UNIT 11

THE CALIFORNIA COORDINATE SYSTEM

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Introduction

A 1986 change to the Public Resources Code of the State of California (§§ 8801 et seq.) created the California Coordinate System of 1983 (CCS83). The implementation is to be phased in by January 1, 1995, when state plane work must refer to it instead of the previous California Coordinate System of 1927 (CCS27). The law defined the zones for CCS27 and CCS83. It specified certain constants for each zone and defined the U.S. Survey ft.

Surveyors who are preparing for the LS examination should be aware that the new coordinate system will be phased in through the beginning of 1995. However, nothing prevents the State Board of Registration from asking questions about the older system on future examinations. Examinees need to prepare for both. While this presentation concentrates on the California Coordinate System of 1983, it is broadly applicable to the System of 1927. Calculations for the older system are not as sophisticated as now required. Therefore, surveyors who understand the System of 1983 will find it much easier to master the older system.

The North American Datum of 1927 (NAD27), developed by the U.S. Coast and Geodetic Survey, was the basis for CCS27. It used Clarke's spheroid of 1866 as its spheroid of reference. The North American Datum of 1983 (NAD83), developed by the National Geodetic Survey (NGS), underlies CCS83. NGS adjusted NAD83 during the period 1975-1986 and based it on a new ellipsoid, the Geodetic Reference System of 1980 (GRS80). To date, NGS has implemented NAD83 as a metric system.

Because the reference spheroids differ, the NAD27 latitude and longitude of a station differ from the NAD83 latitude and longitude for the same station. In fact, the differences between the two systems vary inconsistently making only approximate transforms possible. Rigorous computation of coordinates is only possible by returning to the original field observations, readjusting them and then computing positions from them.

Six Lambert conformal zones, NGS zones 0401 through 0406, comprise CCS83. Each zone is based upon a secant cone whose axis is coincident with the GRS80 axis of rotation. The secant cone intersects the surface of the ellipsoid at two standard parallels. The specification of a central meridian fixes the cone relative to the ellipsoid.

As enacted, units can be either m or U.S. Survey ft. One U.S. Survey ft equals exactly 1200/3937 m. To convert U.S. Survey ft to m, multiply by 12/39.37; to convert m to U.S. Survey ft, multiply by 39.37/12. The NGS has chosen to publish all CCS83 station data in m. State plane coordinates expressed in m or ft can be converted to the other units simply by multiplying the northing and easting by the appropriate conversion factor.

The NGS recommends using polynomial coefficients to simplify conversions between NAD83 geodetic and plane coordinates. They also allow accurate calculation of grid scale factor. Polynomial coefficients are easy to use. Therefore, they are appropriate for both manual and programmed applications. Through their use, projection tables and interpolation become unnecessary. The polynomial coefficients are used in algebraic equations that enable the use of handheld calculators. They can produce millimeter accuracy using calculators capable of carrying 10 significant digits.

The NGS developed the coefficients by polynomial curve fitting. I have translated the NGS metric coefficients into U.S. Survey ft coefficients by using the m-ft factor raised to the appropriate power. The U.S. Survey ft coefficients were not independently determined by curve fitting directly.

The California Land Surveyor's Association (CLSA) has published a book of projection tables as Special Publication No. 55/88. It is similar in form to the tables the USCGS published for NAD27. Written by Ira H. Alexander and Robert J. Alexander, it is a tool for the surveyor who has worked with CCS27 and who wishes to do work in CCS83 with little modification. Calculations for state plane coordinate systems fall into five broad classes:

1. Transformations of coordinates between ellipsoid and grid.
2. Manipulations of observations. The adjustments applied to lengths and azimuths in converting between ground, ellipsoid and grid.
3. Transformations from zone to zone.
4. Coordinate geometry with adjusted observations on the grid.
5. Calculations directly on the ellipsoid.

This presentation deals with the first four types of calculations.

Performance Expected on the Exams

Convert geodetic coordinates to plane coordinates (CCS 83).

Convert plane coordinates (CCS 83) to geodetic coordinates.

Calculate the convergence angle of a station.

Convert geodetic (astronomic azimuth) to grid azimuth (CCS 83).

Calculate scale factors.

Calculate combined scale factor and apply it to field measurements.

Key Terms

Convergence

Geodetic azimuth

Grid azimuth

NAD 27

NAD 83

U.S. Survey ft

Polynomial coefficients

Second term correction

Geodetic height

Geoid separation

Geodetic distance

Scale factor

Grid distance

Combined factor

Video Presentation Outline

The State Plane Coordinate System

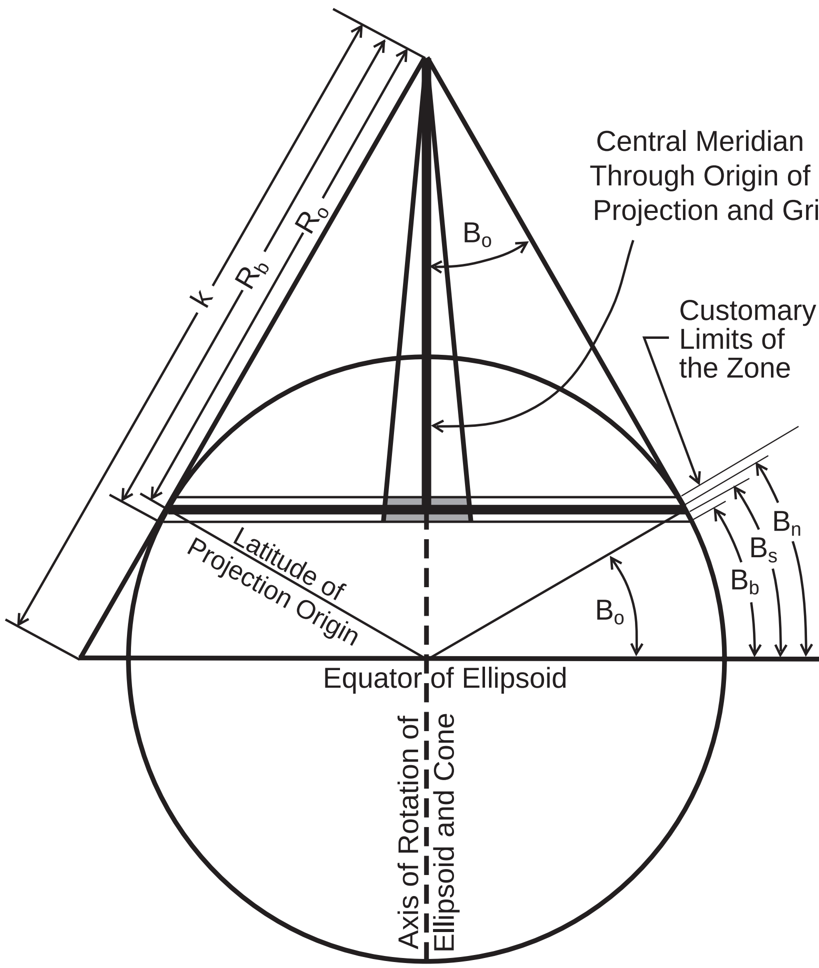

Figure 11-1. *Ellipsoid and secant cone.*

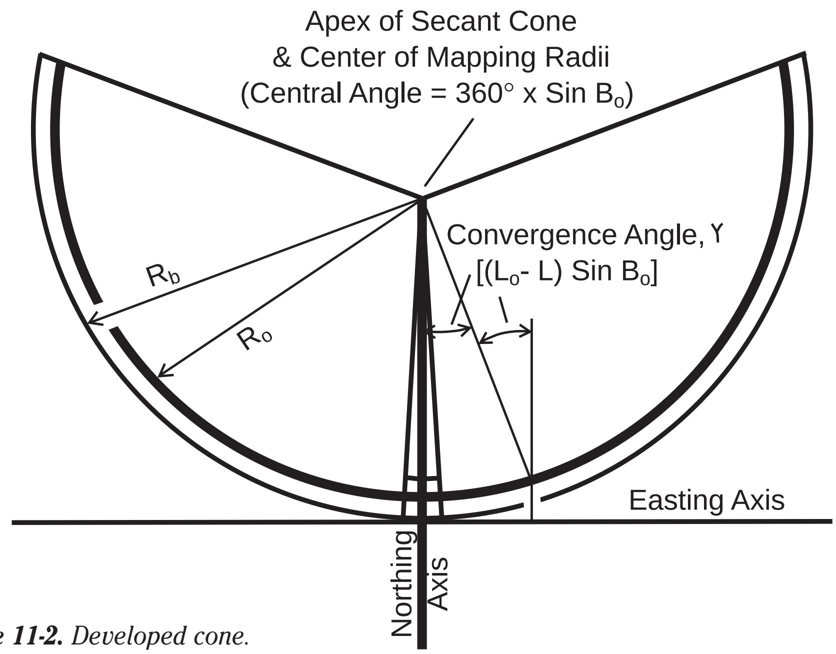

Figure 11-2. *Developed cone.*

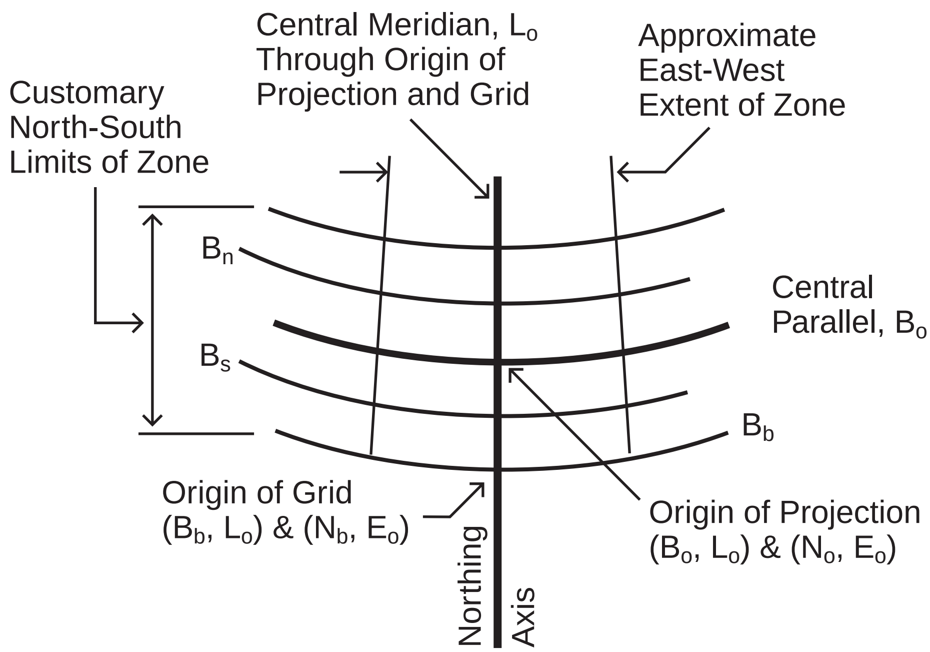

Figure 11-3. *Developed cone detail.*

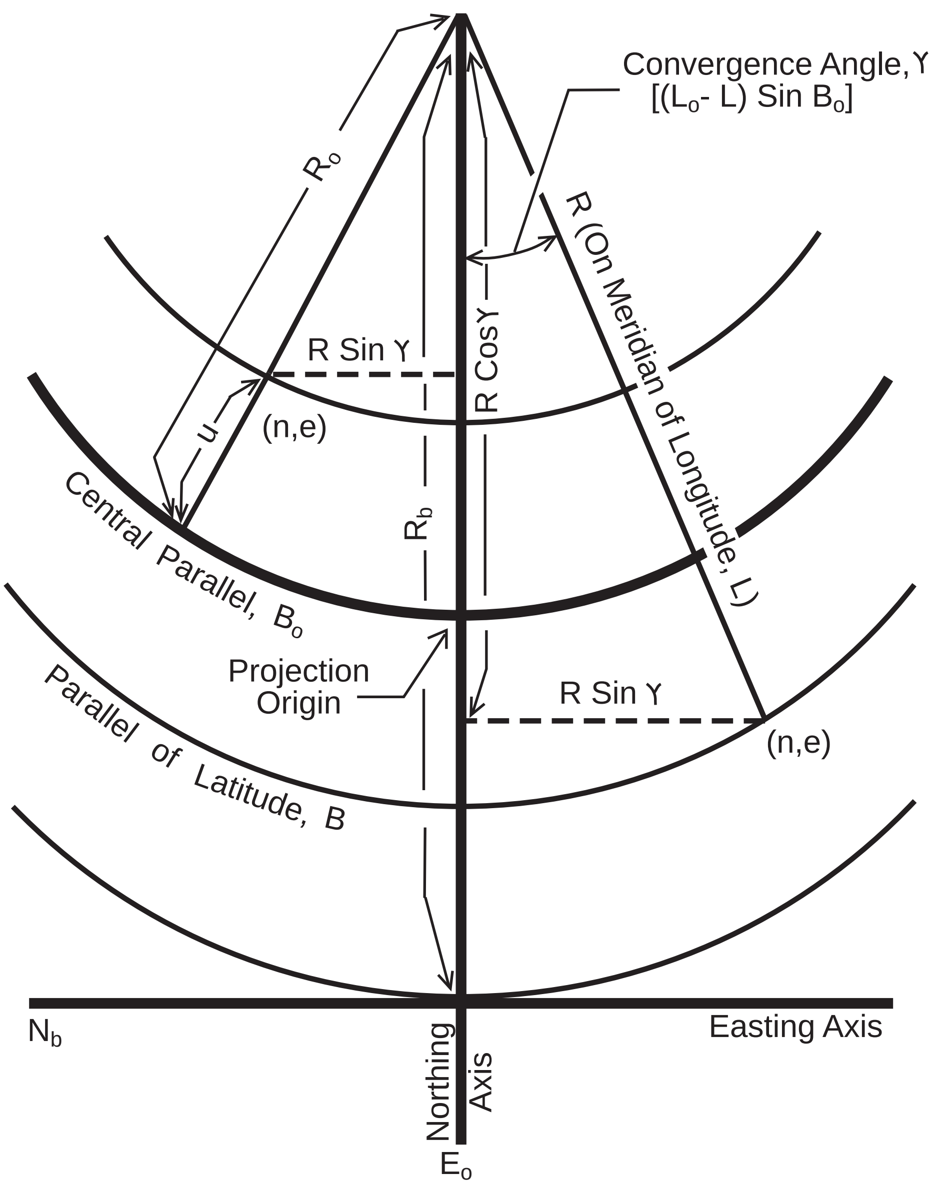

Figure 11-4. *Conversion triangle.*

Example Problems

Direct or Forward Computation: Conversion From Geodetic Latitude and Longitude to Plane Coordinates

NOTE: When working on calculators having only ten significant digits, such as a HP41, it is necessary when adding and subtracting latitudes or longitudes to truncate tens and hundreds of degrees (i.e., truncate 33.040048312° to 3.040048312°).

Direct or forward mapping equations are used to compute state plane coordinates from geodetic coordinates. L polynomial coefficients are used for direct computation. The L coefficients are used to convert the length of the meridian arc between B and B_o to the length "u" which is R_o - R. This permits the calculation of the mapping radius and subsequently the northing and easting.

Conversion of geodetic coordinates, that is latitude B, (ø), and longitude L, (λ), to plane coordinates (n, e) proceeds as follows:

Determine Radial Difference, u, if Projection Tables Will Not be Used:

$\Delta B = B - B_o$ in decimal degrees *(see note above)*

$u = L_1\Delta B + L_2\Delta B^2 + L_3\Delta B^3 + L_4\Delta B^4$
or for hand calculation

$u = \Delta B[L_1 + \Delta B(L_2 + \Delta B(L_3 + L_4\Delta B))]$

Where:
B = north latitude of station, also noted as ø
B_o = latitude of the projection origin, central parallel, a tabled constant
u = radial distance from station to the central parallel, R_o - R
L_1, L_2, L_3, L_4 = polynomial coefficients for direct computation, tabled with the zone constants

Determine Mapping Radius, R:

The mapping radius may be determined in two ways:

1. By the formula:

$R = R_o - u$

Where:
R = mapping radius of station
R_o = mapping radius of the projection origin
u = radial distance from station to the central parallel, R_o - R

2. If projection tables are provided, the mapping radius also may be interpolated by entering the tables with the argument of latitude against mapping radius.

Determine Plane Convergence, γ:

$$\gamma = (L_o - L) \sin B_o$$ *(see note, page 7)*

Where:

γ = convergence angle. *(Carry all significant digits for this calculation.)*
L = west longitude of station, also noted as λ
L_o = longitude of central meridian, longitude of projection and gridorigin, a tabled constant
$\sin B_o$ = a tabled constant, sine of the latitude of the projection origin

Determine Northing and Easting:

For polynomial solutions:

$$n = N_o + u + [(R \sin\gamma) \quad \tan \frac{\gamma}{2}$$

For polynomial solution or projection tables:

$$n = R_b + N_b - R \cos\gamma$$

$$e = E_o + R \sin\gamma$$

Where:

γ = convergence angle
e = easting of station
E_o = easting of projection and grid origin, 6561666.667 ft or 2000000.0000 m in all zones
n = northing of station
N_o = northing of the projection origin, a tabled constant
R = mapping radius of station
u = radial distance from station to the central parallel, $R_o - R$
R_b = mapping radius of the grid base, a tabled constant
N_b = northing of the grid base, 1,640,416.667 ft or 500000.0000 m in all zones

Inverse Computation: Conversion of Plane Coordinate Position to Geodetic Latitude and Longitude

Inverse mapping equations are used to compute geodetic coordinates from state plane coordinates. The geodetic coordinates, latitude and longitude, are on the ellipsoid of reference.

The G coefficients are used for inverse conversion. The G coefficients are used to convert the northing and easting of the station to the length "u" which is R_o - R. This permits the calculation of the length of the meridian arc between B and B_o. Adding that length to B_o, the latitude is obtained. Longitude is calculated with conventional formulas. Be careful to use the correct algebraic sign for each value in the formulas.

The applicable formulas to be solved in the order given are:

Determine Plane Convergence, γ:

NOTE: The value of the convergence angle must be carried to all available digits. Register or stack mathematics is recommended.

$$\gamma = \arctan \frac{e - E_o}{R_b - n + N_b}$$

Where:

- γ = convergence angle at the station, carry to all available digits
- e = easting of station
- E_o = easting of projection and grid origin, 6561666.667 ft or 2000000.0000 m for all zones
- R_b = mapping radius of the grid base, a tabled constant
- n = northing of station
- N_b = northing of the grid base, 1640416.667 ft or 500000.0000 m for all zones

Determine Longitude, L:

$$L = L_o - \frac{\gamma}{\sin B_o} \quad \textit{(see note above)}$$

Where:

- γ = convergence angle at the station
- sin B_o = a tabled constant, sine of the latitude of the projection origin
- L = west longitude of station, also noted as λ
- L_o = central meridian, longitude of projection and grid origin, a tabled constant

Determine Radial Difference, u:

$$u = n - N_o - [(e - E_o) \tan \frac{\gamma}{2}]$$

Where:

γ = convergence angle at the station
e = easting of station
E_o = easting of projection and grid origin, 6561666.667 ft or 2000000.0000 m for all zones
n = northing of station
N_o = northing of the projection origin
u = radial distance from station to the central parallel, $R_o - R$

Determine Latitude, B:

Where:
$B = B_o + G_1u + G_2u^2 + G_3u^3 + G_4u^4$ *(see note on page 7)*
$B = B_o + u [G_1 + u (G_2 + u (G_3 + G_4u))]$ for handheld calculation *(see note on page 7)*
B = north latitude of station, also noted as ϕ
B_o = latitude of the projection origin, central parallel
u = radial distance from station to the central parallel, $R_o - R$
G_1, G_2, G_3, G_4 = polynomial coefficients for inverse conversion, tabled with the zone constants

Only if Projection Tables are to be Used:

Determine mapping radius for interpolation arguing mapping radius against latitude by the following formula:

$$R = \sqrt{(e - E_o)^2 + (R_b - n + N_b)^2}$$

Where:
R = mapping radius of station
e = easting of station
E_o = easting of projection and grid origin, 6561666.667 ft or 2000000.0000 m for all zones
R_b = mapping radius of the grid base, a tabled constant
n = northing of station
N_b = northing of the grid base, 1640416.667 ft or 500000.0000 m in all zones

Determining Plane Convergence (Mapping) Angle and Geodetic Azimuth to Grid Azimuth

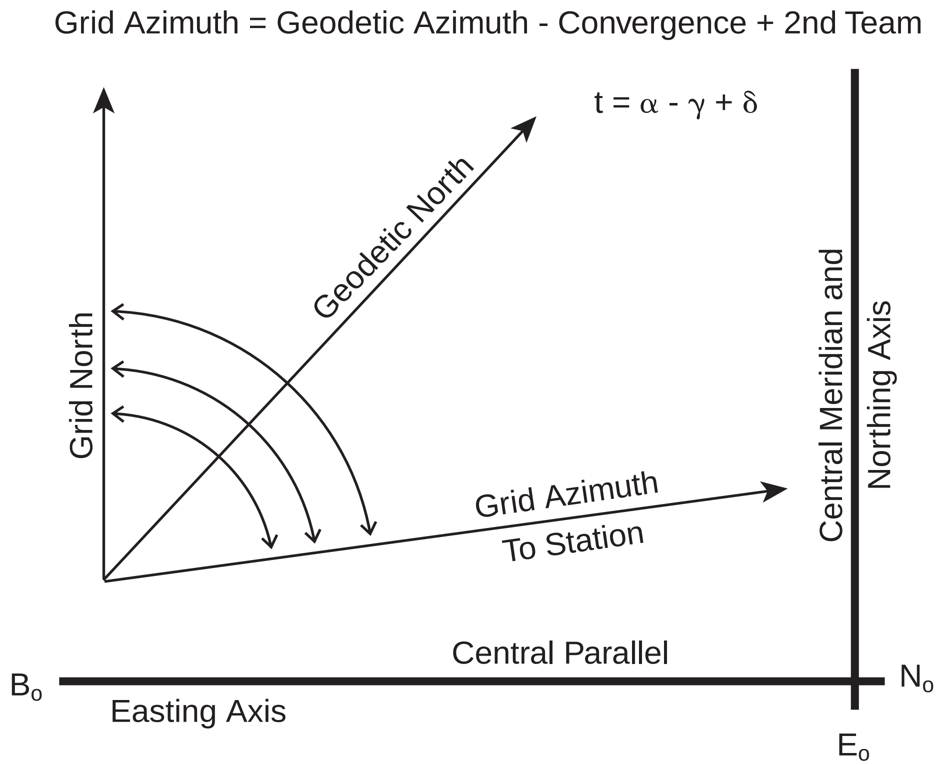

Figure 11-5. *Convergence (mapping angle).*

For CCS83 (NAD83) both geodetic and grid azimuths are reckoned from north. NGS surveys for NAD83 are reckoned from north rather than from south, which had been used for NAD27. Inverses between stations having state plane coordinates give grid azimuth and may be used directly for calculations on the grid. Plane convergence varies with longitude; therefore, the station for which convergence was determined must be specified.

Determine the Plane Convergence, γ:

The plane convergence may be determined in two ways:

1. Where the longitude is not known without calculation, the plane convergence may be calculated from the constants of the projection and the plane coordinates:

$$\gamma = \arctan \frac{e - E_o}{R_{b-n} + N_b}$$

   Where:

   γ = plane convergence angle
   e = easting of station
   E_o = easting of projection origin, 6561666.667 ft or 2000000.0000 m for all zones
   R_b = mapping radius of the grid base, a tabled constant
   n = northing of station
   N_b = northing of the grid base, 1640416.667 ft or 500000.0000 m for all zones

2. The plane convergence also may be calculated using the difference in longitude between the central meridian and the point of question and a tabled constant. Be careful to use the correct algebraic signs.

$$\gamma = \sin B_o (L_o - L)$$

   Where:

   γ = plane convergence
   $\sin B_o$ = sine of the latitude of the projection origin, which is also the ratio between γ and longitude in decimals of a degree, a tabled constant
   L_o = longitude of central meridian, a tabled constant
   L = west longitude of station of the desired plane convergence

The Second Term Correction, δ:

The second term correction is usually minute and can be neglected for most courses under five miles long. This correction is also noted as "t - T." It is the difference between the grid azimuth, "t," and the projected geodetic azimuth, "T." It increases directly with the change in eastings of the line and with the distance of the occupied station from the central parallel. It is neglected in this presentation.

Determine Grid, t, or Geodetic Azimuth, α:

$$t = \alpha - \gamma + \delta$$

Where:

t = grid azimuth, the clockwise angle at a station between the grid meridian (grid north) and the grid line to the observed object. All grid meridians are straight and parallel.

α = geodetic azimuth, the clockwise angle between the geodetic meridian (geodetic north) and the geodetic line to the object observed.

g = plane convergence, mapping angle, the major component of the difference between geodetic azimuth and grid azimuth.

δ = arc to chord correction, known as second term or t - T, a correction applied to long lines of precise surveys to compensate for distortion of straight lines when projected onto a grid. This correction is usually minute and can be neglected for most courses under five miles long. Use 0° in those cases.

Reducing Measured Distance to Geodetic Length and then to Grid Length

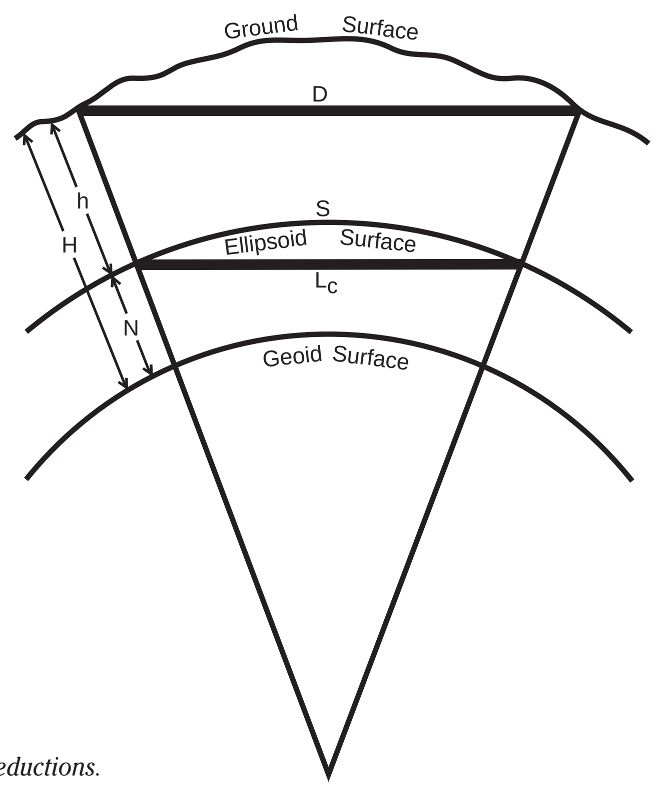

Figure 11-6. *Distance reductions.*

Reduce Measured Length to Ellipsoidal Chord Length:

Surveyors performing high precision work with NAD83 must consider the difference in elevation between the GRS80 ellipsoid and the geoid. Elevation based upon the ellipsoid is geodetic height, h. Elevation based upon the geoid (mean sea-level) is noted as H. The difference between the two surfaces is the geoid separation or height, N.

Determine Approximate Radius of Curvature of the Ellipsoid, R_α:

An approximate radius of the ellipsoid for each zone is the geometric mean radius of curvature at the projection origin. It is close enough for all but the most precise work. It can be obtained by the following formula:

$$R_\alpha \pm = \frac{r_o}{k_o}$$

Where:

$R_\alpha \pm$ = geometric mean radius of curvature of the ellipsoid at the projection origin

r_o = geometric mean radius of the ellipsoid at the projection origin, scaled to the grid

k_o = grid scale factor of the central parallel

Determine Ellipsoidal Reduction Factor, r_e, Also Known as Elevation Factor:

$$r_e = \frac{R_\alpha}{(R_\alpha + N + H)}$$

Where:

r_e = ellipsoidal reduction factor

R_α = radius of curvature of the ellipsoid in the azimuth from equation above

N = geoid separation, which is a negative value within the contiguous 48 states

H = elevation based on mean sea level to which the measured line was reduced to horizontal. The elevation to be used is usually one of the following for measured lengths:

1. Triangulation - average elevation of base line
2. EDM - elevation to which slope length was reduced
3. Taped - average elevation of the line

Each one meter error in N or H contributes 0.16 ppm of error to the distance.

Determine Ellipsoidal Chord Length, L_c:

$$L_c = r_e D$$

Where:
L_c = ellipsoidal chord length
r_e = ellipsoidal reduction factor
D = ground level, horizontal measured distance

Determine Correction of Ellipsoidal Chord Length to Geodetic Length

Geodetic lengths are ellipsoidal arc lengths. When precise geodetic lengths, s, are desired, a correction from ellipsoidal chord length, L_c, to the geodetic length on the ellipsoid surface may be applied to lines generally greater than 5 mi long. Shorter lengths and lines measured in segments are essentially arcs and need not be corrected. The correction to be applied is:

Determine Chord Correction for Geodetic Length:

L_c equals nearly 9 mi before c equals 0.01 ft and over 19 mi before equaling 0.10 ft.

$$c = \frac{L_c^3}{24R_\alpha^2}$$

Where:
c = correction, in same units as used for L_c and R_a
L_c = ellipsoidal chord length
R_a = radius of curvature of the ellipsoid in the azimuth from equation for approximating radius of curvature of ellipsoid above

Determine Geodetic Length, s:

$$s = L_c + c$$

Where:
c = correction, in same units as used for L_c and R_a
L_c = ellipsoidal chord length
s = geodetic length

Project Geodetic Length (or Ellipsoidal Chord Length) to Obtain Grid Length

Grid length, $L_{(grid)}$, is obtained by multiplying the geodetic length by a grid scale factor. Grid scale factor is an expression of the amount of distortion imposed on the length of a line on the ellipsoid as it is projected onto the grid cone. It is represented by the letter "k." It is the ratio of the length on the grid to the length on the ellipsoid. Scale factor is dependent upon latitude. It is less than unity between the standard parallels and greater than unity outside them. Scale factors can be calculated for points or lines. For this presentation, point scale factors are used exclusively. If greater accuracy is needed for a long line, the point scale factor of the mid-point or the mean of the point scale factors at each end may be used.

In a Lambert zone, if the north-south extent is not great, sufficient accuracy often may be obtained by using an average scale factor determined for the average latitude of the survey.

Determine Point Scale Factor, k:

A method which is precise enough for any work performed uses polynomial coefficients. It does not require projection tables and is easy to do using a handheld calculator with storage registers. It also readily lends itself to programmed applications. The coefficients for each zone are tabled at the end of this paper. Approximate point scale factor, k, may be interpolated from projection tables.

if γ and plane coordinates are known:

$$u = n - N_o - [(e - E_o) \tan \frac{\gamma}{2}]$$

when plane coordinates are known:

$$u = R_o - \sqrt{(R_b + N_b - n)^2 + (e - E_o)^2}$$

when geodetic latitude and longitude are known:

$$u = L_1\Delta B + L_2\Delta B^2 + L_3\Delta B^3 + L_4\Delta B^4$$

rearranged for handheld calculators:

$$u = \Delta B[L_1 + \Delta B(L_2 + \Delta B(L_3 + L_4\Delta B))]$$

Where:
- u = radial distance on the projection from the central parallel to the station, R_o - R
- N_o = northing of projection origin
- n = northing of station
- e = easting of station
- E_o = easting of central meridian, 6561666.667 ft or 2000000.0000 m for all zones (also noted as C)
- γ = convergence angle at the station
- R_o = mapping radius through the projection origin, a tabled constant
- R_b = mapping radius through grid base, a tabled constant
- N_b = northing of grid base, a tabled constant equaling 1640416.667 ft or 500000.0000 m
- L_1, L_2, L_3, L_4 = polynomial coefficients tabled with the zone constants
- B = latitude of station (also noted as ø)
- B_o = the latitude of the projection origin, a tabled constant for each zone (also noted as $ø_o$)
- $\Delta B = B - B_o$

$$k = F_1 + F_2u^2 + F_3u^3$$

Where:
- k = point scale factor
- F_1, F_2, F_3 = polynomial coefficients tabled with the zone constants
- u = radial distance on the projection

Determine Grid Length, $L_{(grid)}$:

$$L_{(grid)} = s\,k$$

Where:
- $L_{(grid)}$ = length on grid
- s = geodetic length
- k = point scale factor of midpoint of line or as appropriate

Combined Factor

If no correction from ellipsoidal chord length to geodetic length is warranted and latitude and elevation differences are not great, the measured lengths may be multiplied with a combined factor, cf, to obtain grid lengths, $L_{(grid)}$.

Determine Combined Factor, cf:

$$cf = r_e k$$

Where:

cf = combined factor

r_e = ellipsoidal reduction factor

k = point scale factor calculated for mid-latitude of the survey

Determine Grid Length, $L_{(grid)}$:

$$L_{(grid)} = cf\, D$$

Where:

$L_{(grid)}$ = length on grid

cf = combined factor

D = ground level, horizontal measured distance

Conversion of Grid Length to Ground Length

Grid length, $L_{(grid)}$, may be converted to ground length, D, by reversing the above procedures. For measurements of similar elevation, latitude and short length, a combined correction factor may be divided into the grid length.

Area and State Plane Coordinates

Areas derived from state plane coordinates must be corrected to yield ground-level areas when they are desired.

$$A = \frac{A_{(grid)}}{cf^2}$$

Where:

A = land area at ground level

$A_{(grid)}$ = area of figure on the grid

cf = combined factor

Conversion of Coordinates from One Zone to Another

To convert plane coordinates in the overlap of zones from one zone to the other of the CCS83 system, convert the plane coordinates from the original zone using the constants for that zone to GRS80 geodetic latitude and longitude. Then using the constants for the new zone, convert the geodetic latitude and longitude to plane coordinates in the new zone.

Metric–Foot Equivalency

The U.S. Survey ft, the linear unit of the State coordinate system, is defined by the equivalence: 1 international meter = 39.37 inches, exactly.

A coordinate system in ft may be converted to a coordinate system in m by multiplying the coordinate values by a scale factor of 0.304800609601. An exact conversion can be accomplished by first multiplying by 12 and then dividing by 39.37.

A coordinate system in m may be converted to a system in ft by multiplying the coordinate values by a scale factor of 3.28083333333. An exact conversion can be accomplished by first multiplying by 39.37 and then dividing by 12.

Sample Test Questions

1. Convert the position of station “San Ysidro Levee 1975” from its NAD83 geodetic coordinates of latitude 32° 32' 36.33328" N, longitude 117° 02' 24.17391" W to its CCS83 Zone 6 metric plane coordinates.
2. What are the CCS83 Zone V plane coordinates for a station at latitude 34° 08' 13.1201" N, longitude 118° 19' 32.9502" W? Use projection tables for your solution.
3. Convert the position of station “San Ysidro Levee 1975” from its CCS83 Zone 6 plane coordinates of 542065.352m N, 1925786.624 m E to its equivalent NAD83 geodetic coordinates.
4. In CCS83 Zone V, a station has plane coordinates 1872390.80 ft N, 6463072.10 ft E. What are its NAD83 geodetic coordinates? Use projection tables for your solution.
5. A line has been determined to have a geodetic azimuth of 135° 00' 00" in CCS83 Zone VI. The station is at longitude 117° 25' W. What is its grid azimuth?

6. The geodetic azimuth, *a*, is desired between two stations on the CCS83 Zone VI grid. The azimuth from Station #1 is desired. Given: Station #1 coordinates are: 1660578.090 ft N, 6570078.800 ft E. Station #2 coordinates are: 1653507.022 ft N, 6577149.868 ft E.
7. A survey party occupied station San Javier 1919 which has NGS NAD83 published CCS83 Zone 6 coordinates of 539034.888 m northing, 1977009.714 m easting. San Javier also has a published elevation of 1219.58 m and a geoid height of -33.57 m. All field measured distances were reduced to the elevation of station San Javier 1919. The field distance to a foresighted station is 13000.00 ft. What is the grid distance to the foresighted station? What is the combined factor?
8. In CCS83 Zone VI, a length of 6000.000 ft is measured at 5200 ft elevation. The mid-point of the line is at latitude 32° 36' 20" N. What is the grid length? What is the combined factor?
9. A line in CCS83 Zone VI has a grid length of 200000.000 ft and its mid-point is at latitude 32° 31' N. What is its ground length at 3800 ft elevation?

Answer Key

1. Direct forward computation.

Determine radial difference:

$\Delta B = B - B_o$
$\Delta B = 32.543425911° - 33.333922945°$ (truncate 30° from B and B_o for HP41s)
$\Delta B = -0.790497034°$

$u = \Delta B\,[L_1 + \Delta B\,(L_2 + \Delta B\,(L_3 + L_4\Delta B))]$
$u = \Delta B\,[L1 + \Delta B\,(L_2 + \Delta B\,(L_3 + (0.016171)\,(-0.790497034)))]$
$u = \Delta B\,[L_1 + \Delta B\,(L_2 + (-0.790497034)\,(5.65087 - 0.012783))]$
$u = \Delta B\,[L_1 + (-0.790497034)\,(8.94188 - 4.45689)]$
$u = (-0.790497034)\,(110905.3274 - 3.5454)$
$u = -87667.5297$ m

Determine mapping radius:

$R = R_o - u$
$R = 9706640.076 - (-87667.530)$
$R = 9794307.606$ m

Determine plane convergence:

$\gamma = (L_o - L) \sin B_o$

$\gamma = (116° 15' - 117° 02' 24.17391'') (0.5495175758)$ truncate 110° from L_o and L for HP41s

$\gamma = -0° 26' 02.923552''$ (carry all significant digits)

Determine northing and easting:

$n = N_o + u + [(R \sin\gamma) \tan\frac{\gamma}{2}]$

$n = N_o + u + [(9794307.606) (-0.00757719469) (-0.00378865173)]$

$n = 629451.7134 - 87667.5297 + 281.1686$

$n = 542065.352$ m (exactly as published by NGS)

$e = E_o + R \sin\gamma$

$e = 2000000.0000 + [(9794307.606) (-0.00757719469)]$

$e = 1925786.624$ m (exactly as published by NGS)

2. Direct forward computation (using projection tables).

Determine mapping radius:		R = 30418256.88
for 34° 08' from Projection Table:	101.08844 x 13.1201 =	-1326.29
less diff/1" lat. from column 5:		R = 30416930.59
then for 34° 08' 13.1201" N:		

Determine plane convergence:

$\gamma = (L_o - L) \sin B_o$

$\gamma = (118° 00' - 118° 19' 32.9502'') (0.570011896174)$ truncate 110° from L_o and L for HP41s

$\gamma = -0.18572099101°$

$\gamma = -0° 11' 08.5956''$ (carry to all significant digits)

Determine northing and easting:

$n = R_b + N_b - R \cos\gamma$

$n = 30648744.93 + 1640416.67 - 30416930.59 \cos(-0.18572099101°)$

$n = 1872390.80$ ft N

$e = E_o + R \sin\gamma$

$e = 6561666.67 + 30416930.59 \sin(-0.18572099101°)$

$e = 6463072.10$ ft E

3. Inverse computation.

Determine plane convergence:

$$\gamma = \arctan \left[\frac{e - E_o}{R_b - n + N_b}\right]$$

γ = arctan [(1925786.624 m - 2000000.000 m)/
(9836091.790 m - 542065.352 m + 500000.000 m)]

γ = arctan (-74213.376/9794026.438)

γ = arctan (-0.007577412259)

γ = -0° 26' 02.923559" (carry all significant digits)

Determine longitude:

$$L = L_o - \left(\frac{\gamma}{\sin B_o}\right)$$

L = 116.25° - (-0.4341454331/0.5495175758) truncate 110° from L for HP41s

L = 117.040048312°

L = 117° 02' 24.17392" W (versus 117° 02' 24.17391" published by NGS)

Determine radial difference:

$$u = n - N_o - \left[(e - E_o) \tan \frac{\gamma}{2}\right]$$

$u = n - N_o$ - [(1925786.624m - 2000000.0000 m) (-0.00378865173)]

u = 542065.352 m - 629451.7134 m - 281.1686 m

u = -87667.530 m

Determine latitude:

$B = B_o + u\,[G_1 + u\,(G_2 + u\,(G_3 + G_4 u))]$

$B = B_o + u\,[G_1 + u\,(G_2 + u\,(G_3 + (-8.2753 \times 10^{-28})\,(-87667.530\text{ m})))]$

$B = B_o + u\,[G_1 + u\,(G_2 + (-87667.530\text{ m})\,(-3.73318 \times 10^{-20} + 7.25475 \times 10^{-23}))]$

$B = B_o + u\,[G_1 + (-87667.530\text{ m})\,(-6.55499 \times 10^{-15} + 3.266426 \times 10^{-15})]$

$B = B_o + (-87667.530\text{ m})\,(9.016699372 \times 10^{-6} + 2.8830022 \times 10^{-10})$

B = 33.333922945° - 0.7904970372° truncate 30° from B for HP41s

B = 32° 32' 36.33327" N (versus 32° 32' 36.33328" published by NGS)

4. Inverse computation (using projecting tables).

Determine plane convergence:

$$\gamma = \arctan \left[\frac{e - E_o}{R_b - n + N_b} \right]$$

$$\gamma = \arctan \left[\frac{(6463072.10 \text{ ft} - 6561666.67 \text{ ft})}{(30648744.93 \text{ ft} + 1640416.67 \text{ ft} - 187390.80 \text{ ft})} \right]$$

γ = arctan (-98594.57 / 30416770.80)

γ = arctan (-0.00324145422)

γ = -0.18572099571°

γ = -0° 11' 08.59558" (carry seconds to at least the fourth decimal place)

Determine longitude:

$$L = L_o - \frac{\gamma}{\sin B_o}$$

L = 118° 00' - (-0.18572099571°/0.570011896174) truncate 110° from L for HP41s

L = 118° 00' - (-0.325819500°)

L = 118.325819500° W

L = 118° 19' 32.9502" W

Determine mapping radius for use with projection tables:

$$R = \sqrt{(e - E_o)^2 + (R_b - n + N_b)^2}$$

$$R = \sqrt{-98594.57^2 + 30416770.80^2}$$

R = 30416930.59 ft

Determine the latitude:

Interpolate R in Projection Table to find latitude (ø)

(34° 08')	R =	30418256.88
(unk.)	R =	30416930.59
Diff.		1326.29

using column 5 of Projection Table:

1326.29/101.08844 = 3.1201"

therefore, B = 34° 08' 13.1201" N

5. **Determine the plane convergence.**

$\gamma = \sin B_o(L_o - L)$
$\gamma = 0.549517575763\ (116° 15' - 117° 25')$
$\gamma = -0° 38' 27.97382''$

The second term will be neglected.

Calculate the grid azimuth:

$t = \alpha - \gamma + \delta$
$t = 135° 00' 00'' - (-0° 38' 27.97382'') + 0°$
$t = 135° 38' 27.97382''$

6. **Inverse from Station #1 to Station #2.**

The inversed grid azimuth is: t = 135°00'00".

Calculate the plane convergence:

$$\gamma = \arctan \left[\frac{e - E_o}{R_{b-n} + N_b} \right]$$

$$\gamma = \arctan \left[\frac{(6570078.800 \text{ ft} - 6561666.667 \text{ ft})}{(32270577.813 \text{ ft} - 1660578.090 \text{ ft} + 1640416.667 \text{ ft})} \right]$$

$\gamma = \arctan (0.00026083797)$
$\gamma = +0° 00' 53.8017''$

The second term correction is neglected here.

Calculate the geodetic azimuth:

$t = \alpha - \gamma + \delta$

$135° 00' 00'' = \alpha - (+0° 00' 53.8017'') + 0°)$
$\alpha = 135° 00' 00'' + 0° 00' 53.8017'' + 0°$
$\alpha = 135° 00' 53.8017''$

7. **Convert elevation, geoid height, and measured distance to the same units. Here feet were chosen.**

Determine radius of curvature of ellipsoid:

$$R_\alpha \pm = \frac{r_o}{k_o}$$

$R_\alpha \pm = 20896729.860 \text{ ft} / 0.999954142490$
$R_\alpha \pm = 20897688.176 \text{ ft}$

Determine ellipsoidal reduction factor:

$$r_e = \frac{R_\alpha}{(R_\alpha + N + H)}$$

r_e = 20897688.176 ft / (20897688.176 ft - 110.14 ft + 4001.24 ft)

r_e = 0.999813837

Determine ellipsoidal chord length:

$L_c = r_e D$

L_c = 0.999813837 x 13000.00 ft

L_c = 12997.58 ft

Determine chord correction and geodetic length:

$$c = \frac{L_c^3}{24R_\alpha^2}$$

$c = 12997.58^3 / 24\,(20897688.176^2)$

c = 0.0002 ft

Determine geodetic length:

The chord correction is negligible, therefore the geodetic length,

s = 12997.58 ft

Determine point scale factor:

$u = R_o - \sqrt{(R_b + N_b - n)^2 + (e - E_o)^2}$

u = 31845868.317 ft -

$\sqrt{(322770577.813\text{ ft} + 1640416.667\text{ ft} - 1768483.63)^2 + (6486239.37\text{ ft} - 6561666.67)^2}$

$u = 31845868.317\text{ ft} - [1033141004 \times 10^{15} + 5689277585.29]^2$

u = 31845868.317 ft - 32142599.351 ft

u = -296731.03 ft

$k = F_1 + F_2u^2 + F_3u^3$

$k = 0.999954142490 + [1.14504 \times 10^{-15}\,(8.8049304 \times 10^{10})] +$

$[1.18 \times 10^{-23}\,(-2.6126960 \times 10^{16})]$

k = 0.999954142490 + 0.000100820 - 0.000000308

k = 1.000054654

Determine grid length:

$L_{(grid)} = s\,k$

$L_{(grid)}$ = 12997.58 ft (1.000054654)

$L_{(grid)}$ = 12998.29

Determine combined factor:

$cf = r_e k = (0.999813837)\ 1.000054654 = 0.999868481$

8. Reducing measured distance.

Determine radius of curvature of ellipsoid:

$$R_a \pm = \frac{r_o}{k_o}$$

$R_a \pm = 20896729.860\text{ ft}/0.999954142490$

$R_a \pm = 20897688.176\text{ ft}$

Determine ellipsoidal reduction factor:

$$r_e = \frac{R_\alpha}{(R_\alpha + N + H)}$$

$r_e = 20897688.176\text{ ft}/(20897688.176\text{ ft} + 0 + 5200\text{ ft})$

$r_e = 0.9997512306$

Determine ellipsoidal chord length:

$L_c = r_e D$

$L_c = (0.9997512306)\ (6000.000\text{ ft})$

$L_c = 5998.507\text{ ft}$

Determine chord correction:

$$c = \frac{L_c^3}{24R_\alpha^2}$$

$c = 5998.507^3/(24)(20897688.176^2)$

$c = 0.000021\text{ ft}$

Determine geodetic length:

The chord correction is negligible, therefore the geodetic length,

$s = 5998.507\text{ ft}$

Determine point scale factor:

Scale factor for 32° 36' from Project Table = 1.0000356

20/60 diff. = (0.333) (-0.0000037) = -0.0000012

k = 1.0000344

Determine grid length:

$L_{(grid)} = s\ k$

$L_{(grid)} = (5998.507\text{ ft})\ (1.0000344)$

$L_{(grid)} = 5998.713\text{ ft}$

Determine combined factor:

$cf = r_e k$
$cf = (0.9997512306)(1.0000344)$
$cf = 0.9997856$

Determine grid length:

$L_{(grid)} = cf(D)$
$L_{(grid)} = 0.9997856 \times 6000.00 \text{ ft}$
$L_{(grid)} = 5998.714 \text{ ft}$

9. Convert grid length to ground length.

Determine point scale factor:

$\Delta B = B - B_o$
$\Delta B = 32.5166666667° - 33.3339229447°$
$\Delta B = -0.8172562780°$

$u = \Delta B [L_1 + \Delta B (L_2 + \Delta B (L_3 + (-0.04335871457)))]$
$u = \Delta B [L1 + \Delta B (L_2 + \Delta B(18.4962412854))]$
$u = \Delta B [L_1 + \Delta B (L_2 + (-15.1161693099))]$
$u = \Delta B [L_1 + \Delta B (14.2206306901)]$
$u = \Delta B [L_1 + (-11.6218997086)]$
$u = \Delta B [363850.2731]$
$u = -297358.9199 \text{ ft}$

$k = F_1 + F_2u^2 + F_3u^3$
$k = 0.999954142490 + (1.14504 \times 10^{-15})(-297358.9199^2) + (1.18 \times 10^{-23})(-297358.9199^3)$
$k = 1.00005507933$

Determine geodetic length:

$L_{(grid)} = s\,k$
$200000.000 \text{ ft} = s \times 1.00005507933$
$s = 200000.000 \text{ ft}/1.00005507933$
$s = 199988.985 \text{ ft}$

Determine radius of curvature of the ellipsoid:

$$R_a \pm = \frac{r_o}{k_o}$$
$R_a \pm = 20896729.860 \text{ ft} / 0.999954142490$
$R_a \pm = 20897688.176 \text{ ft}$

Determine chord correction:

$$c = \frac{L_c^3}{24R_a^2}$$

$c = 199988.985^3/[(24)\ (20897688.176^2)]$

use geodetic length as approximate ellipsoidal chord

$c = 0.763$ ft

Determine ellipsoidal chord length:

$s = L_c + c$

$L_c + 0.763$ ft = 199988.985 ft

$L_c = 199988.222$ ft

Determine ellipsoidal reduction factor:

$$r_e = \frac{R_a}{(R_a + N + H)}$$

$r_e = 20897688.176$ ft/(20897688.176 ft + 0 + 3800)

$r_e = 0.99981819476$

Determine level ground length:

$L_c = r_e D$

199988.222 ft = 0.99981819476 x D

D = 199988.222 ft/0.99981819476

D = 200024.587

Appendix 1

State of California Public Resources Code Division 8. Surveying and Mapping

Chapter 1. California Coordinate System

Section 8801. California Coordinate System Defined

(a) The system of plane coordinates which has been established by the United States Coast and Geodetic Survey for defining and stating the positions or locations of points on the surface of the earth within the State of California is based on the North American Datum of 1927 and is identified as the "California Coordinate System." After January 1, 1987, this system shall be known as the "California Coordinate System of 1927."

(b) The system of plane coordinates which has been established by the National Geodetic Survey for defining and stating the positions or locations of points on the surface of the earth within the State of California and which is based on the North American Datum of 1983 shall be known as the "California Coordinate System of 1983."

(c) As used in this chapter:
(1) "NAD27" means the North American Datum of 1927.
(2) "CCS27" means the California Coordinate System of 1927.
(3) "NAD83" means the North American Datum of 1983.
(4) "CCS83" means the California Coordinate System of 1983.
(5) "USC&GS" means the United States Coast and Geodetic Survey.
(6) "NGS" means the National Geodetic Survey.
(7) "FGCC" means the Federal Geodetic Control Committee.

(d) The use of the term "State Plane Coordinates" refers only to CCS27 and CCS83 coordinates.

Section 8802. Delineation of Zones

For CCS27 the state is divided into seven zones. For CCS83, the state is divided into six zones. Zone 7 of CCS27, which encompasses Los Angeles County, is eliminated and the area is included in Zone 5.

Each zone of CCS27 is a Lambert conformal conic projection based on Clarke's Spheroid of 1866, which is the basis of NAD27. The points of control of zones one to six, inclusive, bear the coordinates: Northing (y) = 000.00 feet and Easting (x) = 2,000,000 feet. The point of control of Zone 7 bears the coordinates: Northing (y) = 4,160,926.74 feet and Easting (x) = 4,186,692.58 feet.

Each zone of CCS83 is a Lambert conformal conic projection based on the Geodetic Reference System of 1980, which is the basis of NAD83. The point of control of each of the six zones bear the coordinates: Northing (y) = 500,000 meters and Easting (x) = 2,000,000 meters.

The area included in the following counties constitutes Zone 1 of CCS27 and CCS83: Del Norte, Humboldt, Lassen, Modoc, Plumas, Shasta, Siskiyou, Tehama, and Trinity.

The area included in the following counties constitutes Zone 2 of CCS27 and CCS83: Alpine, Amador, Butte, Colusa, El Dorado, Glenn, Lake, Mendocino, Napa, Nevada, Placer, Sacramento, Sierra, Solano, Sonoma, Sutter, Yolo, Yuba.

The area included in the following counties constitutes Zone 3 of CCS27 and CCS83: Alameda, Calaveras, Contra Costa, Madera, Marin, Mariposa, Merced, Mono, San Francisco, San Joaquin, San Mateo, Santa Clara, Santa Cruz, Stanislaus, and Tuolumne.

The area included in the following counties constitutes Zone 4 of CCS27 and CCS83: Fresno, Inyo, Kings, Monterey, San Benito, and Tulare.

The area included in the following counties and Channel Islands constitutes Zone 5 of CCS27: Kern, San Bernardino, San Luis Obispo, Santa Barbara (excepting Santa Barbara Island), and Ventura (excepting San Nicholas Island) and the Channel Islands of Santa Cruz, Santa Rosa, San Miguel, and Anacapa.

The area included in the following counties and Channel Islands constitutes Zone 5 of CCS83: Kern, Los Angeles (excepting San Clemente and Santa Catalina Islands), San Bernardino, San Luis Obispo, Santa Barbara (excepting Santa Barbara Island), and Ventura excepting San Nicholas Island) and the Channel Islands of Santa Cruz, Santa Rosa, San Miguel, and Anacapa.

The area included in the following counties and Channel Islands constitutes Zone 6 of CCS27 and CCS83: Imperial, Orange, Riverside, and San Diego and the Channel Islands of San Clemente, Santa Catalina, Santa Barbara, and San Nicholas.

The area included in Los Angeles County constitutes Zone 7 of CCS27.

Section 8803. Definition of Zone 1

Zone 1 coordinates shall be named, and, on any map on which they are used, they shall be designated as "CCS27, Zone 1 or CCS83, Zone 1."

On their respective spheroids of reference: (1) the standard parallels of CCS27, Zone 1 and CCS83, Zone 1 are at north latitudes 40 degrees 00 minutes and 41 degrees 40 minutes, along which parallels the scale shall be exact; and (2) the point of control of coordinates is at the intersection of the zone's central meridian, which is at 122 degrees 00 minutes west longitude, with the parallel 39 degrees 20 minutes north latitude.

Section 8804. Definition of Zone 2

Zone 2 coordinates shall be named, and, on any map on which they are used, they shall be designated as "CCS27, Zone 2 or CCS83, Zone 2."

On their respective spheroids of reference: (1) the standard parallels of CCS27, Zone 2 and CCS83, Zone 2 are at north latitudes 38 degrees 20 minutes and 39 degrees 50 minutes, along which parallels the scale shall be exact; and (2) the point of control of coordinates is at the intersection of the zone's central meridian, which is at 122 degrees 00 minutes west longitude, with the parallel 37 degrees 40 minutes north latitude.

Section 8805. Definition of Zone 3

Zone 3 coordinates shall be named, and, on any map on which they are used, they shall be designated as "CCS27, Zone 3 or CCS83, Zone 3."

On their respective spheroids of reference: (1) the standard parallels of CCS27, Zone 3 and CCS83, Zone 3 are at north latitudes 37 degrees 04 minutes and 38 degrees 26 minutes, along which parallels the scale shall be exact; and (2) the point of control of coordinates is at the intersection of the zone's central meridian, which is at 120 degrees 30 minutes west longitude, with the parallel 36 degrees 30 minutes north latitude.

Section 8806. Definition of Zone 4

Zone 4 coordinates shall be named, and, on any map on which they are used, they shall be designated as "CCS27, Zone 4 or CCS83, Zone 4."

On their respective spheroids of reference: (1) the standard parallels of CCS27, Zone 4 and CCS83, Zone 4 are at north latitudes 36 degrees 00 minutes and 37 degrees 15 minutes, along which parallels the scale shall be exact; and (2) the point of control of coordinates is at the intersection of the zone's central meridian, which is at 119 degrees 00 minutes west longitude, with the parallel 35 degrees 20 minutes north latitude.

Section 8807. Definition of Zone 5

Zone 5 coordinates shall be named, and, on any map on which they are used, they shall be designated as “CCS27, Zone 5 or CCS83, Zone 5.”

On their respective spheroids of reference: (1) the standard parallels of CCS27, Zone 5 and CCS83, Zone 5 are at north latitudes 34 degrees 02 minutes and 35 degrees 28 minutes, along which parallels the scale shall be exact; and (2) the point of control of coordinates is at the intersection of the zone’s central meridian, which is at 118 degrees 00 minutes west longitude, with the parallel 33 degrees 30 minutes north latitude.

Section 8808. Definition of Zone 6

Zone 6 coordinates shall be named, and, on any map on which they are used, they shall be designated as “CCS27, Zone 6 or CCS83, Zone 6.”

On their respective spheroids of reference: (1) the standard parallels of CCS27, Zone 6 and CCS83, Zone 6 are at north latitudes 32 degrees 47 minutes and 33 degrees 53 minutes, along which parallels the scale shall be exact; and (2) the point of control of coordinates is at the intersection of the zone’s central meridian, which is at 116 degrees 15 minutes west longitude, with the parallel 32 degrees 10 minutes north latitude.

Section 8809. Definition of Zone 7

Zone 7 coordinates shall be named, and, on any map on which they are used, they shall be designated as “CCS27, Zone 7.”

On its respective spheroid of reference: (1) the standard parallels of CCS27, Zone 7 are at north latitudes 33 degrees 52 minutes and 34 degrees 25 minutes, along which parallels the scale shall be exact; and (2) the point of control of coordinates is at the intersection of the zone’s central meridian, which is at 118 degrees 20 minutes west longitude, with the parallel 34 degrees 08 minutes north latitude.

Section 8810. Definition of U.S. Survey Foot and Coordinates

The plane coordinates of a point on the earth’s surface, to be used in expressing the position or location of the point in the appropriate zone of CCS27 or CCS83, shall consist of two distances, expressed in feet and decimals of a foot or meters and decimals of a meter. When the values are expressed in feet, the “U.S. Survey foot,” (one foot = 1200/3937 meters) shall be used as the standard foot for CCS27 and CCS83. One of these distances, to be known as the “East x-coordinate,” shall give the distance east of the Y axis; the other, to be known as the “North y-coordinate,” shall give the distance north of the X axis. The Y axis of any zone shall be parallel with the central meridian of that zone. The X axis of any zone shall be at right angles to the central meridian of that zone.

Section 8811. Sources of Plane Coordinates

The state plane coordinates of a point in any zone shall be based upon the plane coordinates of published horizontal control stations or derived from published horizontal control stations of the USC&GS and the NGS or their successors.

Section 8813. Requires Accuracy of Stations and Data Required

The CCS27 and CCS83 shall be based on monumented first- and second-order stations which have been published by USC&GS and NGS or their successors. The geodetic positions of CCS27 and CCS83 stations which are used to increase the density of control and which purport to be of first- or second-order accuracy shall have been surveyed in conformity with first- or second-order survey standards and specifications in effect at the time of the survey as defined by the Federal Geodetic Control Committee. Any survey or map which is to be based on state plane coordinates shall show established field-measured connections to at least two stations of corresponding accuracy or better whose credentials are based upon published stations of USC&GS or NGS or their successors. If an FGCC order of accuracy is claimed for a survey or a map, it shall be justified by additional written data that shows equipment, procedures, closures, adjustments, and a control diagram.

Section 8814. Use of Coordinates and Constructive Notice

State plane coordinates may be used for property identification on any map, survey, conveyance, or other instrument which delineates or affects the title to real property or which delineates, describes, or refers to the property, or any part thereof. However, to constitute, when recorded, constructive notice thereof under the recording laws, the delineating, describing, or referring to the property, or part thereof, shall also refer to data appearing of record in any office, the records of which constitute constructive notice under the recording laws. That record data shall be sufficient to identify the property without recourse to those coordinates, and in case of conflict between them, the references to that recorded data shall be controlling for the purpose of determining constructive notice under the recording laws.

Section 8815. Identification of California Coordinate System

The use of the term “California Coordinate System” on any map or document or any field notes shall be suffixed either with “27” (shown as “CCS27”) for coordinates based on NAD27 or with “83” (shown as “CCS83”) for coordinates based on NAD83.

Section 8816. Use Optional

The use of the State Plane Coordinates by any person, corporation, or governmental agency engaged in land surveying or mapping is optional.

Section 8817. CCS83 Use After January 1, 1995

Prior to January 1, 1995, use of State Plane Coordinates for new projects may be based either on CCS27 or CCS83. On or after January 1, 1995, when State Plane Coordinates are used on new surveys and new mapping projects, the use shall be limited to CCS83.

Section 8818. Land Titles Referred to CCS27

This chapter does not impair or invalidate land titles, legal descriptions, or jurisdictional or land boundaries and, further, this chapter does not impair or invalidate references to, or the use of, CCS27 coordinates, except as provided in Section 8817.

Section 8819. Use of New Technologies

This chapter does not prohibit the use of new geodetic surveying technologies for which FGCC specifications have not yet been published, except that if first- or second-order accuracy is claimed for any of the resulting monumented stations, the state plane coordinates shall conform to FGCC accuracy standards.

Appendix 2

Symbols and Notations

α = geodetic azimuth, the clockwise angle between geodetic north and the geodetic line to the object observed

γ = the plane convergence angle, the major component of the difference between geodetic azimuth and grid azimuth, also sometimes called mapping angle

δ = arc to chord correction, also known as second term or "t - T" (A correction applied to long lines of precise surveys to compensate for distortion of straight lines when projected onto the grid. This correction is usually minute and can be neglected for most courses under five miles long.)

a = 6378137 m (exact) or 20925604.4742 ft, the equatorial radius of the GRS80 ellipsoid

b = 6356752.314140347 m = 20855444.8840 ft = the semiminor axis of the GRS80 ellipsoid

B = north geodetic latitude of a station, also noted as ø

B_b = north geodetic latitude of the parallel passing through grid origin, a tabled constant

B_n = north geodetic latitude of the northerly standard parallel where the cone intersects the ellipsoid (Line of exact scale)

B_o = the latitude of the central parallel passing through the projection origin, a tabled constant for each zone, also noted as $\_{o}$

B_s = north geodetic latitude of the southerly standard parallel where the cone intersects the ellipsoid (Line of exact scale)

cf = combined factor for simultaneously applying average ellipsoidal reduction and scale factors

D = ground level, horizontal measured distance

E_o = easting of projection origin and central meridian, 6561666.667 ft or 2000000.0000 m for all zones

e^2 = 0.006694380022903416 = the square of the first eccentricity of the GRS80 ellipsoid

e ft^2 = 0.006739496775481622 = the square of the second eccentricity of the GRS80 ellipsoid

F_1, F_2, F_3 = polynomial coefficients tabled with the zone constants

G_1, G_2, G_3, G_4 = polynomial coefficients for inverse conversion, tabled with the zone constants

h = geodetic height, elevation using the ellipsoid for its datum. Related to MSL datum by the formula, $h = N + H$

H = elevation using the geoid for its datum, this is approximately elevation based on mean sea level

k = point grid scale factor

K = mapping radius on the cone at the equatorial plane of the ellipsoid

k_o = grid scale factor of the central parallel, B_o, a tabled constant

L = west geodetic longitude of station, also noted as λ

L_c = ellipsoidal chord length

$L_{(grid)}$ = grid length, distance between two points on the grid plane

L_o = longitude of the central meridian passing through the projection and grid origin, a tabled constant, also noted as λ_o

L_s = measured slope length

L_1, L_2, L_3, L_4 = polynomial coefficients for direct computation, tabled with the zone constants

M_o = radius of curvature of the ellipsoid in the meridian at the projection origin, scaled to the grid

N = geoid separation or height, the distance at a station from the geoid to the ellipsoid, it is negative within the contiguous 48 states

N_b = northing of grid base, a tabled constant equalling 1640416.667 ft or 500000.0000 m for all zones

N_o = northing of the projection origin, a tabled constant

p = $1/f = \text{flattening}^{-1} = 298.2572221008827$ for GRS80

R = mapping radius through a station

R_a = radius of curvature of the ellipsoid in the azimuth

R_b = mapping radius through the grid base, a tabled constant

r_e = ellipsoidal reduction factor, also known as elevation factor

r_o = geometric mean radius of the ellipsoid at the projection origin, scaled to the grid

R_o = mapping radius through the projection origin, a tabled constant

s = geodetic length, the ellipsoidal arc length of a line

$\sin B_o$ = sine of the latitude of the projection origin, which is also the ratio between γ and longitude in decimals of a degree, a tabled constant

t = grid azimuth, the clockwise angle at a station between the grid meridian (grid north) and the grid line to the observed object (All grid meridians are straight and parallel. Grid azimuths are related to geodetic azimuths by the formula, $t = a - \gamma + \delta$>≥

T = projected geodetic azimuth (Azimuth of a straight line of the ellipsoid when projected onto the grid is slightly curved.)

u = radial distance on the projection from the station to the central parallel, $R_o - R$

Constants for the Geodetic Reference System of 1980 GRS80 Ellipsoid

a = 6378137 m (exact) = 20925604.4742 ft = the equatorial radius of the ellipsoid

b = 6356752.314140347 m = 20855444.8840 ft = the semiminor axis

p = 1/f = 298.2572221008827 = flattening^{-1}

e^2 = 0.006694380022903416 = the square of the first eccentricity

e ft^2 = 0.006739496775481622 = the square of the second eccentricity

Appendix 3

North American Datum 1983 (NAD83) – California Coordinate System 1983 (CCS83)

CALIFORNIA ZONE 1, CA01, ZONE# 0401

Meters			US Survey Feet		
B_s	=	40° 00' N	B_s	=	40° 00' N
B_n	=	41° 40' N	B_n	=	41° 40' N
B_b	=	39° 20' N	B_b	=	39° 20' N
L_o	=	122° 00' W	L_o	=	122° 00' W
N_b	=	500000.0000 m	N_b	=	1640416.667 ft
E_o	=	2000000.0000 m	E_o	=	6561666.667 ft
B_o	=	40.8351061249° N	B_o	=	40.8351061249° N
$SinB_o$	=	0.653884305400	$SinB_o$	=	0.653884305400
R_b	=	7556554.6408 m	R_b	=	24791796.351 ft
R_o	=	7389802.0597 m	R_o	=	24244708.924 ft
N_o	=	666752.5811 m	N_o	=	2187504.093 ft
K	=	12287826.3052 m	K	=	40314310.136 ft
k_o	=	0.999894636561	k_o	=	0.999894636561
M_o	=	6362067.2798 m	M_o	=	20872882.401 ft
r_o	=	6374328. m	r_o	=	20913107.780 ft
L_1	=	111039.0203	L_1	=	364300.5191
L_2	=	9.65524	L_2	=	31.6772
L_3	=	5.63491	L_3	=	18.4872
L_4	=	0.021275	L_4	=	0.069800
G_1	=	9.005843038E-06	G_1	=	2.744986448E-06
G_2	=	-7.05240E-15	G_2	=	-6.55192E-16
G_3	=	-3.70393E-20	G_3	=	-1.04884E-21
G_4	=	-1.1142E-27	G_4	=	-9.6167E-30
F_1	=	0.999894636561	F_1	=	0.999894636561
F_2	=	1.23062E-14	F_2	=	1.14329E-15
F_3	=	5.47E-22	F_3	=	1.55E-23

The customary limits of the zone are from 39° 20' N to 42° 20' N.

North American Datum 1983 (NAD83) – California Coordinate System 1983 (CCS83)

CALIFORNIA ZONE 2, CA02, ZONE# 0402

Meters			US Survey Feet		
B_s	=	38° 20' N	B_s	=	38° 20' N
B_n	=	39° 50' N	B_n	=	39° 50' N
B_b	=	37° 40' N	B_b	=	37° 40' N
L_o	=	122° 00' W	L_o	=	122° 00' W
N_b	=	500000.0000 m	N_b	=	1640416.667 ft
E_o	=	2000000.0000 m	E_o	=	6561666.667 ft
B_o	=	39.0846839219° N	B_o	=	39.0846839219° N
$SinB_o$	=	0.630468335285	$SinB_o$	=	0.630468335285
R_b	=	8019788.9307 m	R_b	=	26311590.850 ft
R_o	=	7862381.4027 m	R_o	=	25795162.985 ft
N_o	=	657407.5280 m	N_o	=	2156844.531 ft
K	=	12520351.6538 m	K	=	41077187.051 ft
k_o	=	0.999914672977	k_o	=	0.999914672977
M_o	=	6360268.3937 m	M_o	=	20866980.555 ft
r_o	=	6373169. m	r_o	=	20909305.294 ft
L_1	=	111007.6240	L_1	=	364197.5131
L_2	=	9.54628	L_2	=	31.3198
L_3	=	5.63874	L_3	=	18.4998
L_4	=	0.019988	L_4	=	0.065577
G_1	=	9.008390180E-06	G_1	=	2.745762818E-06
G_2	=	-6.97872E-15	G_2	=	-6.48347E-16
G_3	=	-3.71084E-20	G_3	=	-1.05080E-21
G_4	=	-1.0411E-27	G_4	=	-8.9858E-30
F_1	=	0.999914672977	F_1	=	0.999914672977
F_2	=	1.23106E-14	F_2	=	1.14370E-15
F_3	=	5.14E-22	F_3	=	1.46E-23

The customary limits of the zone are from 37° 40' N to 40° 30' N.

North American Datum 1983 (NAD83) – California Coordinate System 1983 (CCS83)

CALIFORNIA ZONE 3, CA03, ZONE# 0403

Meters			US Survey Feet		
B_s	=	37° 04' N	B_s	=	37° 04' N
B_n	=	38° 26' N	B_n	=	38° 26' N
B_b	=	36° 30' N	B_b	=	36° 30' N
L_o	=	120° 30' W	L_o	=	120° 30' W
N_b	=	500000.0000 m	N_b	=	1640416.667 ft
E_o	=	2000000.0000 m	E_o	=	6561666.667 ft
B_o	=	37.7510694363° N	B_o	=	37.7510694363° N
$SinB_o$	=	0.612232038295	$SinB_o$	=	0.612232038295
R_b	=	8385775.1723 m	R_b	=	27512330.711 ft
R_o	=	8246930.3684 m	R_o	=	27056804.050 ft
N_o	=	638844.8039 m	N_o	=	2095943.327 ft
K	=	12724574.9735 m	K	=	41747209.726 ft
k_o	=	0.999929178853	k_o	=	0.999929178853
M_o	=	6358909.6841 m	M_o	=	20862522.855 ft
r_o	=	6372292. m	r_o	=	20906428.003 ft
L_1	=	110983.9104	L_1	=	364119.7127
L_2	=	9.43943	L_2	=	30.9692
L_3	=	5.64142	L_3	=	18.5086
L_4	=	0.019048	L_4	=	0.062493
G_1	=	9.010315015E-06	G_1	=	2.746349509E-06
G_2	=	-6.90503E-15	G_2	=	-6.41501E-16
G_3	=	-3.71614E-20	G_3	=	-1.05230E-21
G_4	=	-9.8819E-28	G_4	=	-8.5291E-30
F_1	=	0.999929178853	F_1	=	0.999929178853
F_2	=	1.23137E-14	F_2	=	1.14398E-15
F_3	=	4.89E-22	F_3	=	1.38E-23

The customary limits of the zone are from 36° 30' N to 39° 00' N.

North American Datum 1983 (NAD83) – California Coordinate System 1983 (CCS83)

CALIFORNIA ZONE 4, CA04, ZONE# 0404

Meters			US Survey Feet		
B_s	=	36° 00' N	B_s	=	36° 00' N
B_n	=	37° 15' N	B_n	=	37° 15' N
B_b	=	35° 20' N	B_b	=	35° 20' N
L_o	=	119° 00' W	L_o	=	119° 00' W
N_b	=	500000.0000 m	N_b	=	1640416.667 ft
E_o	=	2000000.0000 m	E_o	=	6561666.667 ft
B_o	=	36.6258593071° N	B_o	=	36.6258593071° N
$SinB_o$	=	0.596587149880	$SinB_o$	=	0.596587149880
R_b	=	8733227.3793 m	R_b	=	28652263.494 ft
R_o	=	8589806.8935 m	R_o	=	28181724.783 ft
N_o	=	643420.4858 m	N_o	=	2110955.377 ft
K	=	12916986.0281 m	K	=	42378478.327 ft
k_o	=	0.999940761703	k_o	=	0.999940761703
M_o	=	6357772.8978 m	M_o	=	20858793.249 ft
r_o	=	6371557. m	r_o	=	20904016.591 ft
L_1	=	110964.0696	L_1	=	364054.6183
L_2	=	9.33334	L_2	=	30.6211
L_3	=	5.64410	L_3	=	18.5174
L_4	=	0.018382	L_4	=	0.060308
G_1	=	9.011926076E-06	G_1	=	2.746840562E-06
G_2	=	-6.83121E-15	G_2	=	-6.34643E-16
G_3	=	-3.72043E-20	G_3	=	-1.05351E-21
G_4	=	-9.4223E-28	G_4	=	-8.1324E-30
F_1	=	0.999940761703	F_1	=	0.999940761703
F_2	=	1.23168E-14	F_2	=	1.14427E-15
F_3	=	4.70E-22	F_3	=	1.33E-23

The customary limits of the zone are from 35° 20' N to 38° 00' N.

North American Datum 1983 (NAD83) – California Coordinate System 1983 (CCS83)

CALIFORNIA ZONE 5, CA05, ZONE# 0405

Meters			US Survey Feet		
B_s	=	34° 02' N	B_s	=	34° 02' N
B_n	=	35° 28' N	B_n	=	35° 28' N
B_b	=	33° 30' N	B_b	=	33° 30' N
L_o	=	118° 00' W	L_o	=	118° 00' W
N_b	=	500000.0000 m	N_b	=	1640416.667 ft
E_o	=	2000000.0000 m	E_o	=	6561666.667 ft
B_o	=	34.7510553142° N	B_o	=	34.7510553142° N
$SinB_o$	=	0.570011896174	$SinB_o$	=	0.570011896174
R_b	=	9341756.1389 m	R_b	=	30648744.932 ft
R_o	=	9202983.1099 m	R_o	=	30193453.753 ft
N_o	=	638773.0290 m	N_o	=	2095707.846 ft
K	=	13282624.8345 m	K	=	43578078.311 ft
k_o	=	0.999922127209	k_o	=	0.999922127209
M_o	=	6355670.9697 m	M_o	=	20851897.173 ft
r_o	=	6370113. m	r_o	=	20899279.068 ft
L_1	=	110927.3840	L_1	=	363934.2590
L_2	=	9.12439	L_2	=	29.9356
L_3	=	5.64805	L_3	=	18.5303
L_4	=	0.017445	L_4	=	0.057234
G_1	=	9.014906468E-06	G_1	=	2.747748987E-06
G_2	=	-6.68534E-15	G_2	=	-6.21091E-16
G_3	=	-3.72796E-20	G_3	=	-1.05565E-21
G_4	=	-8.6394E-28	G_4	=	-7.4567E-30
F_1	=	0.999922127209	F_1	=	0.999922127209
F_2	=	1.23221E-14	F_2	=	1.14477E-15
F_3	=	4.41E-22	F_3	=	1.25E-23

The customary limits of the zone are from 33° 30' N to 36° 20' N.

North American Datum 1983 (NAD83) – California Coordinate System 1983 (CCS83)

CALIFORNIA ZONE 6, CA06, ZONE# 0406

Meters			US Survey Feet		
B_s	=	32° 47' N	B_s	=	32° 47' N
B_n	=	33° 53' N	B_n	=	33° 53' N
B_b	=	32° 10' N	B_b	=	32° 10' N
L_o	=	116° 15' W	L_o	=	116° 15' W
N_b	=	500000.0000 m	N_b	=	1640416.667 ft
E_o	=	2000000.0000 m	E_o	=	6561666.667 ft
B_o	=	33.3339229447° N	B_o	=	33.3339229447° N
$SinB_o$	=	0.549517575763	$SinB_o$	=	0.549517575763
R_b	=	9836091.7896 m	R_b	=	32270577.813 ft
R_o	=	9706640.0762 m	R_o	=	31845868.317 ft
N_o	=	629451.7134 m	N_o	=	2065126.163 ft
K	=	13602026.7133 m	K	=	44625982.642 ft
k_o	=	0.999954142490	k_o	=	0.999954142490
M_o	=	6354407.2007 m	M_o	=	20847750.958 ft
r_o	=	6369336. m	r_o	=	20896729.860 ft
L_1	=	110905.3274	L_1	=	363861.8950
L_2	=	8.94188	L_2	=	29.3368
L_3	=	5.65087	L_3	=	18.5396
L_4	=	0.016171	L_4	=	0.053054
G_1	=	9.016699372E-06	G_1	=	2.748295465E-06
G_2	=	-6.55499E-15	G_2	=	-6.08981E-16
G_3	=	-3.73318E-20	G_3	=	-1.05713E-21
G_4	=	-8.2753E-28	G_4	=	-7.1424E-30
F_1	=	0.999954142490	F_1	=	0.999954142490
F_2	=	1.23251E-14	F_2	=	1.14504E-15
F_3	=	4.15E-22	F_3	=	1.18E-23

The customary limits of the zone are from 32° 10' N to 34° 30' N.

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State Plane Coordinate System of 1983 by James E. Stem is an excellent source of information concerning NAD83 procedures for the surveyor. It thoroughly deals with corrections to lengths and azimuths. It and other publications are available by phone. The National Geodetic Information Center number for phone orders is currently (301) 443-8631. They accept credit cards and have a listing of their publications which they will mail free of charge.