



UNIT
2

BASIC SURVEY MATH

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Introduction

The purpose of this video unit is to present basic math concepts and principles useful to survey computations. It has been assumed that most viewers are already familiar with some or most of the topics presented in the beginning of the unit. It is important to have a developed understanding of the basic operations of arithmetic, algebra, geometry, and trigonometry. This unit is not designed as a complete math course, but rather as an overview and guide to computation processes unique to surveying and mapping. Surveyors who need to work on math operations and fundamental skills addressed in the video will find sources for further study in the reference section at the end of this unit.

Survey mathematics generally consists of applications of formulas and equations that have been adapted to work toward the specific needs of the surveyor such as:

- Units of measurement and conversions
- Check and adjustment of raw field data
- Closure and adjustment of survey figures
- Calculations for missing elements of a figure
- Working with coordinates (COGO)
- Intersections of straight lines and of circles

It is hoped this video unit will help viewers to recognize solution formats for problems and then make correct and effective use of appropriate methods to solve these particular survey problems.

Performance Expected on the Exams

Recognize solution formats, and make correct and effective use of appropriate mathematical solutions to particular survey applications.

Key Terms

Absolute value	Adjacent side
Algebra	Arc
Arithmetic	Azimuth
Bearing	Central angle
Chord	Circular curve
Circumference	Complementary angle
Coordinate conversion	Cosecant
Cosine	Cotangent
Cubes	Decimal system
delta x, delta x	Departure
External distance	Geodetic north
Grads	Grid north

Hexagon	Horizontal curve
Hypotenuse	Intersections
Intersection of straight line and arc	Intersections of straight lines
Inverse processes	Latitude
Law of cosines	Law of sines
Length of arc	Magnetic north
Meter	Mid-ordinate distance
Most probable value	Oblique triangle
Opposite side	Order of operations
Parabola	Parallelogram
Pentagon	Percent of slope
Percentage	pi
Plane geometry	Polar coordinates
Polygon	Pythagorean theorem
Quadrants	Quadratic equation
Quadrilateral	Radian
Radius	Radius point
Random error	Rate of change
Rectangular coordinates	Residual
Rhomboid	Right triangle
Roots	Rounding off
Sag curve	Secant
Sector of a circle	Segment of a circle
Sexagesimal system	Signed numbers
Significant figures	Simultaneous equation
Sine	Square root
Squares	Standard error
Supplementary angles	US survey foot
Tangent	Trigonometry

Video Presentation Outline

Arithmetic

- Decimal system
- Rounding off and significant figures
- Percentage
- Squares, cubes and roots

Conversion of Units of Measure

- Converting lineal units
- Converting angular units
- Converting units of area

Random Error Analysis

- Error definitions
- Error residuals
- Statistical error matrix
- Propagation of error
- Error in summation
- Error in product
- Error in series

Algebra

- Signed numbers
- Equations
- Order of operations
- Parentheses
- Evaluating equations and combining terms
- Solving equations
- The quadratic equation formula

Plane Geometry

- Angles
- Geometrical theorems
- Geometrical figures
- Polygons
- Triangles

Trigonometry

- Right triangles
- Pythagorean theorem
- Trigonometric functions
- Oblique triangles
- Directions: bearings and azimuths
- Latitudes and departures
- Plane coordinates

Coordinate Geometry

- Intersection of straight lines
- Intersection of straight line and arc
- Intersection of two arcs

Sample Test Questions

1. The product of 416.78 multiplied by 210.98 is?
 - A. 879.32
 - B. 8,793.32
 - C. 87,932.24
 - D. 879,322.44
2. The quotient of 36.11 divided by 191.67 is?
 - A. 188.40
 - B. 18.84
 - C. 1.88
 - D. 0.19
3. Square the number 0.713729, showing the results to the nearest five decimal places.
 - A. 0.50941
 - B. 0.50940
 - C. 0.50942
 - D. 0.50943
4. The percentage of slope for a proposed ramp is +3.55%. What is the change in elevation of this ramp for a horizontal length of 356 ft?
 - A. -126.38 ft
 - B. +12.60 ft
 - C. +12.64 ft
 - D. +126.38 ft
5. Where the centerline slope of a highway has a vertical drop of 14.75 ft in 265 ft horizontally, what is the rate of change expressed in percentage?
 - A. 0.55%
 - B. 0.56%
 - C. 5.55%
 - D. 5.57%
6. Determine the square root of 0.6935, showing the result to the nearest five decimal places.
 - A. 0.832776
 - B. 0.83276
 - C. 0.83277
 - D. 0.832766

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7. 24.91 expressed in ft and in, equals:
- A. 24 ft, 10-7/8 in
 - B. 24 ft, 10-3/8 in
 - C. 24 ft, 10-1/4 in
 - D. 24 ft, 11 in
8. 4,178.309 meters equals _____ United States survey ft.
- A. 1,273.56 survey ft
 - B. 1,273.55 survey ft
 - C. 13,708.20 survey ft
 - D. 13,708.34 survey ft
9. 6,172.98 United States survey ft equals _____ meters.
- A. 1,881.528 m
 - B. 1,881.547 m
 - C. 20,252.313 m
 - D. 20,252.519 m
10. When converted to survey ft, 3,421.381 meters equals _____ survey ft.
- A. 1,042.84 survey ft
 - B. 1,042.85 survey ft
 - C. 11,224.87 survey ft
 - D. 11,224.98 survey ft
11. 21.56 chains converts to _____ survey ft.
- A. 1,422.36 survey ft
 - B. 1,386.00 survey ft
 - C. 1293.60 survey ft
 - D. 1,422.96 survey ft
12. $14^{\circ} 34' 37''$ converted to radian measurement is _____?
- A. 0.250345 rad
 - B. 0.254416 rad
 - C. 0.250351 rad
 - D. 0.250337 rad
13. 0.758612 rad, when converted to degrees, minutes, and seconds is _____.
- A. $43^{\circ} 46' 52''$
 - B. $43^{\circ} 41' 35''$
 - C. $43^{\circ} 33' 12''$
 - D. $43^{\circ} 27' 55''$
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14. How many hectares are contained in a rectangular parcel that measures 19.23 ch. x 40.63 ch.?
- A. 78.131 hec
 - B. 781.315 hec
 - C. 31.619 hec
 - D. 193.063 hec
15. An angle has been measured six individual times with the following results: a.) $46^{\circ} 21' 45''$; b.) $46^{\circ} 22' 10''$; c.) $46^{\circ} 22' 05''$; d.) $46^{\circ} 22' 00''$; e.) $46^{\circ} 21' 45''$; f.) $46^{\circ} 21' 55''$. What is the most probable value of the angle?
- A. $46^{\circ} 21' 45''$
 - B. $46^{\circ} 21' 50''$
 - C. $46^{\circ} 21' 57''$
 - D. $46^{\circ} 22' 00''$
16. Determine the standard error for the following group of six measurements: a.) 11,249.71 ft; b.) 11,250.06 ft; c.) 11,249.86 ft; d.) 11,249.99 ft; e.) 11,250.01 ft; f.) 11,249.98 ft.
- A. ± 0.13 ft
 - B. ± 0.12 ft
 - C. ± 0.10 ft
 - D. ± 0.08 ft
17. Determine the standard error of the mean for the measurement set in problem #16.
- A. ± 0.21 ft
 - B. ± 0.13 ft
 - C. ± 0.05 ft
 - D. ± 0.03 ft
18. A rectangular parcel of land was surveyed. The measurement for side X was 339.21 ft with an error of ± 0.05 ft. Side Y was measured as 563.67 ft, with an error of ± 0.09 ft. What is the area of the parcel and what is the expected error in the area?
- A. Area = 191,202 ft² or 4.389 ac.; standard error = ± 41.7 ft²
 - B. Area = 191,202 ft² or 4.389 ac.; standard error = ± 41.5 ft²
 - C. Area = 191,202 ft² or 4.389 ac.; standard error = ± 24.1 ft²
 - D. Area = 191,202 ft² or 4.389 ac.; standard error = ± 53.5 ft²

19. The total length for a highway centerline was measured in four different segments using different equipment and different methods of measurement on different days. The total length of the line was found by totaling the length of each segment. Standard error for each segment was determined to be:

Standard Error of Segment #1 = ± 0.04 ft

Standard Error of Segment #2 = ± 0.03 ft

Standard Error of Segment #3 = ± 0.08 ft

Standard Error of Segment #4 = ± 0.11 ft

The standard error of the total distance of the centerline is _____?

- A. Standard error of the sum = ± 0.14 ft
B. Standard error of the sum = ± 0.26 ft
C. Standard error of the sum = ± 0.02 ft
D. Standard error of the sum = ± 0.07 ft
20. What is the sum of the following five numbers: (-230.67); (+517.39); (+100.26); (-311.47); and (-481.28)?
- A. 405.77
B. 1,641.07
C. -1,641.07
D. -405.77

21. The remainder after -146.11 has been subtracted from -37.82 is _____?

- A. -108.29
B. 108.29
C. -183.93
D. 183.93

22. Write an equation based on the following word statement: "three times a number, plus the number cubed, minus the number multiplied by 87." In the algebraic equation, let b stand for the number referred to in the problem statement.

- A. $3(b + b^3) - (87b)$
B. $3(b + b^3) - 87b$
C. $3b + b^3 - 87b$
D. $(3b) + b^3 - (87b)$

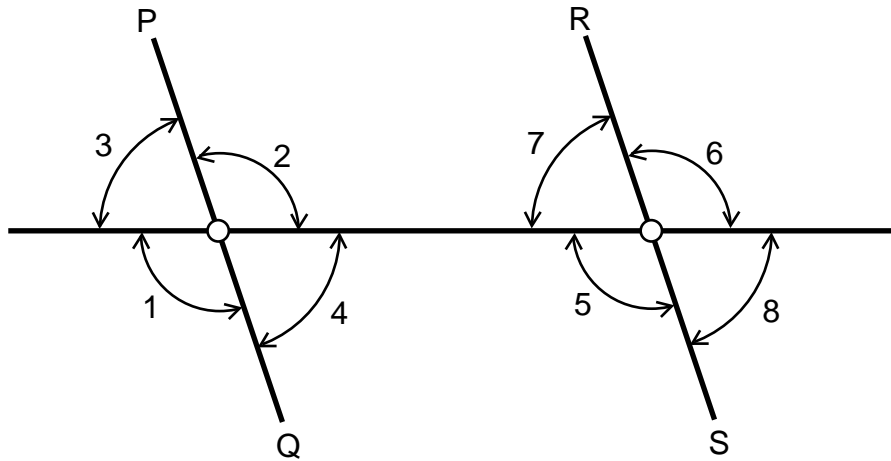
23. Letting $w = 12$ and $z = 3$, evaluate the following equation:

$$5w + (21 - w) 14z + (z - 23).$$

- A. 418
B. 2,878
C. 2,881
D. 19,162

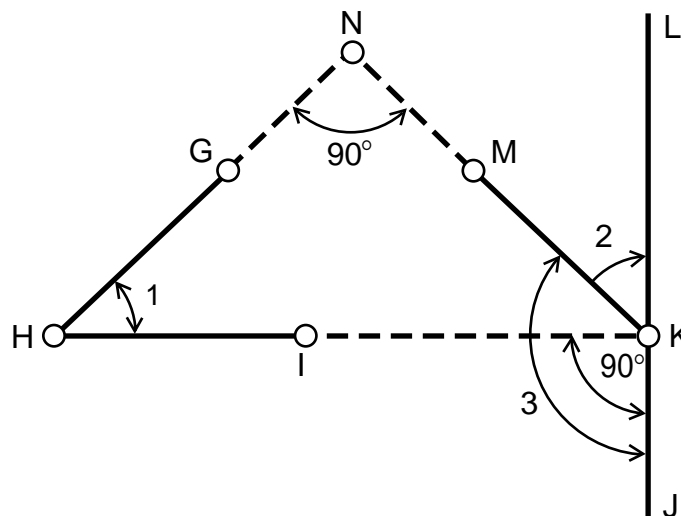
24. If angle 3 in the sketch below is $71^\circ 39' 12''$, calculate the values of angles 1, 2, 5, and 8. Assume lines P-Q and R-S are parallel.

- A. $\angle 1 = 108^\circ 20' 48''$; $\angle 2 = 108^\circ 20' 48''$; $\angle 5 = 108^\circ 20' 48''$; $\angle 8 = 71^\circ 39' 12''$
- B. $\angle 1 = 108^\circ 20' 48''$; $\angle 2 = 71^\circ 39' 12''$; $\angle 5 = 71^\circ 39' 12''$; $\angle 8 = 108^\circ 20' 48''$
- C. $\angle 1 = 108^\circ 20' 48''$; $\angle 2 = 71^\circ 39' 12''$; $\angle 5 = 108^\circ 20' 48''$; $\angle 8 = 71^\circ 39' 12''$
- D. $\angle 1 = 108^\circ 20' 48''$; $\angle 2 = 108^\circ 20' 48''$; $\angle 5 = 71^\circ 39' 12''$; $\angle 8 = 71^\circ 39' 12''$



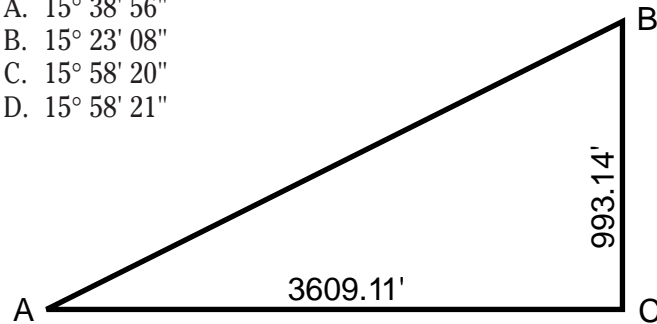
25. If angle 1 in the sketch below is $46^\circ 11' 20''$, calculate the values of angles 2 and 3.

- A. $\angle 2 = 46^\circ 11' 20''$; $\angle 3 = 133^\circ 48' 40''$
- B. $\angle 2 = 43^\circ 48' 40''$; $\angle 3 = 133^\circ 48' 40''$
- C. $\angle 2 = 46^\circ 11' 20''$; $\angle 3 = 136^\circ 11' 20''$
- D. $\angle 2 = 43^\circ 48' 40''$; $\angle 3 = 136^\circ 11' 20''$



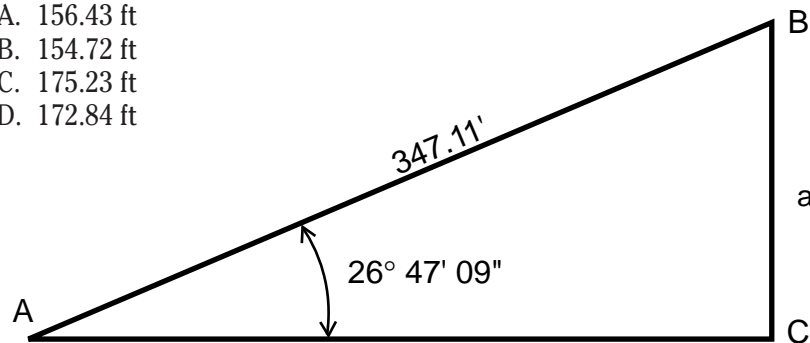
26. Solve for angle A in the triangle shown below:

- A. $15^{\circ} 38' 56''$
- B. $15^{\circ} 23' 08''$
- C. $15^{\circ} 58' 20''$
- D. $15^{\circ} 58' 21''$



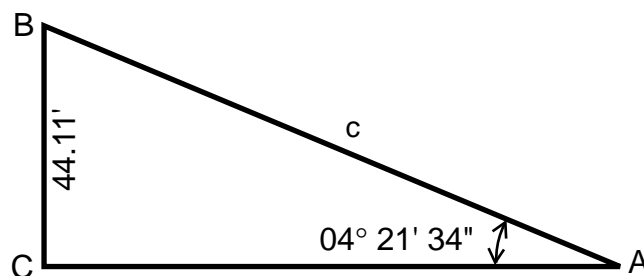
27. Solve for the missing side "a" of the triangle in the sketch below.

- A. 156.43 ft
- B. 154.72 ft
- C. 175.23 ft
- D. 172.84 ft



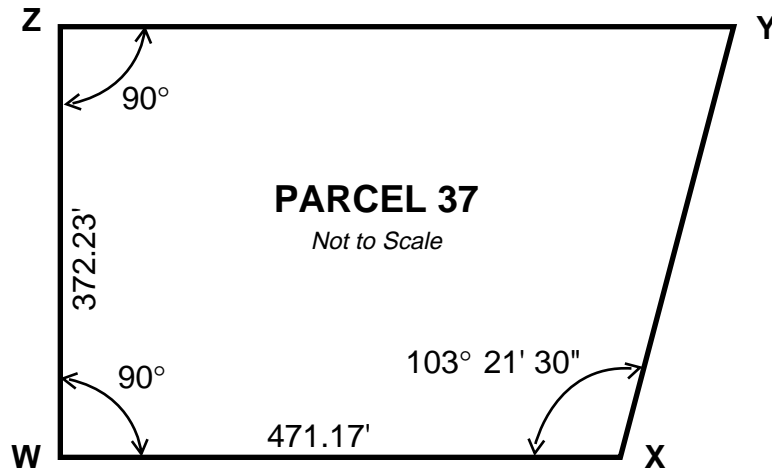
28. What is length of side "c" in the triangle shown in the sketch below?

- A. 578.61 ft
- B. 598.75 ft
- C. 600.36 ft
- D. 580.29 ft



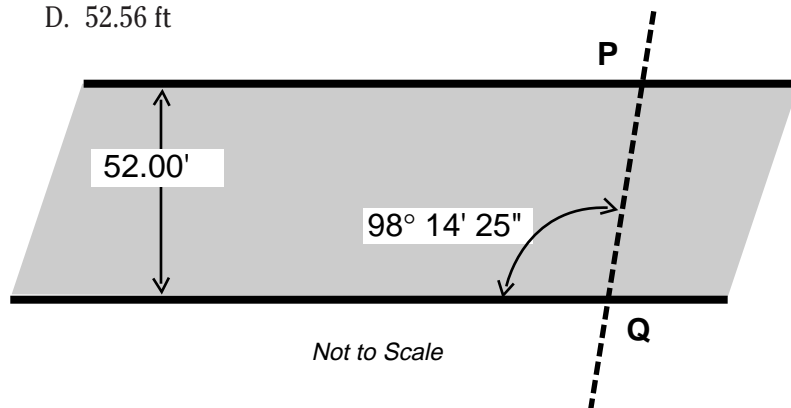
29. What is the length of side Y to Z in the sketch of parcel #37 shown below?

- A. 557.17 ft
- B. 559.56 ft
- C. 558.56 ft
- D. 556.25 ft



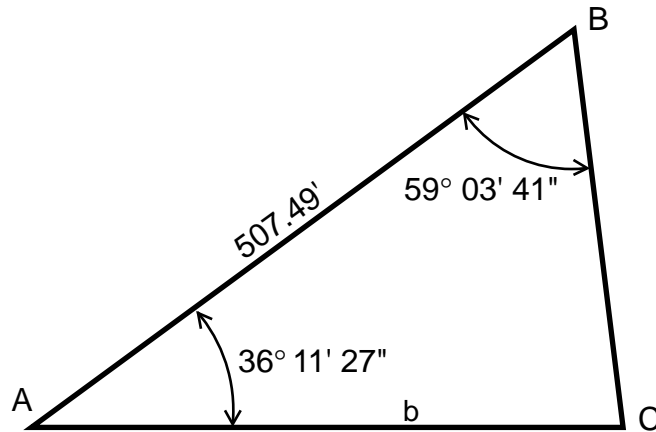
30. Line P to Q intersects the street alignment as shown in the sketch below. What is the length of line P-Q?

- A. 52.50 ft
- B. 52.52 ft
- C. 52.54 ft
- D. 52.56 ft



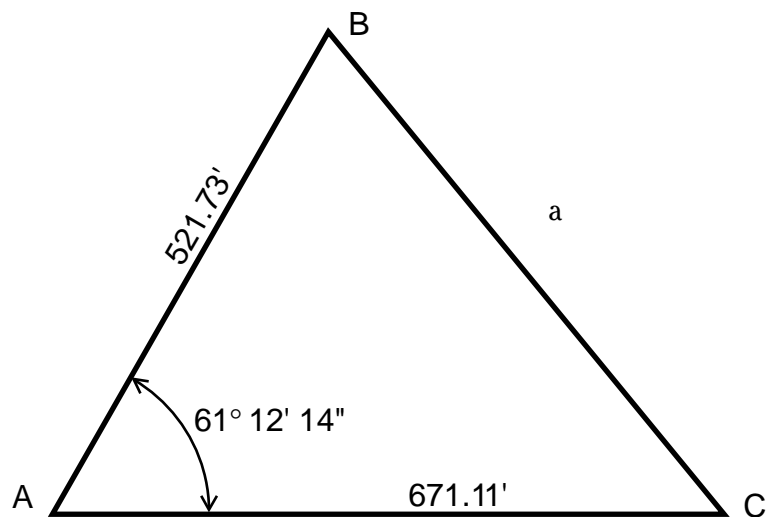
31. Given the following dimensions shown for the oblique triangle in the sketch below, solve for the length of side "b."

- A. 855.58 ft
- B. 857.02 ft
- C. 438.34 ft
- D. 37.12 ft



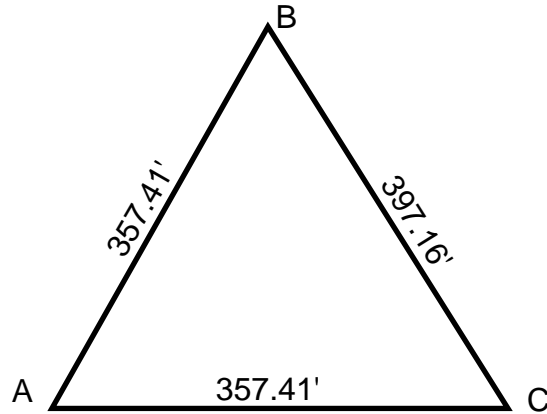
32. Solve for side "a" using the elements given for the oblique triangle shown in the sketch below.

- A. 751.27 ft
- B. 618.43 ft
- C. 744.27 ft
- D. 620.70 ft



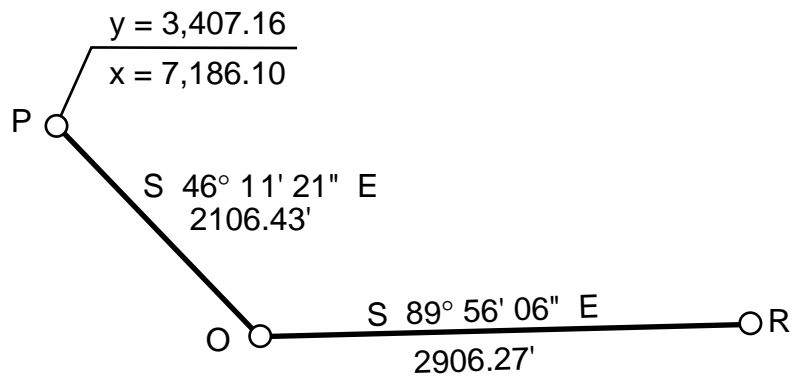
33. From the elements of the oblique triangle given in the sketch below, solve for angle A.

- A. $68^{\circ} 06' 10''$
- B. $67^{\circ} 54' 41''$
- C. $67^{\circ} 50' 53''$
- D. $67^{\circ} 30' 19''$



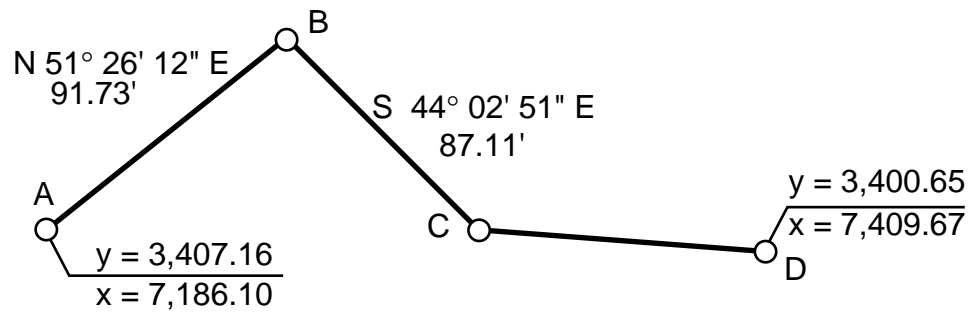
34. Using information given in the sketch below, calculate the coordinates for point R.

- A. $y = 1,945.62$; $x = 11,612.43$
- B. $y = 1,945.73$; $x = 11,612.39$
- C. $y = 1,945.61$; $x = 11,612.33$
- D. $y = 1,945.68$; $x = 11,612.39$



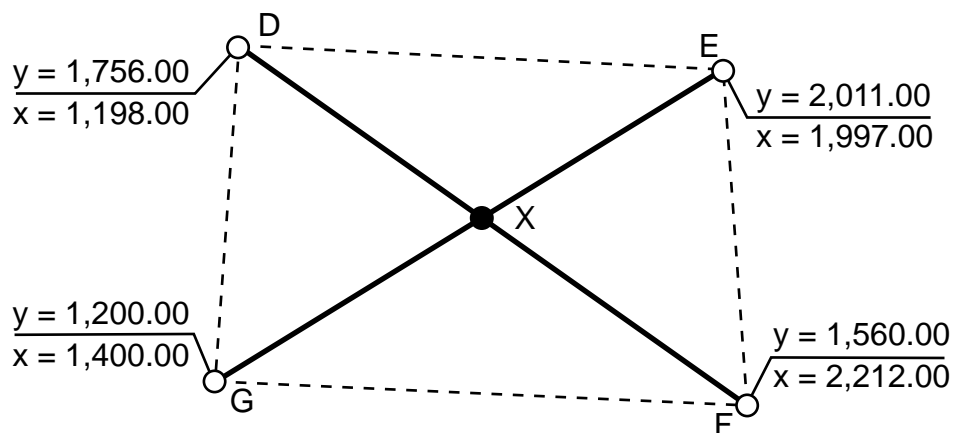
35. After looking at the sketch below, determine the bearing and distance of the line from point C to point D.

- A. N 89° 19' 27" W; 91.33 ft
- B. S 89° 49' 06" W; 91.31 ft
- C. S 89° 19' 27" E; 91.33 ft
- D. S 89° 19' 17" E; 91.29 ft

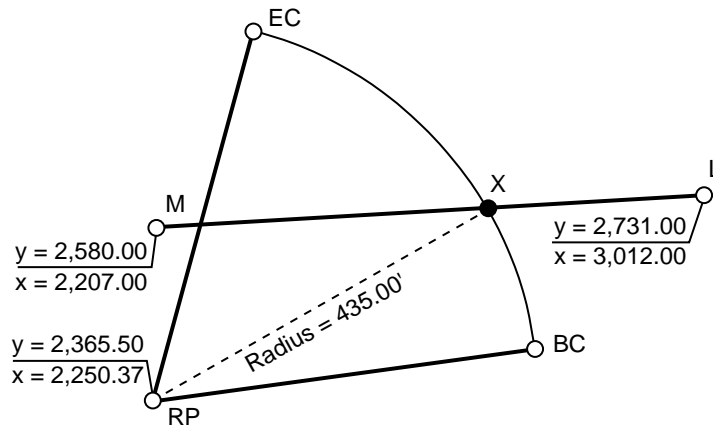


36. Determine the coordinates of the point "x" where the two lines intersect as shown in the sketch below.

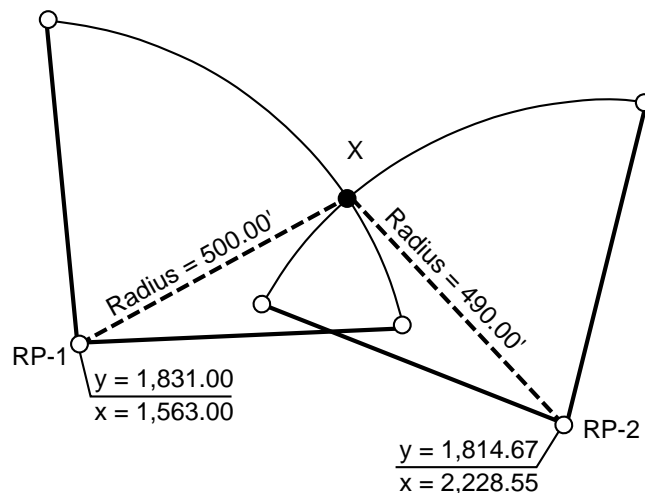
- A. $y = 1,652.56$; $x = 1,733.14$
- B. $y = 1,652.47$; $x = 1,732.99$
- C. $y = 1,652.64$; $x = 1,733.24$
- D. $y = 1,652.53$; $x = 1,733.19$



37. Using information given in the sketch, calculate the bearing of the line from the "RP" to point X. Also determine the distance from L to X.
- A. Bearing "RP" to X = N 48° 51' 10" E; Distance L to X= 438.71 ft
 - B. Bearing "RP" to X = N 50° 51' 50" E; Distance L to X= 439.85 ft
 - C. Bearing "RP" to X = N 49° 10' 30" E; Distance L to X= 440.00 ft
 - D. Bearing "RP" to X = N 49° 12' 40" E; Distance L to X= 441.07 ft

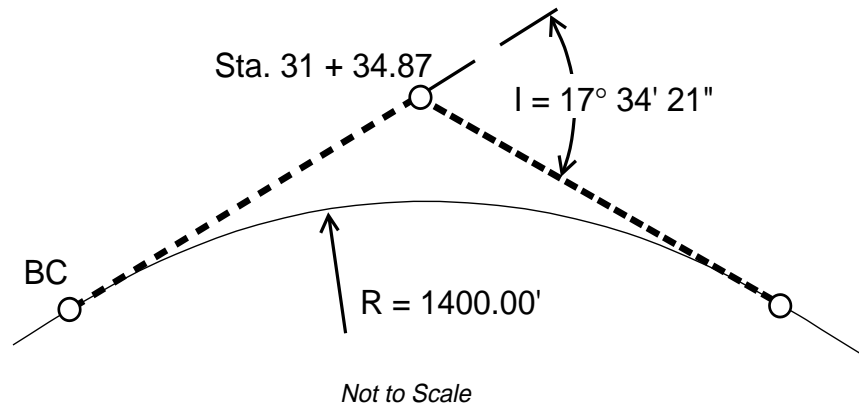


38. Determine the bearing of lines RP-1 to X and line RP-2 to X from the survey data shown in the sketch below.
- A. Bearing Line RP-1 to X = N 44° 17' 50" W;
Bearing Line RP-2 to X = N 40° 12' 45" E
 - B. Bearing Line RP-1 to X = 40° 12' 45" E;
Bearing Line RP-2 to X = N 40° 12' 45" W
 - C. Bearing Line RP-1 to X = N 44° 17' 50" E;
Bearing Line RP-2 to X = N 40° 12' 45" W
 - D. Bearing Line RP-1 to X = N 40° 12' 45" W;
Bearing Line RP-2 to X = N 44° 17' 50" W



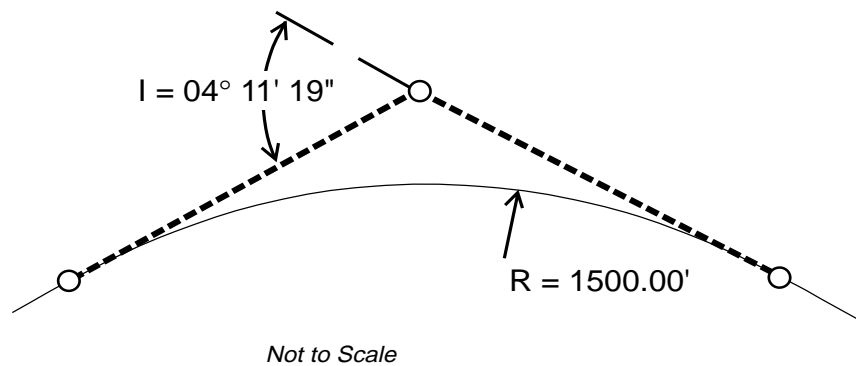
39. Determine the deflection angle and the sub-chord length (from beginning of curve) required to locate sta. 30+74.50 on its correct position on the arc using data given in the sketch below.

- A. Deflection = $03^{\circ} 11' 33''$; Sub-chord = 155.94 ft
- B. Deflection = $06^{\circ} 23' 06''$; Sub-chord = 155.69 ft
- C. Deflection = $06^{\circ} 23' 06''$; Sub-chord = 311.48 ft
- D. Deflection = $03^{\circ} 11' 33''$; Sub-chord = 152.07 ft



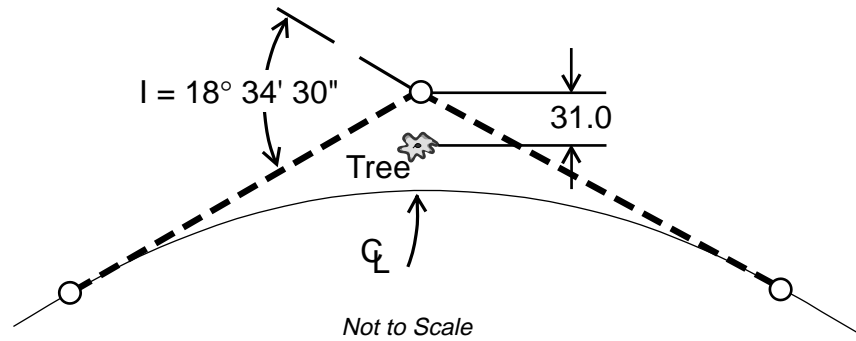
40. Using curve information given in the sketch, calculate the external and mid-ordinate distances for the curve.

- A. External = 0.37 ft; Mid-ordinate = 0.37 ft
- B. External = 1.00 ft; Mid-ordinate = 1.00 ft
- C. External = 1.04 ft; Mid-ordinate = 1.04 ft
- D. External = 4.15 ft; Mid-ordinate = 4.14 ft



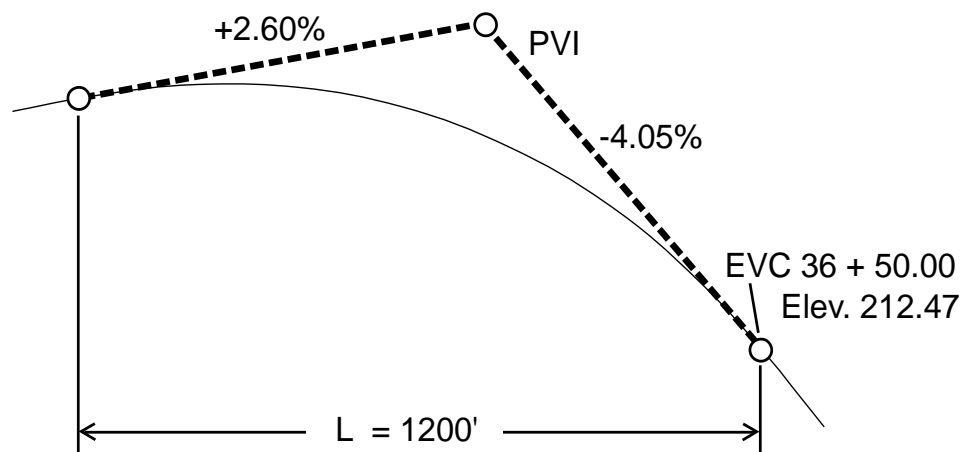
41. Using the data given in the sketch below, calculate the centerline radius that will allow the outside edge of a 42-ft roadway (overall width) to clear the center of the tree by 6 ft.

- A. C/L radius = 4,366.42 ft
- B. C/L radius = 4,478.80 ft
- C. C/L radius = 4,424.92 ft
- D. C/L radius = 4,436.80 ft

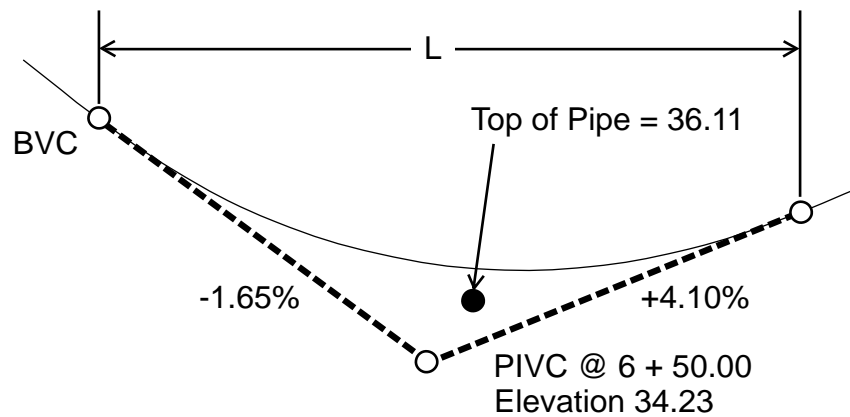


42. From data given on the sketch of the vertical curve below, calculate the elevation at station 31+56.

- A. Elev. @ sta. 31+56 = 225.02 ft
- B. Elev. @ sta. 31+56 = 225.25 ft
- C. Elev. @ sta. 31+56 = 225.47 ft
- D. Elev. @ sta. 31+56 = 225.72 ft



43. Referring back to Problem #42 (above), calculate the station and elevation of the high point of the curve.
- A. Sta. = 29+19; Elev. = 205.74 ft
 - B. Sta. = 29+19; Elev. = 227.27 ft
 - C. Sta. = 30+50; Elev. = 226.80 ft
 - D. Sta. = 33+88; Elev. = 221.17 ft
44. Using the information given in the diagram below, calculate the station and elevation of the BVC of curve designed to provide a minimum of 3.0 ft clearance at the top of pipe located at station 6+87. Determine "L" to the nearest half station.
- A. Sta. @ BVC = 3+50; Elev. = 39.18 ft
 - B. Sta. @ BVC = 4+00; Elev. = 38.36 ft
 - C. Sta. @ BVC = 4+25; Elev. = 37.94 ft
 - D. Sta. @ BVC = 3+25; Elev. = 39.59 ft



Answer Key

1. C. 87,932.24
2. D. 0.19
3. A. 0.50941
4. C. +12.64 ft
5. D. -5.57%
6. C. 0.83277
7. A. 24 ft – 10 7/8 in
8. D. 13,708.34 survey ft
9. A. 1,881.528 meters
10. D. 11,224.98 survey ft
11. D. 1,422.96 survey ft
12. B. 0.254416 rad
13. D. 43° 27' 55"
14. C. 31.619 hec
15. C. 46° 21' 57"
16. B. ±0.12 ft
17. C. ±0.05 ft
18. B. 191,202 ft² or 4.389 ac.; std. error = ±41.5 ft²
19. A. ±0.14 ft
20. D. -405.77
21. B. 108.29
22. D. 3b – 87b
23. A. 418
24. A. <1 = 108° 20' 48"; <2 = 108° 20' 48"; <5 = 108° 20' 48"; <8 = 71° 39' 12"
25. A. <2 = 46° 11' 20"; <3 = 133° 48' 40"
26. B. 15° 23' 08"
27. A. 156.43 ft
28. D. 580.29 ft
29. B. 559.56 ft
30. C. 52.54 ft
31. D. 437.12 ft
32. D. 620.70 ft
33. D. 67° 30' 19"
34. A. y = 1,945.62; x = 11,612.43
35. D. S 89° 19' 17" E; 91.29 ft
36. A. y = 1,652.56; x = 1,733.14
37. C. Bearing "RP" to X = N 49° 10' 30" E; Distance L to X = 440.00 ft
38. C. Bearing Line RP-1 to X = N 44° 17' 50" E;
Bearing Line RP-2 to X = N 40° 12' 45" W
39. A. Deflection = 03° 11' 33"; Sub-chord = 155.94 ft
40. C. External = 1.00 ft; Mid-ordinate = 1.00 ft
41. A. C/L radius = 4,366.42 ft
42. D. Elev. @ sta. 31+56 = 225.72 ft
43. B. Sta. = 29+19; Elev. = 227.27 ft
44. D. Sta. @ BVC = 3+25; Elev. = 39.59 ft

References

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