
UNIT 8

# AREAS AND VOLUMES

**Don Hunter, PLS**
**California Department of Transportation**

## Introduction

Area is the surface within a set of lines, expressed in square units, as in "square feet." The area within a triangle is the surface within three sides. The area of a circle is the surface within the circumference. One of the objectives of land surveys is to determine the area of a parcel of land. There are a number of methods used for calculating areas, including the use of double meridian distances and coordinates.

Volume is defined as the amount of material occupying a certain space, expressed in cubic units, as in "cubic yards." Data obtained from engineering design surveys are used to calculate earthwork quantities. The standard method for calculating earthwork quantities is the average end area method.

# Performance Expected on the Exam

Calculate the areas of circles, triangles, quadrilaterals and regular polygons.

Calculate the areas of circular sectors and segments.

Calculate areas using the trapezoidal rule and Simpsons 1/3 rule.

Calculate the areas of irregular polygons by double meridian distance and coordinate methods.

Calculate volume by average end area method.

# Key Terms

Sector
Segment
Polygon
Regular polygon
Quadrilateral
Parallelogram
Trapezoid
Trapezium
Trapezoidal rule
Simpson's 1/3 rule
Latitudes
Departures
Northings
Eastings
DMD
Prism
End area

# Video Presentation Outline

## Standard Geometric Figures and Area Formulas

- Circle, sector, segment
- Polygon
- Triangle
- Quadrilateral

## Irregular Polygons with More than Four Sides and Curved Sides

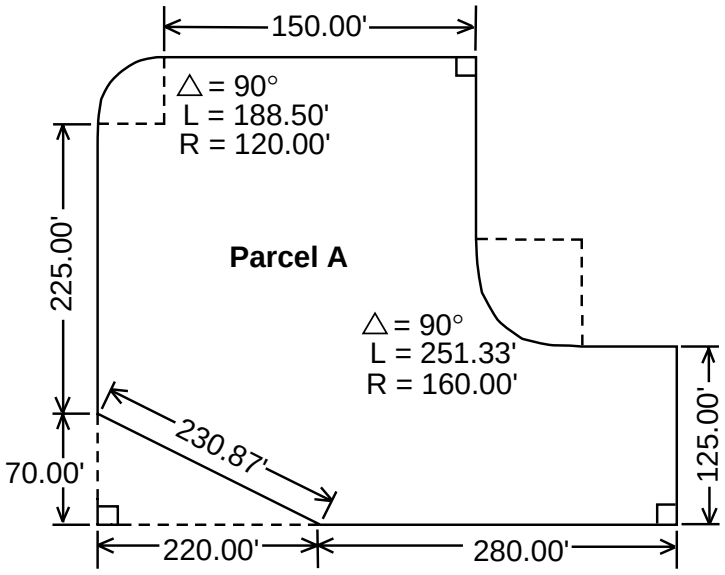



***Figure 8-1.*** *Division into simple geometric shapes.*

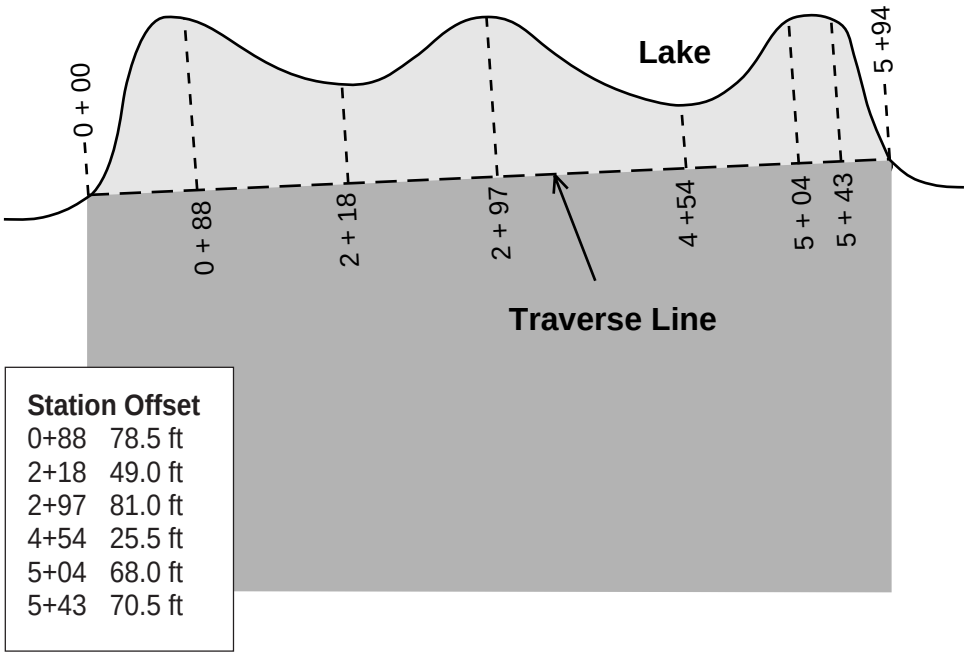
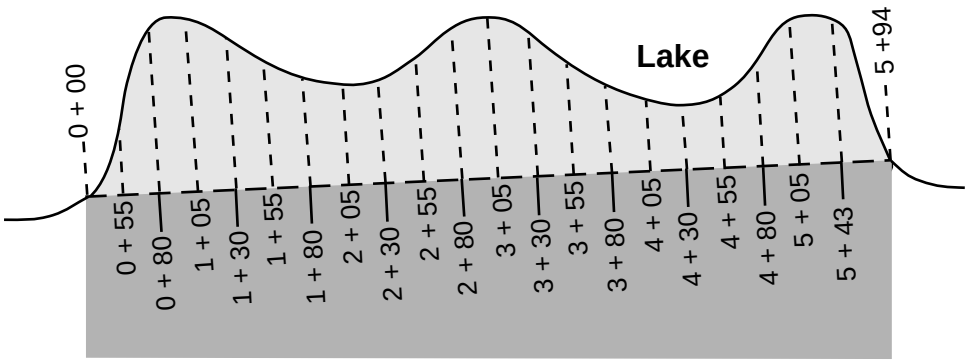



***Figure 8-2.*** *Offsets to traverse line at irregular intervals.*




| Station | Offset | Station | Offset |
|---|---|---|---|
| 0+55 | 52.5 ft | 3+05 | 79.5 ft |
| 0+80 | 88.0 ft | 3+30 | 69.0 ft |
| 1+05 | 87.0 ft | 3+55 | 58.5 ft |
| 1+30 | 78.5 ft | 3+80 | 50.0 ft |
| 1+55 | 69.5 ft | 4+05 | 40.5 ft |
| 1+80 | 59.0 ft | 4+30 | 29.5 ft |
| 2+05 | 50.5 ft | 4+55 | 27.0 ft |
| 2+30 | 52.5 ft | 4+80 | 46.0 ft |
| 2+55 | 65.0 ft | 5+05 | 68.0 ft |
| 2+80 | 78.0 ft | 5+43 | 71.0 ft |

***Figure 8-3.*** *Offsets to traverse line at regular intervals.*

- Trapezoidal rule

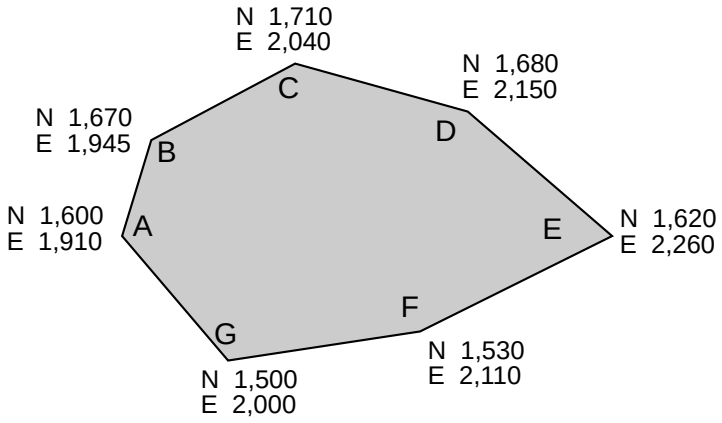
$$A = S\ (\frac{O_1}{2} + O_2 + O_3 + \cdots + \frac{O_n}{2})$$

- Simpson's 1/3 rule

$$A = \frac{S}{3}\ [O_1 + O_n + 4\ (\Sigma\ O_{even}) + 2\ (\Sigma\ O_{odd})$$

## Areas of Polygons Formed by Traverses

Determine the area of Parcel A, B, C, D, E, F, G.




- Area of a rectangle using maximum and minimum values of northings and eastings.
- Area by coordinates

$$A = \frac{1}{2}\left[N_1 (E_n - E_2) + N_2 (E_1 - E_3) + \cdots + N_n (E_{n-1} - E_1)\right]$$

- Area by DMD

$$\frac{1}{2} A = \sum \text{DMD x LAT for each line}$$

Area equals the absolute value of $\frac{1}{2} A$

Rule:

1. Start DMD calculations from the most westerly point in the traverse.
2. The DMD of the first course is equal to the departure of the first course.
3. The DMD of any course is equal to the DMD of the preceding course, plus the departure of the preceding course, plus the departure of the course itself. Alegbraic signs must be considered.
4. The DMD of the last course is equal to the departure of the last course with the opposite sign.

## Volumes

- Parrallelopipeds
- Prisms
- Volume by average end area

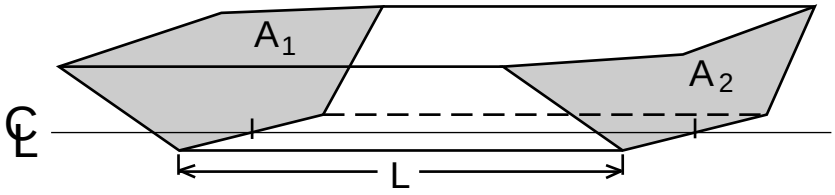
Volume in cu yd

$$2V = \frac{L\,(A_1 + A_2)}{27}$$

Where:

L = Distance between end areas, $A_1$ and $A_2$, in ft

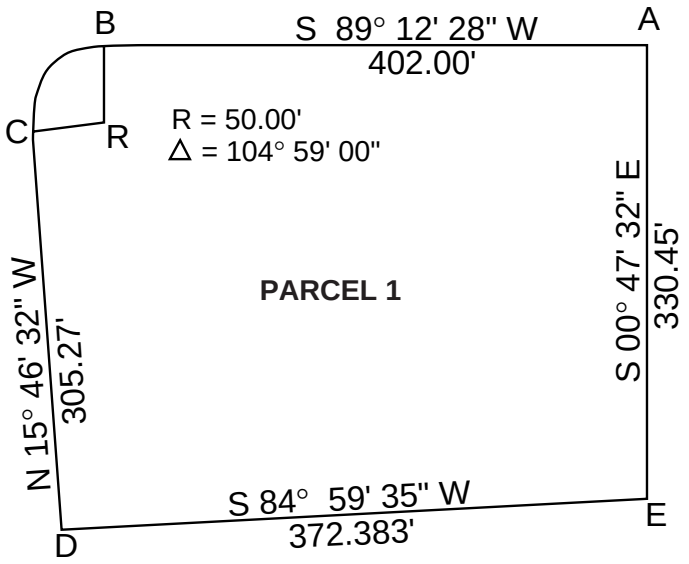
$A_1$ $A_2$ = end areas



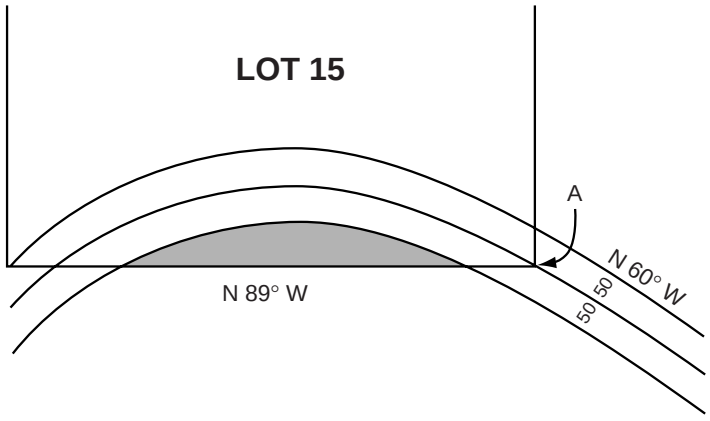

# Sample Test Questions

1. The area of a parcel shown on a map is 6800 sq ft. The map has a note stating “the bearings distances and coordinates are based on California Coordinate System, 1927, Zone 2.” The combination factor is O.999567. What is the actual ground area?
2. The corners of a parcel of land have the following coordinate values: (1) N 6472.88, E 7939.12; (2) N 6538.55, E 7802.41; (3) N 8447.65, E 8117.64 (4) N 8457.11, E 9190.96; (5) N 7128.07, E 9201.77; (6) N 7116.88, E 7933.88. What is the area, in acres, of the parcel?
3. The area on a map with a scale of 1:12000 as measured with a planimeter is 2.54 sq in. What is the area in acres?

4. Calculate the area of Parcel 1.




5. A highway, R/W 100 ft wide, is to be acquired from the owner of Lot 15 as shown on the sketch. Point "A" is the centerline B. C. of a curve with a radius of 1000 ft. Point "A" is also the SE corner of Lot 15. What is the area of that portion of Lot 15 south of the R/W?



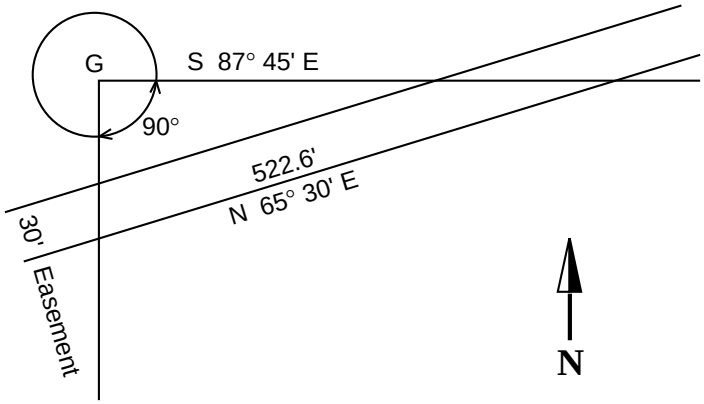
6. Problem C-4, 1983 LS

   A utility company proposes to acquire an easement across the land of a local property owner, as indicated in the plat shown below. The property owner has agreed to accept $7500 per acre in severance fees for the area that lies northwest of the northwest line of the easement. The settlement for the area within the easement is a separate consideration and it may be neglected.

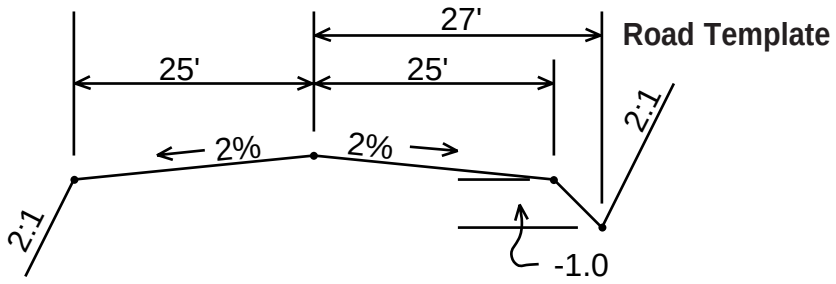
   Required:

   A. What is the area for which compensation to the landowner is appropriate?

   B. What fee is ultimately to be paid to the landowner?




7. Use the following road template and cross section notes to calculate the earth work quantity for the roadway between station 35 + 50 and 36 + 00. Centerline grade at 35 + 50 is 887.0 and at 36 + 00 is 888.5.




| | | | | | |
|---|---|---|---|---|---|
| | 93.0 / 50 | 91.6 / 25 | 890.4 / 36+00 | 90.0 / 15 | 89.0 / 50 |
| **X-Section Notes** | 92.3 / 50 | 90.0 / 20 | 889.5 / 35+50 | 90.0 / 20 | 88.0 / 50 |

# Answer Key

1. Divide the grid area by the combination factor squared.
   6806 sq ft

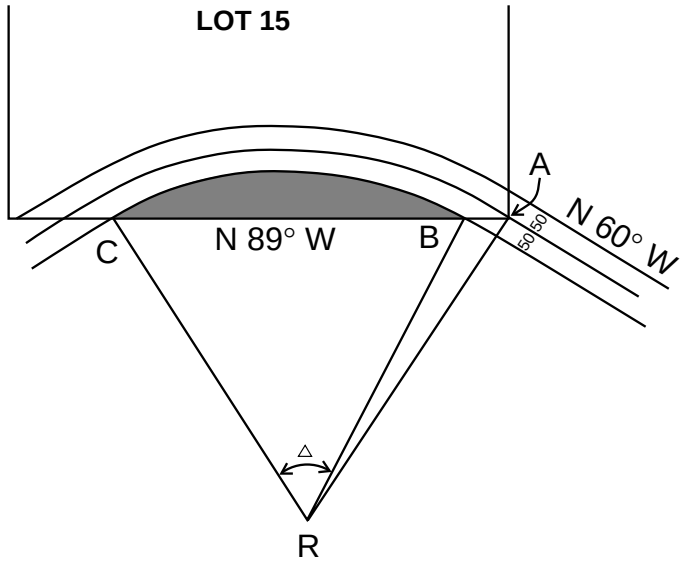
2. Use the formula for computing area by coordinates.
   37.513 acres

3. 1 in = 1000 ft scale, therefore, each sq in of the map is 1000 x 1000 ft or 1,000,000 sq ft per sq in.

   $$\frac{1{,}000{,}000 \times 2.54}{43{,}560\ \text{ft}^2} = 58.3 \text{ acres}$$

4. Calculate the area by either coordinate or DMD method for polygon ABRCDE and add the sector RBC.
   3.310 acres

5. Angle BAR = Difference in bearings between the south line of Lot 15 and the radial to A = 61°.



5. (Cont.)

By law of sines angle ABR = 112° 58' 42"

Angle ARB = 180 - (112° 58' 42" + 61°)

= 5° 21' 36"

Bearing of RB = Bearing RA - Angle ARB

= N24° 38' 24"E

Angle RBC = Difference in bearings between south line of Lot 15 and RB.

= 66° 21' 36"

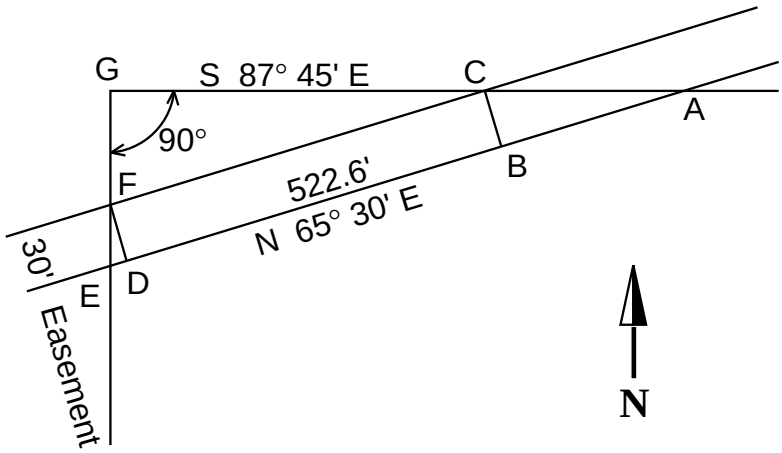
= 180° - 2 (66° 21' 36")

= 47° 16' 48"

Area of segment = $\frac{\pi r^2 \Delta}{360} - \frac{r^2 \sin^2 \Delta}{2}$

= .938 acres

6. Solve right triangles ABC and DEF for AB and DE



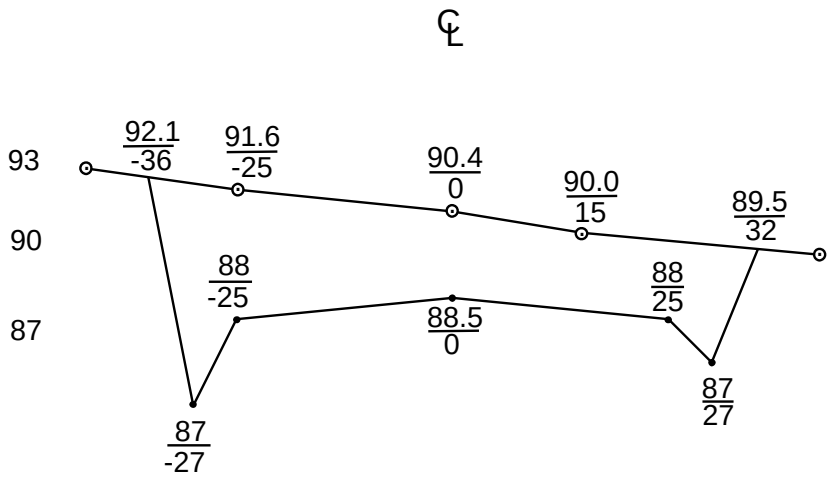

$$CF = 522.6 - AB - DE$$
$$= 522.6 - 15.12 - 59.52$$
$$= 447.96 \text{ ft}$$

A. Solve right triangle CFG for base FG and height GC

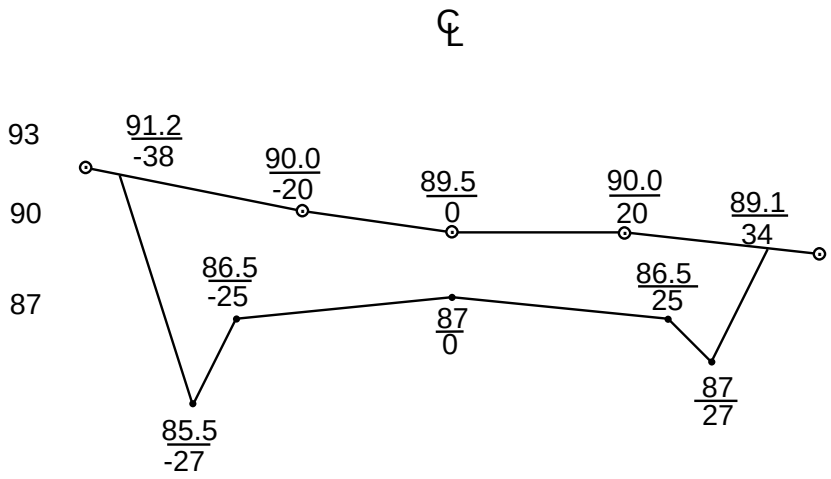
$$\text{Area} = \frac{1}{2}\left(\frac{FG \times GC}{43560}\right)$$
$$= \frac{1}{2}\left(\frac{201.63 \times 400.02}{43560}\right)$$
$$= .926 \text{ acres}$$

B. $$\text{Payment} = .926 \times \$7500$$
$$= \$6943.37$$

7. Volume to be calculated by average end area.




36 + 00




35 + 50

7. (Cont.)

First, place the road template at the correct elevation on each cross section. Generate the intersection of the 2:1 side slopes with the ground line.

The two end areas can be calculated using the coordinate method for determining the area of a polygon. Coordinate pairs are composed of elevation (N) and distance left (-) or right (+) of the centerline of the template (E).

For the end area at station 35 + 50 ($A_1$)

$$\begin{aligned}
87\ (25 - (25)) &= 4350.0 \\
86.5\ (0 - (-27)) &= 2335.5 \\
85.5\ (-25 - (-38)) &= 1111.5 \\
91.2\ (-27 - (-20)) &= -638.4 \\
90\ (-38 - 0) &= -3420.0 \\
89.5\ (-20 - 20) &= -3580.0 \\
90\ (0 - 34) &= -3060.0 \\
89.1\ (20 - 27) &= -623.0 \\
85.5\ (34 - 25) &= 783.0 \\
86.5\ (27 - 0) &= 2335.5 \\
\hline
\Sigma &= -405.90 \\
\text{Area} &= \frac{405.90}{2} \\
&= 203 \text{ sq ft}
\end{aligned}$$

End area at 36 + 00 ($A_2$) by coordinate method = 154 sq ft

$$\begin{aligned}
2 \text{ volume in cu yd} &= \frac{L\,(A_1 + A_2)}{27} \\
&= \frac{50\,(203 + 154)}{27} \\
2V &= 661 \\
V &= 330 \text{ cu yd cut}
\end{aligned}$$

# References

_____, *California Department of Transportation Survey Manual,* Appendix.

Davis, Raymond E., Francis S. Foote, et. al., *Surveying Theory and Practice,* Sixth Edition, McGraw Hill, New York, 1981.

Moffitt, Francis H. and Harry Bouchard, *Surveying,* Eighth Ed., International Textbook Co., Scranton, PA, 1987.