2.45318	-1.40071	1.91942	-.645504	34.46
0.5	2.89298	-1.73757	2.17751	-.692299	30.85
0.6	3.47293	-2.16185	2.48793	-.735459	27.43
0.7	4.24075	-2.69858	2.86318	-.774443	24.20
0.8	5.2612	-3.3807	3.3192	-.80899	21.19
0.9	6.6229	-4.2517	3.8765	-.83912	18.41
1.0	8.4477	-5.3696	4.5614	-.86502	15.69
1.1	10.903	-6.8116	5.4082	-.88703	13.57
1.2	14.224	-8.6818	6.4616	-.90557	11.51
1.3	18.735	-11.121	7.7804	-.92109	9.68
1.4	24.892	-14.319	9.4423	-.93401	8.08
1.5	33.339	-18.539	11.550	-.94473	6.68
1.6	44.986	-24.139	14.243	-.95361	5.48

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Table II
Variance Factors for Censored Samples

	μ_{11} (k')	μ_{12}	μ_{22}	φ	ρ
1.7	61.132	-31.616	17.706	-.96097	4.46
1.8	83.638	-41.664	22.193	-.96706	3.59
1.9	115.19	-55.252	28.046	-.97211	2.87
2.0	159.66	-73.750	35.740	-.97630	2.28
2.1	222.74	-99.100	45.930	-.97979	1.79
2.2	312.73	-134.08	59.526	-.98270	1.39
2.3	441.92	-182.68	77.810	-.98514	1.07
2.4	628.58	-250.68	102.59	-.98718	0.82
2.5	899.99	-346.53	136.44	-.98890	0.62

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TABLE III TEST DATA FOR DAYS TO POTENTIAL DROP

5 Sack				6 Sack							
Moist		Steam		Moist				Steam			
I.D.	Days	I.D.	Days	I.D.	Days	I.D.	Days	I.D.	Days	I.D.	Days
7-38	26	4-37	20	1-4	18	13-21	110	13-46	34	8-3	71
1-39	34	4-40	24	13-23	30	14-2	110	6-1	37	8-5	71
5-37	34	8-36	25	1-3	32	3-4	111	8-4	39	6-3	72
3-40	35(35)*	6-40	26	7-4	35	13-24	112	6-2	44	6-5	72
3-36	38	2-40	28	13-13	40	13-16	114	13-35	44	13-47	72
1-38	39	2-38	29	7-2	48(50)	13-2	118	14-8	44	4-1	73
5-39	40	6-36	29(30)	7-3	48	13-19	121	4-3	45	13-27	77
5-38	41	4-39	31	7-1	54	13-9	128	13-40	48	14-6	77
3-39	45	8-40	32	3-1	55	13-10	128	13-48	48	14-7	77
1-36	49(50)	6-37	33	13-15	55	13-18	131	13-32	49	14-9	77
7-36	54	6-38	33	5-4	56	13-7	135	4-4	50(50)	14-10	77
7-37	54	8-39	33	7-5	61	14-1	137	13-49	51	13-26	82
7-40	54	4-38	34	13-20	61	3-5	146	13-33	55	13-29	91
1-40	56	6-39	40(40)	5-1	62	13-8	152	13-36	55	13-30	103
5-36	56	2-37	41	5-3	62	13-6	163	13-43	55	8-1	104
5-40	56	2-36	42	1-5	67			13-44	55		
7-39	57(60)	8-38	43	3-2	70			2-1	56		
3-38	62	4-36	44	13-14	70			13-41	57		
3-37	64	2-39	48	13-22	75(75)			6-4	58(60)		
1-37	71	8-37	54	14-4	77			2-5	61		
				13-5	77			13-28	61		
				1-2	78			13-34	61		
				5-2	79			13-37	61		
				5-5	79			13-38	61		
				3-3	80			4-2	62		
				13-12	89			13-45	63		
				13-25	91			2-4	64		
				13-11	93			4-5	64		
				14-3	96			8-2	64		
				13-17	98			13-50	65		
				13-3	98			13-31	68		
				13-4	100(100)			13-39	68		
				14-5	103			13-42	70(70)		
				1-1	104			2-2	71		
				13-1	107			2-3	71		

* X_k - termination point

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Table III TEST DATA FOR DAYS TO POTENTIAL DROP (cont.)

7 Sack				6 Sack Moist					
Moist		Steam		2-day cure		8-day cure		32-day cure	
I.D.	Days	I.D.	Days	I.D.	Days	I.D.	Days	I.D.	Days
1-45	53	4-43	70	9-22	2	10-41	26	12-12	60
7-44	109	6-45	112	9-32	2	10-31	29	12-43	61
3-14	132	4-45	118	9-34	2	11-11	38	12-44	75
7-43	176	4-44	120	9-2	5	11-1	42	12-25	80 (100)
5-42	177	8-42	124	9-11	5	11-2	44	12-21	101
5-43	177	6-43	124	9-21	5	11-12	44	12-15	103
5-44	177	8-45	125	9-24	5	10-32	47	12-24	103
3-45	199 (200)	4-42	129	9-25	5	10-43	48	12-35	109
5-41	201	6-44	135	9-33	5	11-3	48	12-45	109
5-45	208	2-43	139	9-35	5	11-5	48	12-11	115
3-44	223	6-41	145	9-14	6	11-4	49	12-14	117
7-45	232	2-45	146 (150)	9-5	8	10-35	50 (50)	12-31	117
3-42	251	2-41	158	9-1	9 (10)	11-14	52	12-42	119
3-43	276	8-43	162	9-4	12	11-15	52	12-41	122 (125)
1-41	287	6-42	166	9-13	12	11-13	55	12-13	129
1-42	287 (300)	2-44	174	9-31	19 (20)	10-44	65	12-22	129
7-41	316	4-41	174	9-12	21	10-45	65	12-23	138
7-42	316	8-44	188 (200)	9-23	22	10-34	68 (75)	12-32	140
1-44	317	8-41	207	9-3	23	10-42	76	12-34	143
		2-42	326	9-15	26	10-33	83	12-33	154

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Table IV
Summary of Data Evaluation of Censored Data

Test and Sample Size n	Termination Point X_k	Percent Tested P	Estimated Mean (1) $\hat{\mu}_1$	Estimated S.D.(1) $\hat{\sigma}_1$	Estimated mean(2) $\hat{\mu}$	Estimated S.D.(2) $\hat{\sigma}_2$
5-sk.-moist n=20	35	20	40.33	5.95		
	50	50	49.05	12.93	49.03	12.92
	60	85	48.26	11.81	48.34	11.86
	71	100	48.20	11.94	48.20	11.94
5-sk.-steam n=20	30	35	32.21	5.94	33.76	5.94
	40	70	34.36	8.38	34.35	8.38
	54	100	34.45	8.78	34.45	8.78
6-sk.-moist n=50	50	14	78.79	26.54	78.46	26.45
	75	38	83.56	30.35	84.17	30.57
	100	64	87.11	33.72	86.81	33.57
	163	100	87.28	34.24	87.28	34.24
6-sk.-steam n=50	50	22	58.20	10.64	58.33	10.79
	60	38	64.06	14.93	63.63	14.77
	70	66	64.99	15.34	63.37	14.53
	104	100	63.10	14.95	63.10	14.95
7-sk.-moist n=19	200	42	217.00	73.72	223.00	75.92
	300	84	220.16	74.53	220.48	78.58
	317	100	216.53	73.49	216.53	73.49
7-sk.-steam	150	60	152.73	29.29	142.17	29.28
	200	90	145.95	33.96	146.97	34.86
	326*	100	152.10	51.18*	152.10	51.18*
6-sk-moist 2-day cure n=20	10	65	7.57	4.19	7.62	4.22
	20	80	10.12	8.03	10.13	8.04
	26	100	9.95	7.84	9.95	7.84
6-sk.-moist 8-day cure n=20	50	60	48.66	10.01	48.66	10.01
	75	90	51.57	14.20	51.59	14.22
	83	100	51.45	14.23	51.45	14.23
6-sk.-moist 32-day cure	100	20	166.03	46.99	137.48	55.53
	125	70	112.44	27.59	112.47	27.61
	154	100	111.20	26.11	112.20	26.11

*Due to extreme value of Sample 20 being 119 days longer than Sample 19.

Note: Estimate $\hat{\mu}_1 = \bar{X} + (\sigma^2 - S^2)/d$; $\hat{\sigma}_1 = d/Z$ $Z_2(\psi, p)$ obtained from Table I where $\psi' = S^2/(S^2+d^2)$, $d = (X_k - \bar{X})$

Estimate $\hat{\mu} = \bar{X} + \lambda d$; $\hat{\sigma}_2 = S^2 + \lambda d^2$; (σ, h) is obtained from Table V where $\gamma = S^2/d^2$, $h = 1-P$

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TABLE V

Determination of λ from γ and h.

$\gamma \backslash h$	.01	.02	.03	.04	.05	.06	.07	.08	.09
.00	.010100	.020400	.030902	.041583	.052507	.063627	.074953	.086488	.09824
.05	.010551	.021294	.032292	.043350	.054670	.066189	.077909	.089834	.10197
.10	.010950	.022082	.033398	.044902	.056596	.068483	.080568	.092852	.10534
.15	.011310	.022798	.034466	.046318	.058356	.070586	.083009	.095629	.10845
.20	.011642	.023459	.035453	.047629	.059990	.072539	.085280	.098216	.11135
.25	.011952	.024076	.036377	.048858	.061522	.074372	.087413	.10065	.11408
.30	.012243	.024658	.037249	.050018	.062969	.076106	.089433	.10295	.11667
.35	.012520	.025211	.038077	.051120	.064345	.077756	.091355	.10575	.11914
.40	.012784	.025738	.038866	.052173	.065660	.079332	.093193	.10725	.12150
.45	.013036	.025243	.039624	.053182	.066921	.080845	.094958	.10926	.12377
.50	.013279	.026728	.040352	.054153	.068135	.082301	.096657	.11121	.12595
.55	.013513	.027196	.041054	.055089	.069306	.083708	.098298	.11308	.12806
.60	.013739	.027649	.041733	.055595	.070439	.085068	.099887	.11490	.13011
.65	.013958	.028087	.042391	.056874	.071538	.086388	.10143	.11666	.13209
.70	.014171	.028513	.043030	.057726	.072605	.087670	.10292	.11837	.13402
.75	.014378	.028927	.043652	.058356	.073643	.088917	.10438	.12004	.13590
.80	.014579	.029330	.044258	.059364	.074655	.090133	.10580	.12167	.13773
.85	.014775	.029723	.044848	.060153	.075642	.091319	.10719	.12325	.13952
.90	.014967	.030107	.045425	.060923	.076606	.092477	.10854	.12480	.14126
.95	.015154	.030483	.045989	.061676	.077549	.093611	.10987	.12632	.14297
1.00	.015338	.030850	.046540	.062413	.078471	.094720	.11116	.12780	.14465

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TABLE V (cont.)

δ \ h	.10	.15	.20	.25	.30	.35	.40	.50	.55	.60	.65	.70	.80	.90
.00	.11020	.17342	.24268	.31682	.4021	.4941	.5961	.8368	.9808	1.145	1.336	1.561	2.176	3.283
.05	.11431	.17935	.25033	.32793	.4130	.5066	.6101	.8540	.9994	1.166	1.358	1.585	2.203	3.314
.10	.11804	.18479	.25741	.33662	.4233	.5184	.6234	.8703	1.017	1.185	1.379	1.608	2.229	3.345
.15	.12148	.18985	.26405	.34480	.4330	.5296	.6361	.8860	1.035	1.204	1.400	1.630	2.255	3.376
.20	.12469	.19460	.27031	.35255	.4422	.5403	.6483	.9012	1.051	1.222	1.419	1.651	2.280	3.405
.25	.12722	.19910	.27626	.35993	.4510	.5506	.6600	.9158	1.067	1.240	1.439	1.672	2.305	3.435
.30	.13059	.20338	.28193	.36700	.4595	.5604	.6713	.9300	1.083	1.257	1.457	1.693	2.329	3.464
.35	.13333	.20747	.28737	.37379	.4676	.5699	.6821	.9437	1.098	1.274	1.476	1.713	2.353	3.492
.40	.13595	.21139	.29260	.38033	.4755	.5791	.6927	.9570	1.113	1.290	1.494	1.732	2.376	3.520
.45	.13847	.21517	.29765	.38665	.4831	.5880	.7029	.9700	1.127	1.306	1.511	1.751	2.399	3.547
.50	.14090	.21882	.30253	.39276	.4904	.5967	.7129	.9826	1.141	1.321	1.528	1.770	2.421	3.575
.55	.14325	.22235	.30725	.39870	.4976	.6051	.7225	.9950	1.155	1.337	1.545	1.788	2.443	3.601
.60	.14552	.22578	.31184	.40447	.5045	.6133	.7320	1.007	1.169	1.351	1.561	1.806	2.465	3.628
.65	.14773	.22910	.31630	.41008	.5114	.6213	.7412	1.019	1.182	1.366	1.577	1.824	2.486	3.654
.70	.14987	.23234	.32065	.41555	.5180	.6291	.7502	1.030	1.195	1.380	1.593	1.841	2.507	3.679
.75	.15196	.23550	.32489	.42090	.5425	.6367	.7509	1.042	1.207	1.394	1.608	1.858	2.528	3.705
.80	.15400	.23858	.32903	.42612	.5308	.6441	.7676	1.053	1.220	1.408	1.624	1.875	2.548	3.738
.85	.15599	.24158	.33307	.43122	.5370	.6515	.7761	1.064	1.232	1.422	1.639	1.892	2.568	3.754
.90	.15793	.24452	.33703	.43622	.5430	.6586	.7844	1.074	1.244	1.435	1.653	1.908	2.588	3.779
.95	.15983	.24740	.34091	.44112	.5490	.6656	.7925	1.085	1.255	1.448	1.668	1.924	2.607	3.803
1.00	.16170	.25022	.34471	.44592	.5548	.6724	.8005	1.095	1.267	1.461	1.682	1.940	2.626	3.827

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Table VI

Error Analysis of Potential Jump Test Data

Test	Percent of Sample	$\hat{\mu}$	$\hat{\sigma}$	Coefficient % of Variation	Standard Error of $\hat{\mu}$	Standard Error of $\hat{\sigma}$
					(days)	(days)
5-sk.-moist	20	40.3	6.0	.15	3.5	3.1
	50	49.0	12.9	.26	2.7	2.1
	85	48.3	11.9	.25	2.7	1.9
	100	48.2	11.9	.25		
5-sk.-steam	35	32.2	5.9	.18	2.6	2.1
	70	34.4	8.4	.24	2.0	1.6
	100	34.5	8.8	.25	1.9	1.3
6-sk.-moist	14	78.8	26.5	.33	16.9	11.0
	38	83.6	30.4	.36	6.3	5.6
	64	87.1	33.7	.38	5.2	4.5
	100	87.3	34.2	.39	4.7	3.3
6-sk.-steam	22	58.2	10.6	.18	3.4	2.7
	38	64.1	14.9	.23	3.0	2.7
	66	65.0	15.3	.24	2.2	1.9
	100	63.1	15.0	.24	2.1	1.5
7-sk.-moist	42	217.0	73.7	.33	24.7	22.4
	84	220.2	74.5	.36	19.6	15.7
	100	216.5	73.5	.34	19.0	13.3
7-sk.-steam	60	152.7	29.3	.19	7.5	6.7
	90	146.0	34.0	.23	8.0	6.0
	100	152.1	51.2	.24	8.0	5.6
6-sk.moist 2-day cure	65	7.6	4.2	.55	1.0	0.8
	80	10.1	8.0	.79	1.8	1.4
	100	10.0	7.8	.78	1.8	1.3
6-sk.-moist 8-day cure	60	48.7	10.0	.21	2.6	2.3
	90	51.6	14.2	.28	3.2	2.3
	100	51.4	14.2	.28	3.2	2.2
6-sk.-moist 32-day cure	20	166.0	47.0	.28		
	70	112.4	27.6	.25	6.2	4.4
	100	111.2	26.1	.23	6.2	4.2

Coefficient of Variation = $\hat{\sigma}/\hat{\mu}$

Standard error of $\hat{\mu} = \sqrt{\mu_{11}\ \hat{\sigma}^2/n}$

Standard error of $\hat{\sigma} = \sqrt{\mu_{22}\ \ ^2/n}$

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TABLE VII-1

COEFFICIENTS FOR ESTIMATING THE MEAN AND STANDARD DEVIATION IN CENSORED SAMPLES OF SIZES ≦10 FROM A NORMAL POPULATION

n	n-k	$x_{(1)}$	$x_{(2)}$	$x_{(3)}$	$x_{(4)}$	$x_{(5)}$	$x_{(6)}$	$x_{(7)}$	$x_{(8)}$	$x_{(9)}$	$x_{(10)}$
2	0	.50000000	.50000000								
		-.88622693	.88622693								
3	0	.33333333	.33333333	.33333333							
		-.59081795	.00000000	.59081795							
	1	.00000000	1.00000000								
		-1.18163590	1.18163590								
4	0	.25000000	.25000000	.25000000	.25000000						
		-.45394040	-.11018073	.11018073	.45394040						
	1	.11606577	.24083805	.64309618							
		-.69713303	-.12681665	.82394968							
	2	-.40555159	1.40555159								
		-1.36544125	1.36544125								
5	0	.20000000	.20000000	.20000000	.20000000	.20000000					
		-.37238157	-.13521392	.00000000	.13521392	.37238157					
	1	.12515679	.18304590	.21471643	.47708089						
		-.51173274	-.16678091	.02740065	.65111300						
	2	-.06377484	.14982836	.91394649							
		-.76958387	-.21211572	.98169958							
	3	-.74110683	1.74110683								
		-1.49712813	1.49712813								
6	0	.16666667	.16666667	.16666667	.16666667	.16666667	.16666667				
		-.31752484	-.13855961	-.04321165	.04321165	.13855961	.31752484				
	1	.11828773	.15097353	.16803141	.18280232	.37990501					
		-.40969394	-.16845737	-.04061162	.07395248	.54481044					

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Table VII-2

n	n-k	$x_{(1)}$	$x_{(2)}$	$x_{(3)}$	$x_{(4)}$	$x_{(5)}$	$x_{(6)}$	$x_{(7)}$	$x_{(8)}$	$x_{(9)}$	$x_{(10)}$
6	2	.01848367	.12260668	.17614875	.68276091						
		-.55281996	-.20913743	-.02897078	.79092817						
	3	-.21591800	.06485167	1.15106633							
		-.82435700	-.27604235	1.10039935							
	4	-1.02606712	2.02606712								
		-1.59884545	1.59884545								
7	0	.14285714	.14285714	.14285714	.14285714	.14285714	.14285714	.14285714			
		-.27781036	-.13509780	-.06246312	.00000000	.06246312	.13509780	.27781036			
	1	.10882014	.12954538	.13997050	.14873929	.15705206	.31587262				
		-.34400143	-.16098444	-.06807697	.01143886	.09006788	.47155609				
	2	.04654966	.10721153	.13748095	.16260139	.54615647					
		-.43696302	-.19432593	-.07179355	.03213315	.67094936					
	3	-.07380239	.06771901	.13752310	.86856027						
		-.58481466	-.24284221	-.07174176	.89939863						
	4	-.34744564	-.01345544	1.36090107							
		-.86817366	-.32689877	1.19507242							
	5	-1.27331716	2.27331716								
		-1.68122579	1.68122579								
8	0	.12500000	.12500000	.12500000	.12500000	.12500000	.12500000	.12500000	.12500000		
		-.24758623	-.12944776	-.07130849	-.02295726	.02295726	.07130849	.12944776	.24758623		
	1	.09966946	.11386201	.12079601	.12649213	.13176484	.13698715	.27042840			
		-.29775817	-.15150866	-.07963530	-.02000181	.03635632	.09505131	.41749631			
	2	.05691876	.09621316	.11531993	.13090112	.14512418	.45552284				
		-.36375811	-.17875554	-.08808945	-.01319507	.05698091	.58681726				
	3	-.01672011	.06765099	.10840499	.14131770	.69934643					
		-.45862177	-.21555013	-.09699747	.00022386	.77094550					
	4	-.15491146	.01760383	.10013416	1.03717346						
		-.61096114	-.27072110	-.10611506	.98779730						

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Table VII 3

n	n-k	$x_{(1)}$	$x_{(2)}$	$x_{(3)}$	$x_{(4)}$	$x_{(5)}$	$x_{(6)}$	$x_{(7)}$	$x_{(8)}$	$x_{(9)}$	$x_{(10)}$
8	5	-.46316724	-.08553809	1.54870533							
		-.90454196	-.36895282	1.27349478							
	6	-1.49153218	2.49153218								
		-1.75016272	1.75016272								
9	0	.11111111	.11111111	.11111111	.11111111	.11111111	.11111111	.11111111	.11111111	.11111111	
		-.22373410	-.12326850	-.07509922	-.03596908	.00000000	.03596908	.07509922	.12326850	.22373410	
	1	.09148453	.10175129	.10665020	.11060016	.11419180	.11765844	.12118875	.23647484		
		-.26325227	-.14211324	-.08407936	-.03699545	.00618347	.04915546	.09539434	.37570705		
	2	.06023974	.08756442	.10056358	.11099937	.12038626	.12937290	.39087375			
		-.31289447	-.16465459	-.09375143	-.03636313	.01604052	.06776492	.52385821			
	3	.01040388	.06597348	.09230224	.11331998	.13204377	.58595666				
		-.37968567	-.19359128	-.10482346	-.03325300	.03166420	.67968924				
	4	-.07313367	.03155502	.08087391	.11994592	.84075879					
		-.47658635	-.23351551	-.11807995	-.02556713	.85374893					
	5	-.22717960	-.02842070	.06443680	1.19116347						
		-.63301232	-.29441786	-.13477100	1.06220121						
	6	-.56642662	-.15208218	1.71850879							
		-.93553052	-.40469106	1.34022157							
	7	-1.68675768	2.68675768								
		-1.80924841	1.80924841								
10	0	.10000000	.10000000	.10000000	.10000000	.10000000	.10000000	.10000000	.10000000	.10000000	.10000000
		-.20438349	-.11719379	-.07625921	-.04358325	-.01421667	.01421667	.04358325	.07625921	.11719379	.20438349
	1	.08432557	.09206394	.09568489	.09856130	.10112450	.10356728	.10600993	.10852533	.21013724	
		-.23641943	-.13341377	-.08507858	-.04652372	-.01191838	.02150761	.05586656	.09368358	.34229613	
	2	.06045239	.08044709	.08978431	.09718132	.10374300	.10994657	.11065505	.34239029		
		-.27530689	-.15233666	-.09469015	-.04877160	-.00765306	.01389894	.07222921	.47463020		

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SUBJECT: ANALYSIS OF CENSORED DATA (SMALL CENSORED SAMPLE FROM NORMAL DISTRIBUTION)

RM

PAGE OF

Table VII-4

n	n-k	$x_{(1)}$	$x_{(2)}$	$x_{(3)}$	$x_{(4)}$	$x_{(5)}$	$x_{(6)}$	$x_{(7)}$	$x_{(8)}$	$x_{(9)}$	$x_{(10)}$
10	3	.02442821	.06355637	.08178041	.09616708	.10885704	.12073778	.50447311			
		-.32524451	-.17575100	-.10578536	-.05017756	-.00056384	.04685799	.61066429			
	4	-.03158434	.03829283	.07072145	.09620404	.11851783	.70784819				
		-.39304766	-.20633250	-.11917222	-.05013282	.01113054	.75755466				
	5	-.12396225	-.00163103	.05485447	.09896448	.97177434					
		-.49191253	-.24905990	-.13615341	-.04717854	.92430437					
	6	-.29230821	-.07092971	.03053696	1.33270096						
		-.65203499	-.31497342	-.15928304	1.12629145						
	7	-.65962405	-.21376659	1.87339063							
		-.96246073	-.43568764	1.39814837							
	8	-1.86335148	2.86335148								
		-1.86082625	1.86082625								

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TABLE VIII

Variance and Co-variance of Censored Data Estimates

P	$\sigma 11$	$\sigma 12$	$\sigma 22$
.05	51.57600	27.22429	15.68796
.10	17.79459	10.62002	7.51418
.15	9.25870	5.85259	4.84903
.20	5.78039	3.71733	3.53748
.25	4.02381	2.54931	2.76048
.30	3.01994	1.83219	2.24800
.35	2.39819	1.35740	1.88531
.40	1.99085	1.02593	1.61549
.45	1.71281	0.78527	1.40712
.50	1.51709	0.60523	1.24145
.55	1.37607	0.46736	1.10662
.60	1.27266	0.35982	0.99476
.65	1.19583	0.27471	0.90043
.70	1.13826	0.20657	0.81975
.75	1.09492	0.15156	0.74989
.80	1.06232	0.10690	0.68869
.85	1.03797	0.07058	0.63449
.90	1.02008	0.04113	0.58592
.95	1.00752	0.01759	0.54174
.96	1.00559	0.01355	0.05330
.97	1.00384	0.00974	0.52496
.98	1.00230	0.00617	0.51669
.99	1.00099	0.00287	0.50843

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TABLE IX

Computation of Censored Sampling Procedure to Jump Potential Data

	X_k	$\overline{X}$	$d=X_k-\overline{X}$	d^2	S^2	$\gamma=S^2/d^2$	$\psi=\frac{S^2}{S^2+d^2}$	$p=k/n$	$h=\frac{n-k}{n}$	$\lambda=f(\sigma_1', h)$	$z=f(\psi, p)$	$\hat{\mu}=\overline{X}+\lambda d$	$\hat{\mu}=\overline{X}+\frac{\sigma^2-S^2}{d}$	$\hat{\sigma}=S^2+\lambda\hat{\mu}^2$	$\hat{\sigma}=\frac{d}{z}$
5-sk.moist n=20	35	32.25	2.75	7.56	13.19	1.745	.636	.20	.80		.4621		40.33		5.95
	50	38.10	11.90	141.61	36.89	.261	.207	.50	.50	.9189	.9206	49.03	49.05	12.92	12.93
	60	45.18	14.82	219.63	93.79	.427	.299	.85	.15	.2134	1.2545	48.34	48.26	11.86	11.81
5-sk.-steam n=20	30	25.86	4.14	17.14	8.98	.524	.344	.35	.65	1.536	.6967	33.76	32.21	5.94	5.94
	40	29.79	10.21	104.24	23.60	.226	.185	.70	.30	.4468	1.2188	34.35	34.36	8.38	8.38
6-sk.moist n=50	50	35.86	14.14	199.94	97.27	.486	.327	.14	.86	3.013	.5328	78.46	78.79	26.45	26.54
	75	52.58	22.42	502.66	226.35	.450	.310	.38	.62	1.409	.7386	84.17	83.56	30.57	30.35
	100	66.69	33.31	1109.56	456.69	.412	.292	.64	.36	.6040	.9879	86.81	87.11	33.57	33.72
6-sk.steam n=50	50	43.82	6.18	38.19	24.33	.637	.389	.22	.78	2.349	.5806	58.33	58.20	10.79	10.64
	60	48.63	11.37	129.28	47.50	.367	.269	.38	.62	1.319	.7616	63.63	64.06	14.77	15.93
	70	55.06	14.94	223.20	87.03	.390	.281	.66	.34	.5564	.9739	63.37	64.99	14.53	15.34
7-sk.moist n=19	200	150.00	50.00	2500.00	2084.75	.834	.455	.42	.58	1.472	.6782	223.00	217.00	75.92	73.72
	300	197.81	102.19	10442.8	3858.38	.369	.270	.84	.16	.2219	1.3040	220.48	220.16	78.58	74.53
7-sk.steam n=20	150	123.92	26.08	680.17	367.34	.540	.351	.60	.40	.7206	.8903	142.17	152.73	29.28	29.29
	200	139.39	60.61	3673.57	755.93	.206	.171	.90	.10	.1251	1.7848	146.97	145.95	34.86	33.96
6-sk.moist 2-day cure	10	4.92	5.08	25.81	4.07	.158	.136	.65	.35	.5313	1.2113	7.62	7.57	4.22	4.19
	20	6.69	13.31	177.16	18.84	.106	.096	.80	.20	.2582	1.6576	10.13	10.12	8.04	8.03
6-sk.moist 8-day cure	50	42.75	7.25	52.56	57.35	1.091	.522	.60	.40	.8145	.7242	48.66	48.66	10.01	10.01
	75	48.33	26.67	711.29	115.33	.162	.140	.90	.10	.1223	1.8784	51.59	51.57	14.22	14.20
6-sk.moist 32-day cure	100	69.00	31.00	961.00	75.50	.078	.0724	.20	.80	2.209	.6597	137.48	166.03	55.53	46.99
	125	99.36	25.64	657.41	426.37	.649	.393	.70	.30	.5113	.9293	112.47	112.44	27.61	27.59

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