APPENDIX C

MEMBER PROPERTIES

CONTENTS

APPENDIX C

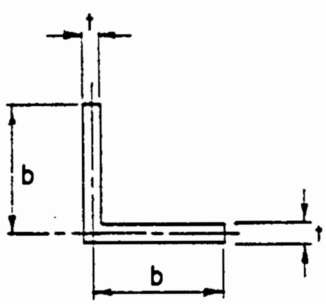
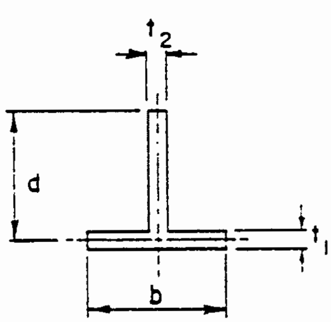
MEMBER PROPERTIES

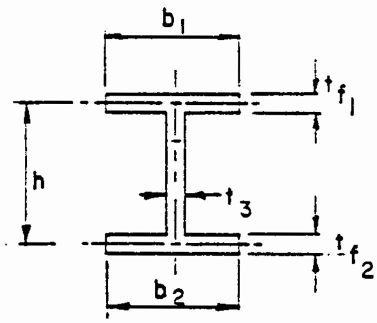
C.1 Torsional Constants

I. Formulas for Standard Sections

For the proper interaction between torsional and bending moments in a STRUDL analysis, the torsional properties of the members must be specified. The torsional rigidities IX for many standard shapes of members have been documented in many texts and will be included in this appendix for your convenience.

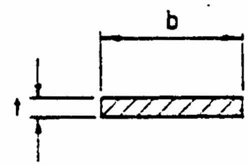
TORSIONAL CONSTANTS IX

Section	Formulas for IX in $\theta = \frac{M_x L}{IX\,G}$
General	$IX = \sum_i b_i t_i^3 / 3$
	$IX = \frac{2bt^3}{3}$
	$IX = \frac{bt_1^3 + dt_2^3}{3}$

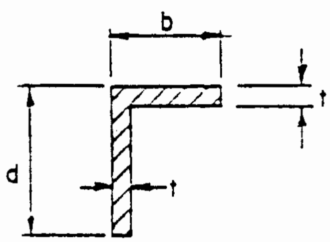


$$IX = \frac{b_1 t_{f_1}^3 + b_2 t_{f_2}^3 + h t_3^3}{3}$$

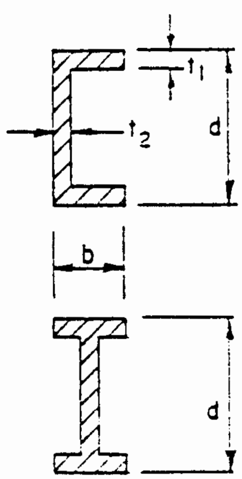
if $t_{f_1} = t_{f_2} = t_3$ $IX = \frac{t^3}{3}(b_1 + b_2 + h)$



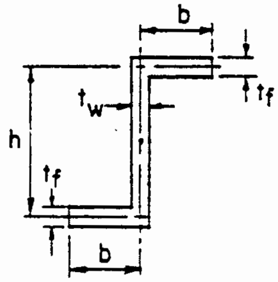
$$IX = \frac{b\ t^3}{3}$$



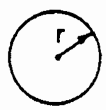
$$IX = \frac{(b + d)\ t^3}{3}$$



$$IX = \frac{2b\ t_1^3 + d\ t_2^3}{3}$$

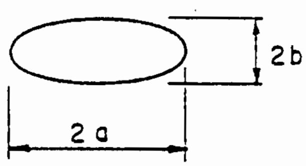


$$IX = \frac{2bt_f^3 + ht_w^3}{3}$$



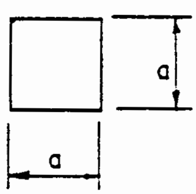
$$IX = \frac{\pi r^4}{2}$$

Solid circular



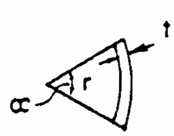
$$IX = \frac{\pi a^3 b^3}{a^2 + b^2}$$

Solid elliptical



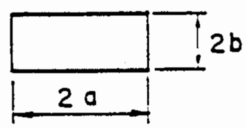
$$IX = 0.1406\ a^4$$

Solid square



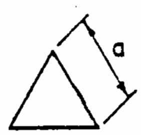
$$IX = \frac{\alpha r t^3}{3}$$

Open circular tube



$$IX = ab^3 \left[\frac{16}{3} - 3.36 \frac{b}{a} \left(1 - \frac{b^4}{12a^4}\right) \right]$$

Solid rectangle

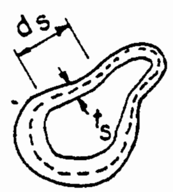


$$IX = \frac{a^4 \cdot \sqrt{3}}{80}$$

Equilateral triangle

Standard Closed Section Formulas:

$$\theta = \frac{T\,L}{E_s\,IX}$$



$$IX = \frac{4\,[A]^2}{\int \frac{d_s}{t_s}}$$

$$\tau_s = \frac{T}{2\,[A]\,t_s}$$

$$f = \frac{T}{2\,A}$$

A = area enclosed within mean dimensions.

d_s = length of particular segment of section

t_s = average thickness of segment at point (s)

τ_s = shear stress at point (s)

IX = torsional resistance, in^4

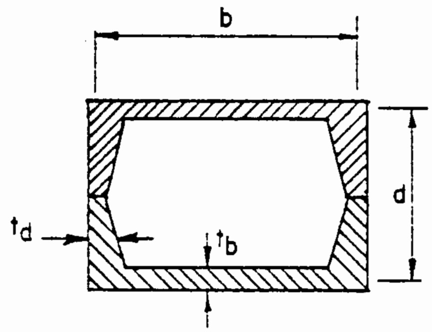
E_s = modulus of elasticity in shear

(steel = 12,000,000)

θ = angular twist (radians)

L = length of member (inches)

f = unit shear force

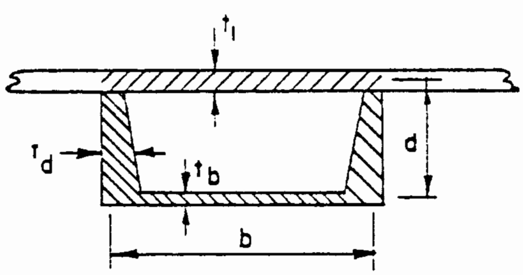


$$\int \frac{d_s}{t_s} = \frac{2b}{t_b} + \frac{2d}{t_d}$$

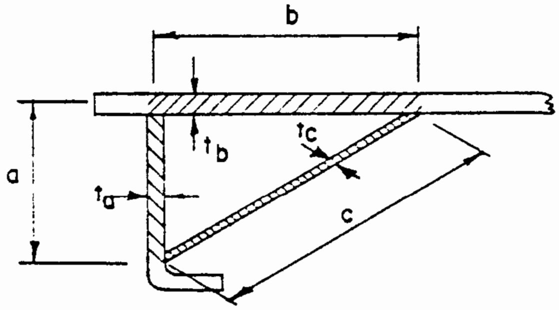
$$IX = \frac{4\,[A]^2}{\int \frac{d_s}{t_s}} = \frac{4(bd)^2}{\frac{2b}{t_b} + \frac{2d}{t_d}} = \frac{2\,b^2\,d^2}{\frac{b}{t_b} + \frac{d}{t_d}}$$

stress at ℄ of b:

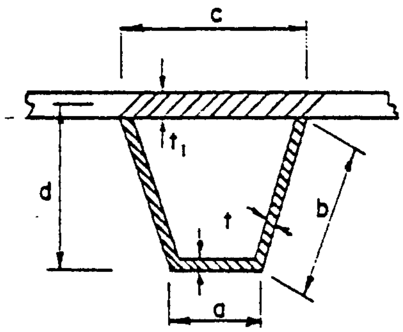
$$\tau_b = \frac{T}{2\,[A]\,t_b} = \frac{T}{2\,bd\,t_b}$$



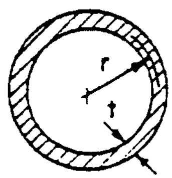
$$IX = \frac{4\,b^2\,d^2}{\frac{b}{t_b} + \frac{2d}{t_d} + \frac{b}{t_1}}$$



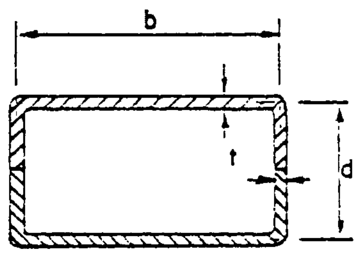
$$IX = \frac{a^2\;b^2}{\frac{a}{t_a} + \frac{b}{t_b} + \frac{c}{t_c}}$$



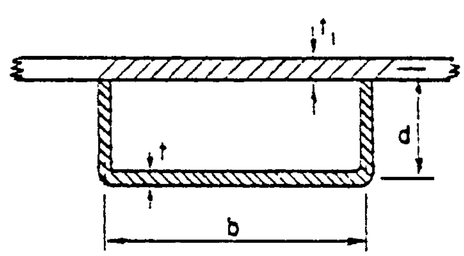
$$IX = \frac{(a + c)^2 \; d^2}{\frac{a + 2b}{t} + \frac{c}{t_1}}$$



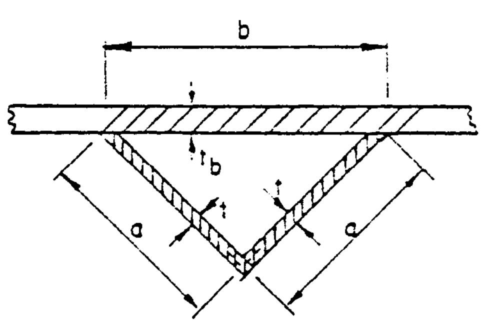
$$IX = 2 \pi r^3 t$$



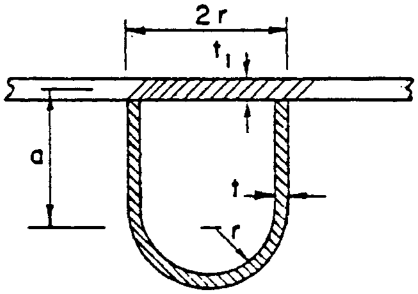
$$IX = \frac{2 t \; b^2 \; d^2}{b + d}$$



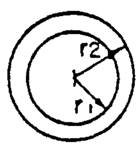
$$IX = \frac{4 \; b^2 \; d^2}{\frac{b + 2d}{t} + \frac{b}{t_1}}$$



$$IX = \frac{a^4}{\frac{2a}{t} + \frac{b}{t_b}}$$

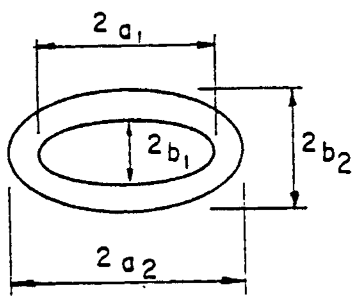


$$IX = \frac{4r^2\left(\frac{\pi r}{2} + 2a\right)}{\frac{2a + \pi r}{t} + \frac{2r}{t_1}}$$



Hollow circle

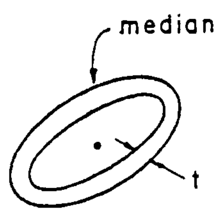
$$IX = \frac{\pi(r_2^4 - r_1^4)}{2}$$



Hollow ellipse

$$IX = \frac{\pi^3 a_2^3 b_2^3}{a_2^2 + b_2^2}(1 - k^4)$$

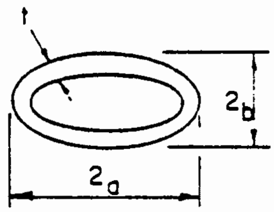
where = $k = \frac{a_1}{a_2} = \frac{b_1}{b_2}$



Thin walled tube of arbitary shape

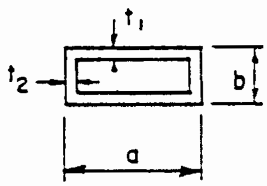
$$IX = \frac{4A_m^2 t}{L}$$

where: A_m = approximate area within median
L = length of median



Thin walled elliptical tube

$$IX = \frac{4\pi^2 t\left[(a-\frac{1}{2}t^2)^2(b-\frac{1}{2}t)^2\right]}{\pi(a+b-t)\left[1+0.27(a-b)^2/(a+b)^2\right]}$$



Rectangular tube

$$IX = \frac{2t_1t_2(a-t_2)^2(b-t_1)^2}{at_2+bt_1-t_2^2-t_1^2}$$

II. Formulas for Built Up Sections

The problem of finding torsional rigidities for some Bridge Department standard cross-sectional shapes becomes difficult when standard formulas do not apply. In the following discussion methods are developed to obtain torsional rigidities for shapes related to Bridge Design.

The following assumptions are made in developing the equations used.

(1) Plane sections remain plane

(2) The material is homogeneous, isotropic and linearly elastic

(3) Saint Venants principle applies

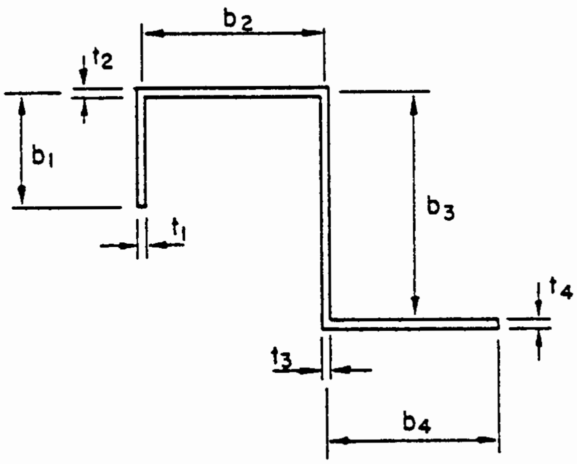
Torsional Rigidities

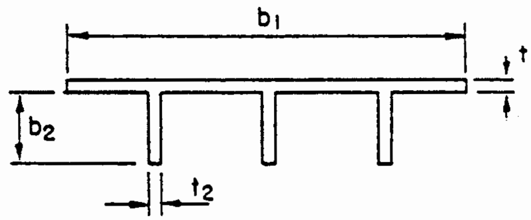
The torsional rigidity of an open thin walled section may be calculated from the following equation.

$$IX = 1/3 \sum_{i=1}^{n} b_i t_i^3 \qquad \text{(Ref. B.1)}$$

This equation is for the general case of a section with n elements.

Examples





$$IX = \frac{b_1 t_1^3 + b_2 t_2^3 + b_3 t_3^3 + b_4 t_4^3}{3}$$

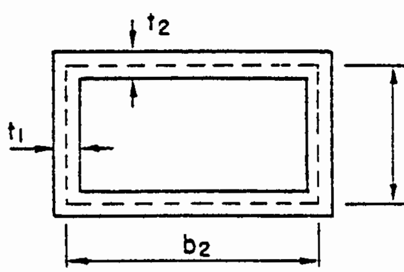
$$IX = \frac{b_1 t_1 + 3 b_2 t_2}{3}$$

The torsional constant for a single thin walled closed section may be obtained from:

$$IX = \frac{4\,\Omega^2}{\oint \frac{ds}{t}} \qquad \text{(Ref. B.1)}$$

where Ω is the total area enclosed by the center line of the walls of the closed section. The integral term represents the sum of the various lengths of wall section divided by their respective thicknesses.

Example

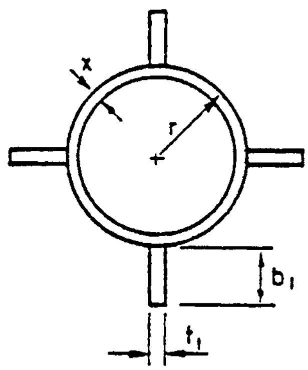


$$IX = \frac{4\,(b_1\ b_2)^2}{\frac{2b_1}{t_1} + \frac{2b_2}{t_2}} = \frac{2\,(b_1\ b_2)^2}{\frac{b_1}{t_1} + \frac{b_2}{t_2}}$$

For a hybrid section of a closed section plus outstanding fins, the formula for IX becomes:

$$IX = \frac{4\Omega^2}{\oint \frac{ds}{t}} + \frac{1}{3}\sum_{i=1}^{n} b_i t_i^3 \qquad \text{(Ref. B.1)}$$

Examples

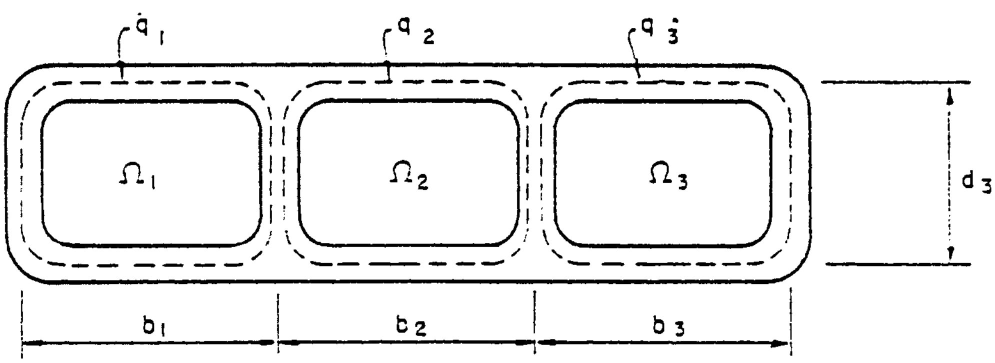


$$IX = \frac{4(\pi r^2)^2}{2\pi \frac{r}{t}} + \frac{4}{3}\, b_1 t_1$$

$$IX = 2\pi r^3 t + \frac{4}{3}\, b_1 t_1^3$$

III. Multi Celled Sections

Torsion of two or more cells connected at the walls is a statically indeterminate problem. The general method to find the torsional rigidity IX (Ref. B.1) is as follows. Assume an n celled closed thin walled section.



The equation of equilibrium for n cells is:

(1) $$M_t = 2 \sum_{i=1}^{n} q_i \Omega_i$$ (1)

where q_i is the shear flow in cell i, and Ω_i is the area inclosed by the center line of the walls inclosing the cell, and M_t is the twisting moment applied to the cell.

The equations of consistent deformation are:

(2) $$S_{ji}\ q_i + S_{jj}\ q_j + S_{jk}\ q_k = 2\Omega_j \theta$$ (2)

where: $S_{ji} = -\frac{1}{G} \int_{S_{ji}} \frac{ds}{t}$ $\qquad S_{jk} = -\frac{1}{G} \int_{S_{jk}} \frac{ds}{t}$

$$S_{jj} = \frac{1}{G} \oint_{S_{jj}} \frac{ds}{t}$$

G is the shear modulus of elasticity.

$\int_{S_{ji}}$ ds/t is the sum of the length of cell wall, common to cells j and i, divided by its thickness.

$\int_{S_{jk}}$ ds/t is the sum of the length of cell wall common to cells j and k, divided by its thicknesses.

$\oint_{S_{jj}}$ ds/t is the sum of the lengths of cell walls common to cell j, divided by their respective thicknesses.

θ is the angle of twist in radians

Equation (2) will yield n equations for n unknown shear flows and can be solved for the shear flows q_i in terms of G and the angle of twist θ. Knowing q_i and Ω_i the torsional constant IX may be calculated from:

(3) $$IX = \frac{2}{G\theta} \sum_{i=1}^{ni} q_i \Omega_i$$ (3)

The following examples are attached to show four methods of solution that may be used for a box girder section. The example section chosen was a standard three celled box girder with sloping exterior girders.

a. Solution by Method of Simultaneous Equations

The torsional constant IX for a three celled box girder may be calculated by the method of simultaneous equations which is based upon the following facts:

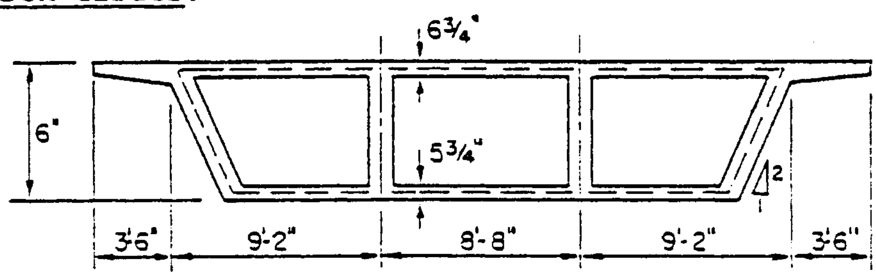
(1) The summation of external torsional moments and the internal resisting shear flow system must be equal to zero.

(2) The angle of twist must be the same for each cell.

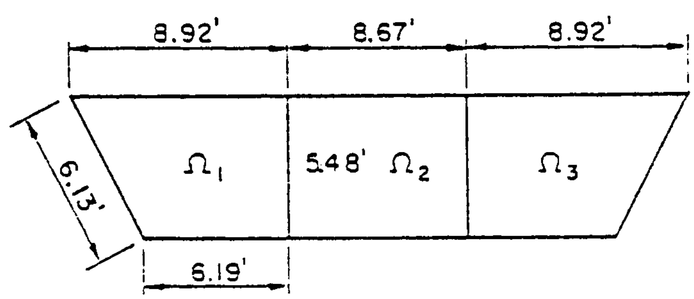
These facts are used to write one equation for each cell in terms of the shear flow q for that particular cell. The resulting shear flows are then used to calculate the Torsional Constant IX. The method used in the following example uses three simultaneous equations to solve for the unknown shear flows q.

EXAMPLE CALCULATION FOR TORSIONAL CONSTANT IX

Assume Box Girder:



Idealize Box Girder:



$\Omega_1 = \Omega_3 = \frac{(6.19 + 8.92)(5.48)}{2} = 41.40$ sq. ft. Top slab thickness = .56 ft.

$\Omega_2 = (5.48)(8.67) = 47.51$ sq. ft. Bottom slab thickness = .48 ft.

Using equations for multi-celled sections we may obtain the following: (See Ref. 1 for additional details)

$$\delta_{33} = \delta_{11} = \frac{1}{G}\left(\frac{8.92}{.56} + \frac{6.19}{.48} + \frac{5.48}{1.0} + \frac{6.13}{1.0}\right) = \frac{40.44}{G}$$

$$\delta_{12} = \delta_{21} = \delta_{23} = \delta_{32} = \frac{-5.48}{G}$$

$$\delta_{22} = \frac{\left(\frac{5.48}{1} + \frac{5.48}{1} + \frac{8.67}{.56} + \frac{8.67}{.48}\right)}{G} = \frac{44.50}{G}$$

From Equation 2 we obtain :

$$40.44\,q_1 - 5.48\,q_2 + 0\,q_3 = 82.8\ G\theta$$

$$-5.48\,q_1 + 44.50\,q_2 - 5.48\,q_3 = 95.02\ G\theta$$

$$0\,q_1 - 5.48\,q_2 + 40.44\,q_3 = 82.8\ G\theta$$

$$q_1 = 2.42\ G\theta$$

$$q_2 = 2.73\ G\theta$$

$$q_3 = 2.42\ G\theta$$

And from Equation 3 we obtain :

$$IX = \frac{2}{G\theta}(2.42 \cdot 41.40 + 2.73 \cdot 47.51 + 2.42 \cdot 41.40)$$

$$IX = \underline{660.1\ ft^4}$$

b. Solution by Method of Successive Corrections

The Torsional Constant IX for a multiple cell box girder may be calculated by the method of successive corrections. This method is similar to the moment distribution method used in frame analysis. The method is based on the following facts:

(1) The summation of the external torsional moment and the internal resisting shear flow force system must equal 0.

(2) The angle of twist must be the same for each cell.

These facts are used to write one equation for each cell in terms of the shear flow q. The resulting equations are then solved by the method of successive corrections.

The relation between shear flow and twist per unit length is given by,

(3) $$q = \frac{2AG\theta}{\Sigma \frac{L}{T}}$$ (Ref. 2)

Where

G = Modulus of rigidity

L = Length of any cell wall of constant thickness

t = Thickness

θ = Twist per unit length

A = Area of cell interior

Assuming $G\theta = 1$, equation (3) can be written

$$q_i = \frac{2A}{\Sigma \frac{L}{T}}$$

This equation then solves for q for each cell independently. The resulting q for each cell is the assumed q that is adjusted by the successive approximations method.

Carry over factors are determined for each cell from the following equations,

$$C.O.F._{(2-1)} = \frac{\left(\frac{L}{T}\right) \text{web } (1-2)}{\left(\Sigma \frac{L}{T}\right) \text{cell } (1)}$$

$$C.O.F._{(1-2)} = \frac{\left(\frac{L}{T}\right) \text{web } (2-1)}{\left(\Sigma \frac{L}{T}\right) \text{cell } (2)}$$

$$C.O.F._{(2-3)} = \frac{\left(\frac{L}{T}\right) \text{web } (3-2)}{\left(\Sigma \frac{L}{T}\right) \text{cell } (3)}$$

$$C.O.F._{(3-2)} = \frac{\left(\frac{L}{T}\right) \text{web } (2-3)}{\left(\Sigma \frac{L}{T}\right) \text{cell } (2)}$$

$$C.O.F._{(i-j)} = \frac{\left(\frac{L}{T}\right) \text{web } (J-I)}{\left(\Sigma \frac{L}{T}\right) \text{cell } (J)}$$

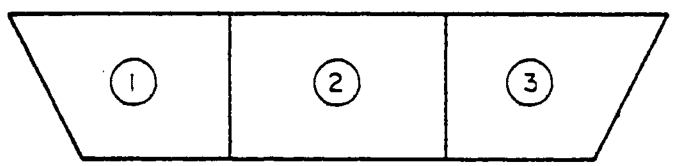
The carry over operation is performed until the desired precision is reached. The final q for each cell is equal to the initial q plus all the carry overs from adjacent cells. It will be noted in the example problem that the carry over from cell 2 in the third step is computed from the sum of carry overs to cell 2 in the previous step. The torsional constant IX is then computed from the following equation.

$$IX = 2 \sum_{j=1} q_j A_j \qquad (4)$$

<u>Example Problem</u>

Given: Box girder section with cell areas and wall thicknesses taken from previous example problem.

Required: Compute the torsional constant IX by the method of successive corrections and compare with results obtained from solving the simultaneous equations.



$A_1 = 41.40$ sq. ft. $\qquad \Sigma \frac{L}{T} = 40.44$

$A_2 = 47.51 \qquad \Sigma \frac{L}{T} = 44.50$

$A_3 = 41.40 \qquad \Sigma \frac{L}{T} = 40.44$

Assuming $G\theta = 1$, equation 3 may be written:

$$q_3 = q_1 = \frac{2A_1}{\Sigma \frac{L}{T}} \doteq \frac{2 \cdot 41 \cdot 40}{40.44} = 204.7 \times 10^{-2}$$

$$q_2 = \frac{2A_2}{\Sigma \frac{L}{T}} = \frac{2 \cdot 47.51}{44.50} = 213.5 \times 10^{-2}$$

$$C.O.F._{1-2} = \frac{5.48}{40.44} = .123$$

$$C.O.F._{2-1} = C.O._{2-3} = \frac{5.48}{44.50} = .136$$

$$C.O.F._{3-2} = \frac{5.48}{40.44} = .123$$

	CELL 1	CELL 2		CELL 3
C.O.F.	.123	.136		.123
q	204.7	213.5		204.7
C.O.	29.0	25.2	25.2	29.0
C.O.	6.8	3.6	3.6	6.8
C.O.	1.0	.8	.8	1.0
Total	241.5×10^{-2}	272.7×10^{-2}		241.5×10^{-2}

From equation (4) we may obtain:

$$IX = 2\,(2.415 \cdot 41.40 + 2.727 \times 47.51 + 2.415 \cdot 41.40)$$

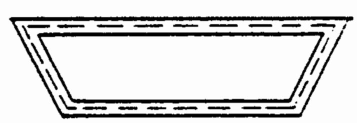
$$IX = 659.044 \text{ ft.}^4$$

c. Solution by Approximate Method

An approximate method to find the torsional constant of a multiple box girder would be to assume that the interior web members were not effective in torsion. The torsional constant could then be calculated from standard formulas published in Engineering Handbooks. A good reference for torsional constants is "Design of Welded Structures" by Blodgett. The following example uses the dimensions as stated in the other methods but neglects the effects of interior web members for torsional considerations.

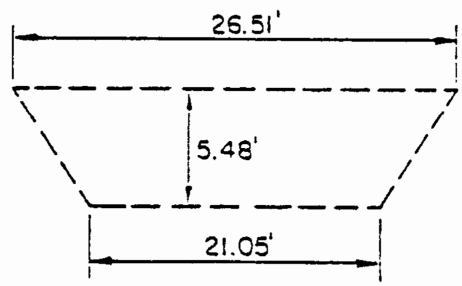
Approximate Method Example:

Box Idealized
(No webs)



from "Design of Welded Structures", Page 2.10-4

$$R = \frac{4\,(A)^2}{\int ds/t_s}$$



$$A = \left(\frac{26.51 + 21.05}{2}\right) 5.48 = 130.31$$

$$A^2 = 16{,}980.70$$

$$4\,A^2 = 67{,}922.80$$

$$\int \frac{ds}{T} = \frac{26.51}{.56} + \frac{12.26}{1} + \frac{21.05}{.48} = 103.45$$

$$R = \frac{67{,}922.80}{103.45} = 656.6 \text{ ft.}^4$$

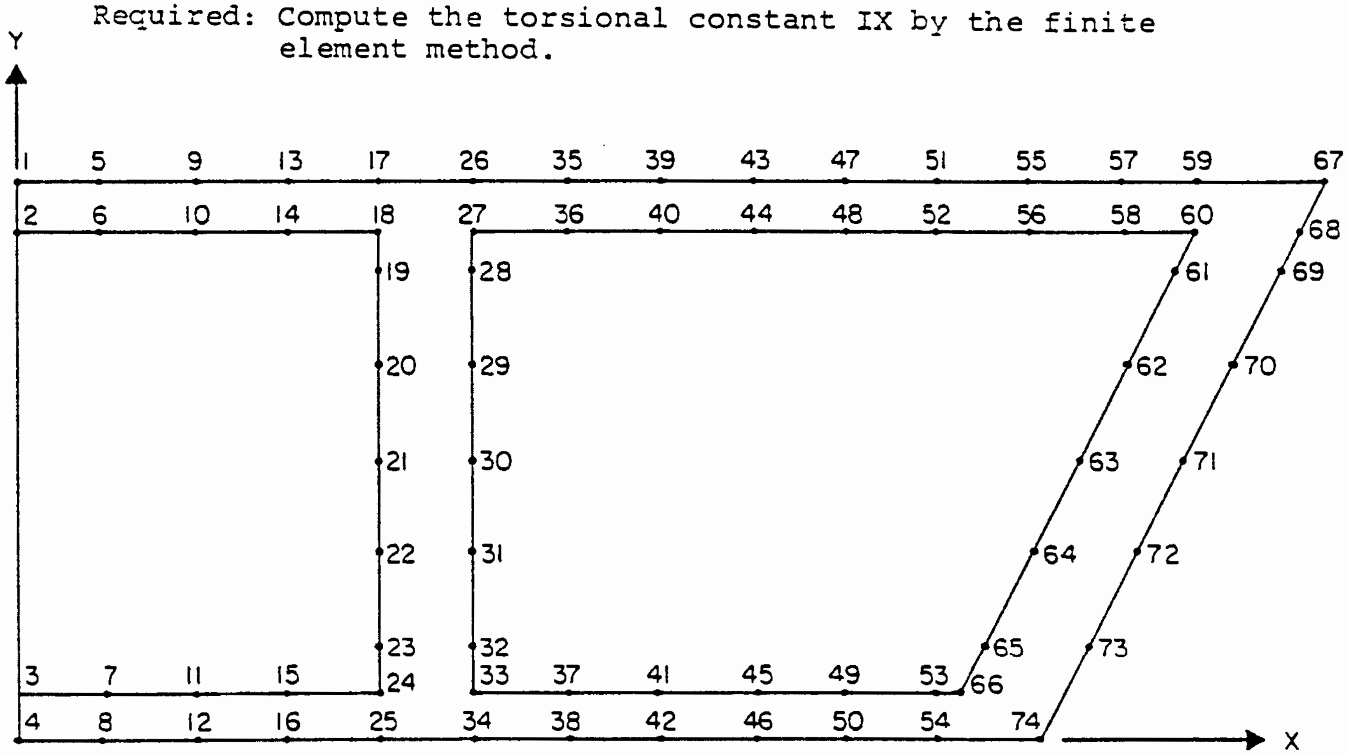
d. Solution by Finite Element Analysis

The solution by finite elements requires the use of a torsional analysis program available in Bridge Computer Services. A complete discussion of the theory and use of the program is also available in Bridge Computer Services. The following example shows the steps needed to obtain a torsional constant by the finite element method.

Example Problem

Given: Box girder section with cell areas and wall thickness taken from previous example problem. (See sketch)

Required: Compute the torsional constant IX by the finite element method.



FINITE ELEMENT MODEL

The following coding describes the elements that make up the cross section to be analyzed. The first four cards give the title of the problem, the loads to be applied, the material properties, and the number of node and element cards to follow in the input. The subsequent cards describe the position of each node in XY coordinates and the last group of cards describes how the elements are connected at the nodes.

The attached output listing shows the results that may be expected from the input data shown. For additional information on input and output for this program contact Bridge Computer Services.

THREE CELLED BOX GIRDER TORSIONAL CONSTANT EXAMPLE

MAGNITUDE OF SHEAR FORCE IN THE X-DIRECTION...   10.00000

MAGNITUDE OF SHEAR FORCE IN THE Y-DIRECTION...   10.00000

X-COORDINATE OF SHEAR FORCE...   0.0

Y-COORDINATE OF SHEAR FORCE...   0.0

TWISTING MOMENT...   10.00000

ELASTIC PROPERTIES OF THE MATERIAL

MODULUS OF ELASTICITY...   3000000.

POISSONS RATIO...   0.150

SHEAR MODULUS...   1304348.

NODE	1	X =	0.0	Y =	6.0000
NODE	2	X =	0.0	Y =	5.4400
NODE	3	X =	0.0	Y =	0.4800
NODE	4	X =	0.0	Y =	0.0
NODE	5	X =	0.8300	Y =	6.0000
NODE	6	X =	0.8300	Y =	5.4400
NODE	7	X =	0.8300	Y =	0.4800
NODE	8	X =	0.8300	Y =	0.0
NODE	9	X =	1.8300	Y =	6.0000
NODE	10	X =	1.8300	Y =	5.4400
NODE	11	X =	1.8300	Y =	0.4800
NODE	12	X =	1.8300	Y =	0.0
NODE	13	X =	2.8300	Y =	6.0000
NODE	14	X =	2.8300	Y =	5.4400
NODE	15	X =	2.8300	Y =	0.4800
NODE	16	X =	2.8300	Y =	0.0
NODE	17	X =	3.8300	Y =	6.0000
NODE	18	X =	3.8300	Y =	5.4400
NODE	19	X =	3.8300	Y =	5.0000
NODE	20	X =	3.8300	Y =	4.0000
NODE	21	X =	3.8300	Y =	3.0000
NODE	22	X =	3.8300	Y =	2.0000
NODE	23	X =	3.8300	Y =	1.0000
NODE	24	X =	3.8300	Y =	0.4800
NODE	25	X =	3.8300	Y =	0.0
NODE	26	X =	4.8300	Y =	6.0000
NODE	27	X =	4.8300	Y =	5.4400
NODE	28	X =	4.8300	Y =	5.0000
NODE	29	X =	4.8300	Y =	4.0000
NODE	30	X =	4.8300	Y =	3.0000
NODE	31	X =	4.8300	Y =	2.0000
NODE	32	X =	4.8300	Y =	1.0000
NODE	33	X =	4.8300	Y =	0.4800
NODE	34	X =	4.8300	Y =	0.0
NODE	35	X =	5.8300	Y =	6.0000
NODE	36	X =	5.8300	Y =	5.4400
NODE	37	X =	5.8300	Y =	0.4800
NODE	38	X =	5.8300	Y =	0.0
NODE	39	X =	6.8300	Y =	6.0000
NODE	40	X =	6.8300	Y =	5.4400
NODE	41	X =	6.8300	Y =	0.4800
NODE	42	X =	6.8300	Y =	0.0
NODE	43	X =	7.8300	Y =	6.0000
NODE	44	X =	7.8300	Y =	5.4400
NODE	45	X =	7.8300	Y =	0.4800
NODE	46	X =	7.8300	Y =	0.0
NODE	47	X =	8.8300	Y =	6.0000
NODE	48	X =	8.8300	Y =	5.4400
NODE	49	X =	8.8300	Y =	0.4800
NODE	50	X =	8.8300	Y =	0.0
NODE	51	X =	9.8300	Y =	6.0000
NODE	52	X =	9.8300	Y =	5.4400
NODE	53	X =	9.8300	Y =	0.4800
NODE	54	X =	9.8300	Y =	0.0
NODE	55	X =	10.8300	Y =	6.0000
NODE	56	X =	10.8300	Y =	5.4400
NODE	57	X =	11.8300	Y =	6.0000
NODE	58	X =	11.8300	Y =	5.4400
NODE	59	X =	12.5600	Y =	6.0000
NODE	60	X =	12.5610	Y =	5.4400
NODE	61	X =	12.3400	Y =	5.0000
NODE	62	X =	11.8400	Y =	4.0000
NODE	63	X =	11.3400	Y =	3.0000
NODE	64	X =	10.8400	Y =	2.0000
NODE	65	X =	10.3400	Y =	1.0000
NODE	66	X =	10.0800	Y =	0.4800
NODE	67	X =	13.2500	Y =	6.0000
NODE	68	X =	13.6700	Y =	5.4400
NODE	69	X =	13.4600	Y =	5.0000
NODE	70	X =	12.9600	Y =	4.0000
NODE	71	X =	12.4600	Y =	3.0000
NODE	72	X =	11.9600	Y =	2.0000
NODE	73	X =	11.4600	Y =	1.0000
NODE	74	X =	10.9600	Y =	0.0

NUMBER OF NODAL POINTS... 74

NUMBER OF ELEMENTS........ 37

1	2	6	5
5	6	10	9
9	10	14	13
13	14	18	17
17	18	27	26
26	27	36	35
35	36	40	39
39	40	44	43
43	44	48	47
47	48	52	51
51	52	56	55
55	56	58	57
57	58	60	59
59	60	68	67
60	61	69	68
61	62	70	69
62	63	71	70
63	64	72	71
64	65	73	72
65	66	74	73
66	53	54	74
53	49	50	54
45	46	50	49
41	42	46	45
37	38	42	41
33	34	38	37
18	19	28	27
19	20	29	28
20	21	30	29
21	22	31	30
22	23	32	31
23	24	33	32
24	25	34	33
15	16	25	24
11	12	16	15
7	8	12	11
3	4	8	7

DUE TO THE X-COMPONENT OF THE SHEARING FORCE

NODE	WARPING FUNCTION
1	0.0
2	0.0
3	0.0
4	0.0
5	3.22495D-07
6	3.22471D-07
7	3.31974D-07
8	3.32218D-07
9	7.09429D-07
10	7.09388D-07
11	7.30379D-07
12	7.30393D-07
13	1.09412D-06
14	1.09011D-06
15	1.12258D-06
16	1.12654D-06
17	1.45261D-06
18	1.48427D-06
19	1.56637D-06
20	1.61158D-06
21	1.62522D-06
22	1.62919D-06
23	1.60640D-06
24	1.52565D-06
25	1.49905D-06
26	1.68985D-06
27	1.66685D-06
28	1.61156D-06
29	1.61319D-06
30	1.62687D-06
31	1.63072D-06
32	1.64180D-06
33	1.70874D-06
34	1.73256D-06
35	1.97379D-06
36	1.97482D-06
37	2.05280D-06
38	2.05139D-06
39	2.25800D-06
40	2.25627D-06
41	2.36945D-06
42	2.37133D-06
43	2.52542D-06
44	2.52374D-06
45	2.67242D-06
46	2.67419D-06
47	2.77535D-06
48	2.77343D-06
49	2.95767D-06
50	2.95946D-06
51	3.00538D-06
52	3.00323D-06
53	3.22188D-06
54	3.22597D-06
55	3.21329D-06
56	3.21093D-06
57	3.39709D-06
58	3.39374D-06
59	3.50626D-06
60	3.51904D-06
61	3.54761D-06
62	3.56054D-06
63	3.53446D-06
64	3.47525D-06
65	3.38119D-06
66	3.28721D-06
67	3.57900D-06
68	3.56375D-06
69	3.55845D-06
70	3.57139D-06
71	3.56068D-06
72	3.51604D-06
73	3.44007D-06
74	3.37097D-06

DUE TO THE X-COMPONENT OF THE SHEARING FORCE

X	Y	WF	TZX	TZY
4.150000-01	5.720000 00	1.613230-07	5.075460-01	5.738670-05
1.330000 00	5.720000 00	5.161960-07	5.049990-01	-3.649840-04
2.330000 00	5.720000 00	9.009550-07	4.992490-01	3.567370-03
3.330000 00	5.720000 00	1.290560-06	4.903530-01	-3.387850-02
4.330000 00	5.720000 00	1.573760-06	2.724960-01	-1.279020-02
5.330000 00	5.720000 00	1.826770-06	3.837390-01	2.298450-02
6.329999 00	5.720000 00	2.114250-06	3.654200-01	-2.397200-03
7.329999 00	5.720000 00	2.391470-06	3.439550-01	2.447300-04
8.329999 00	5.720000 00	2.650180-06	3.193470-01	-2.890060-05
9.329999 00	5.720000 00	2.890120-06	2.915880-01	1.190460-05
1.033000 01	5.720000 00	3.109040-06	2.606730-01	-8.443020-05
1.133000 01	5.720000 00	3.304680-06	2.264120-01	8.150680-04
1.219520 01	5.720000 00	3.454470-06	1.944900-01	-1.696950-02
1.319520 01	5.720000 00	3.542510-06	4.365910-02	-1.883270-02
1.300770 01	5.220000 00	3.545180-06	1.560190-02	-5.544610-02
1.265000 01	4.500000 00	3.561710-06	-3.424410-03	-2.642270-02
1.215000 01	3.500000 00	3.560610-06	6.448130-03	1.258220-02
1.165000 01	2.500000 00	3.525280-06	2.516190-02	5.001500-02
1.115000 01	1.500000 00	3.457110-06	4.560890-02	8.587530-02
1.071000 01	6.200010-01	3.375400-06	1.065010-01	9.844360-02
1.017500 01	2.400010-01	3.284620-06	2.167670-01	8.080350-02
9.329999 00	2.400010-01	3.091490-06	3.381290-01	-2.212880-03
8.329999 00	2.400010-01	2.816470-06	3.658950-01	3.141390-04
7.329999 00	2.400010-01	2.522120-06	3.909710-01	-4.292310-04
6.329999 00	2.400010-01	2.211650-06	4.120020-01	3.292900-03
5.330000 00	2.400010-01	1.886710-06	4.303250-01	-2.713490-02
4.330000 00	5.220000 00	1.582510-06	1.470170-01	-4.146490-02
4.330000 00	4.500000 00	1.601430-06	2.875720-02	-3.168040-02
4.330000 00	3.500000 00	1.619970-06	2.111930-04	-1.904270-02
4.330000 00	2.500000 00	1.628760-06	2.111010-04	-4.425150-03
4.330000 00	1.500000 00	1.627780-06	2.247720-02	9.192520-03
4.330000 00	7.400010-01	1.620970-06	1.412150-01	1.954130-02
4.330000 00	2.400010-01	1.616780-06	2.707900-01	6.467180-03
3.330000 00	2.400010-01	1.318470-06	5.056130-01	3.282040-02
2.330000 00	2.400010-01	9.274240-07	5.145090-01	-3.944640-03
1.330000 00	2.400010-01	5.313290-07	5.202580-01	4.708990-04
4.150000-01	2.400010-01	1.660730-07	5.229030-01	-7.545850-05

DUE TO THE Y-COMPONENT OF THE SHEARING FORCE

NODE	WARPING FUNCTION
1	1.342810D-06
2	1.338740D-06
3	-1.748940D-06
4	-1.753850D-06
5	1.316770D-06
6	1.313200D-06
7	-1.717690D-06
8	-1.722140D-06
9	1.216750D-06
10	1.214050D-06
11	-1.596420D-06
12	-1.600320D-06
13	1.037770D-06
14	1.044170D-06
15	-1.387870D-06
16	-1.383040D-06
17	8.168870D-07
18	7.663620D-07
19	6.110980D-07
20	2.410700D-07
21	-1.486400D-07
22	-5.307000D-07
23	-8.806110D-07
24	-1.058960D-06
25	-1.103340D-06
26	8.027160D-07
27	7.628390D-07
28	6.211030D-07
29	2.479240D-07
30	-1.509520D-07
31	-5.421770D-07
32	-8.895680D-07
33	-1.023640D-06
34	-1.048260D-06
35	9.779690D-07
36	9.992390D-07
37	-1.164680D-06
38	-1.148510D-06
39	1.108050D-06
40	1.131080D-06
41	-1.195020D-06
42	-1.176700D-06
43	1.159490D-06
44	1.191510D-06
45	-1.136880D-06
46	-1.110580D-06
47	1.135800D-06
48	1.177930D-06
49	-9.875230D-07
50	-9.528910D-07
51	1.036600D-06
52	1.089830D-06
53	-7.476370D-07
54	-7.029580D-07
55	8.621170D-07
56	9.272500D-07
57	6.103460D-07
58	6.922060D-07
59	4.140280D-07
60	4.381280D-07
61	3.130610D-07
62	7.909330D-08
63	-1.619690D-07
64	-3.855600D-07
65	-5.575670D-07
66	-6.662080D-07
67	3.852130D-07
68	4.644490D-07
69	4.729540D-07
70	2.785390D-07
71	1.290330D-08
72	-2.510860D-07
73	-4.740320D-07
74	-5.564760D-07

DUE TO THE Y-COMPONENT OF THE SHEARING FORCE

X	Y	WF	TZX	TZY
4.15000D-01	5.72000D 00	1.33038D-06	-4.33595D-02	7.29816D-04
1.33000D 00	5.72000D 00	1.26796D-06	-1.38955D-01	1.33246D-03
2.33000D 00	5.72000D 00	1.13094D-06	-2.43423D-01	-5.29491D-03
3.33000D 00	5.72000D 00	9.19057D-07	-3.47895D-01	5.82317D-02
4.33000D 00	5.72000D 00	7.89971D-07	-4.10429D-02	1.22756D-01
5.33000D 00	5.72000D 00	8.48461D-07	7.32152D-01	5.24808D-02
6.32999D 00	5.72000D 00	1.05685D-06	1.27684D-01	-4.68816D-03
7.32999D 00	5.72000D 00	1.15037D-06	7.32113D-02	1.29767D-03
8.32999D 00	5.72000D 00	1.16903D-06	-8.12566D-02	6.79263D-04
9.32999D 00	5.72000D 00	1.11781D-06	-1.85729D-01	7.04419D-04
1.03300D 01	5.72000D 00	9.81719D-07	-2.90206D-01	1.05708D-03
1.13300D 01	5.72000D 00	7.75753D-07	-3.94687D-01	-2.37954D-03
1.21992D 01	5.72000D 00	5.60983D-07	-4.85180D-01	7.31445D-02
1.31852D 01	5.72000D 00	4.30931D-07	-8.95179D-02	1.12604D-01
1.30077D 01	5.22000D 00	4.34397D-07	3.81070D-02	3.47556D-01
1.26500D 01	4.50000D 00	2.93273D-07	1.65459D-01	3.93523D-01
1.21500D 01	3.50000D 00	4.99773D-08	2.09450D-01	4.25183D-01
1.16500D 01	2.50000D 00	-2.02718D-07	2.04374D-01	4.14507D-01
1.11500D 01	1.50000D 00	-4.26885D-07	1.80887D-01	3.61495D-01
1.07100D 01	6.20001D-01	-5.63815D-07	2.29712D-01	2.50996D-01
1.01750D 01	2.40001D-01	-6.98617D-07	3.40814D-01	1.35429D-01
9.32999D 00	2.40001D-01	-8.50408D-07	3.97041D-01	-1.11532D-04
8.32999D 00	2.40001D-01	-1.04962D-06	2.69529D-01	4.81750D-04
7.32999D 00	2.40001D-01	-1.15745D-06	1.41999D-01	1.01596D-03
6.32999D 00	2.40001D-01	-1.17388D-06	1.44761D-02	-4.07778D-03
5.33000D 00	2.40001D-01	-1.09892D-06	-1.13040D-01	3.41745D-02
4.33000D 00	5.22000D 00	6.91852D-07	-1.92984D-02	4.60754D-01
4.33000D 00	4.50000D 00	4.33233D-07	-3.92284D-03	5.08634D-01
4.33000D 00	3.50000D 00	4.79334D-08	1.07102D-08	5.40096D-01
4.33000D 00	2.50000D 00	-3.44896D-07	-7.91624D-08	5.29418D-01
4.33000D 00	1.50000D 00	-7.14884D-07	7.62116D-03	4.74406D-01
4.33000D 00	7.40001D-01	-9.65710D-07	4.72308D-02	4.09097D-01
4.33000D 00	2.40001D-01	-1.06120D-06	9.49701D-02	1.07995D-01
3.33000D 00	2.40001D-01	-1.23596D-06	4.24415D-01	5.69099D-02
2.33000D 00	2.40001D-01	-1.49457D-06	2.97097D-01	-4.29274D-03
1.33000D 00	2.40001D-01	-1.66190D-06	1.69593D-01	1.24152D-03
4.15000D-01	2.40001D-01	-1.73810D-06	5.29238D-02	4.40369D-04

DUE TO THE TOTAL APPLIED TWISTING MOMENT

NODE	WARPING FUNCTION
1	0.0
2	0.0
3	0.0
4	0.0
5	-2.434900D 00
6	-1.960170D 00
7	2.250190D 00
8	2.657690D 00
9	-5.355180D 00
10	-4.335170D 00
11	4.974460D 00
12	5.846560D 00
13	-8.303420D 00
14	-6.682270D 00
15	7.666020D 00
16	9.068230D 00
17	-1.103990D 01
18	-9.241200D 00
19	-8.201990D 00
20	-3.980790D 00
21	5.701520D-01
22	5.121100D 00
23	9.383310D 00
24	1.058530D 01
25	1.296220D 01
26	-1.260390D 01
27	-9.639290D 00
28	-7.113710D 00
29	-3.232960D 00
30	3.179820D-01
31	3.868930D 00
32	7.708600D 00
33	1.071960D 01
34	1.332440D 01
35	-1.488990D 01
36	-1.165230D 01
37	1.301560D 01
38	1.577940D 01
39	-1.733450D 01
40	-1.350680D 01
41	1.513180D 01
42	1.841430D 01
43	-1.976260D 01
44	-1.537810D 01
45	1.776920D 01
46	2.102780D 01
47	-2.219250D 01
48	-1.724770D 01
49	1.940770D 01
50	2.364040D 01
51	-2.462240D 01
52	-1.911750D 01
53	2.151650D 01
54	2.678270D 01
55	-2.705160D 01
56	-2.098800D 01
57	-2.948760D 01
58	-2.285170D 01
59	-3.100390D 01
60	-2.447730D 01
61	-2.069590D 01
62	-1.114810D 01
63	-1.448310D 00
64	8.251510D 00
65	1.795910D 01
66	2.206880D 01
67	-2.969900D 01
68	-2.276530D 01
69	-1.799520D 01
70	-9.017750D 00
71	-4.377130D-01
72	8.142170D 00
73	1.675730D 01
74	2.638560D 01

DUE TO THE TOTAL APPLIED TWISTING MOMENT

X	Y	WR	TZX	TZY
4.150000D-01	5.720000 00	-1.098770 00	-2.766420D-01	-4.793250D-04
1.330000 00	5.720000 00	-3.521350 00	-2.766420D-01	-2.478990D-04
2.330000 00	5.720000 00	-6.169000 00	-2.766440D-01	-1.523650D-03
3.330000 00	5.720000 00	-8.816680 00	-2.766450D-01	1.495830D-02
4.330000 00	5.720000 00	-1.063100 01	-1.865140D-01	4.168530D-03
5.330000 00	5.720000 00	-1.219630 01	-2.497030D-01	-1.123150D-02
6.329990 00	5.720000 00	-1.434580 01	-2.497070D-01	1.176220D-03
7.329990 00	5.720000 00	-1.649550 01	-2.497120D-01	-1.226050D-04
8.329990 00	5.720000 00	-1.864520 01	-2.497180D-01	1.396370D-05
9.329990 00	5.720000 00	-2.079500 01	-2.497220D-01	-6.588620D-06
1.033000 01	5.720000 00	-2.294480 01	-2.497230D-01	5.034940D-05
1.133000 01	5.720000 00	-2.509470 01	-2.497230D-01	-4.797960D-04
1.219520 01	5.720000 00	-2.695510 01	-2.497550D-01	2.385780D-02
1.318520 01	5.720000 00	-2.695240 01	-4.725150D-02	4.699370D-02
1.300770 01	5.220000 00	-2.140510 01	5.855360D-04	1.254760D-01
1.265000 01	4.500000 00	-1.477800 01	4.915330D-02	1.248700D-01
1.215000 01	3.500000 00	-5.638160 00	6.242860D-02	1.248690D-01
1.165000 01	2.500000 00	3.501700 00	6.243570D-02	1.248690D-01
1.115000 01	1.500000 00	1.264160 01	4.551980D-02	1.248690D-01
1.071000 01	6.200010D-01	2.382370 01	1.167200D-01	1.062120D-01
1.017500 01	2.400010D-01	2.418370 01	2.068370D-01	7.021530D-02
9.329990 00	2.400010D-01	2.271180 01	2.913650D-01	-2.371040D-03
8.329990 00	2.400010D-01	2.033630 01	2.913640D-01	3.112900D-04
7.329990 00	2.400010D-01	1.796070 01	2.913440D-01	-2.418830D-04
6.329990 00	2.400010D-01	1.558520 01	2.913650D-01	1.719370D-03
5.330000 00	2.400010D-01	1.320970 01	2.913610D-01	-1.418530D-02
4.330000 00	5.220000 00	-8.549060 00	-8.775760D-02	1.509090D-02
4.330000 00	4.500000 00	-5.632350 00	-1.783520D-02	1.509080D-02
4.330000 00	3.500000 00	-1.581400 00	-3.223390D-09	1.509100D-02
4.330000 00	2.500000 00	2.469540 00	4.512790D-08	1.509100D-02
4.330000 00	1.500000 00	6.520480 00	1.561450D-02	1.509110D-02
4.330000 00	7.400010D-01	9.599200 00	9.420410D-02	1.509090D-02
4.330000 00	2.400010D-01	1.167290 01	2.306570D-01	4.229090D-03
3.330000 00	2.400010D-01	9.869440 00	3.227910D-01	1.799640D-02
2.330000 00	2.400010D-01	6.888810 00	3.227990D-01	-2.112430D-03
1.330000 00	2.400010D-01	3.932220 00	3.227870D-01	-1.578430D-04
4.150000D-01	2.400010D-01	1.226970 00	3.227960D-01	-5.130590D-04

```
AREA OF SECTION...     4.71290E 01
X-COORDINATE OF CENTROID...     0.0
Y-COORDINATE OF CENTROID...     3.25217E 00
X-MOMENT OF INERTIA...     2.36206E 02
Y-MOMENT OF INERTIA...     3.17953E 03
PRODUCT OF INERTIA....     0.0
ANGLE TO PRINCIPAL AXES...     0.0
X-COORDINATE OF SHEAR CENTER...     0.0
Y-COORDINATE OF SHEAR CENTER...     2.82140E 00
SHEAR COEFFICIENT AXX...     1.85269E 00
SHEAR COEFFICIENT AYY...     2.68815E 00
SHEAR COEFFICIENT AXY...     0.0
TOTAL TWISTING MOMENT...     3.82140E 01
TORSIONAL CONSTANT...     7.06625E 02
```

C.2 Shear Constants - Standard Shapes

The following tables are provided to determine the shear constants that may be needed for a STRUDL analysis.

SHEAR SHAPE FACTORS f

$$\Delta = \int \frac{f V dx}{G A_x}$$

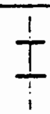
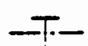
where:

V = shearing force

G = shearing modulus

A = cross-sectional area

f = shear shape factor

Section	f
 solid rectangular	$f = 1.2$
 solid circular	$f = 1.11$
 WF bending about minor axis	$f = \frac{1.2A}{A_f}$ where: A = total area A_f = flange area
 WF bending about major axis	$f = \frac{A}{A_w}$ where: A = total area A_w = web area

REFERENCES

1. Mechanics of Elastic Structures by J. T. Oden, 1967

2. Analysis and Design of Airplane Structures by E. F. Bruhn

3. Design of Welded Structures by O. W. Blodgett