# APPENDIX E – EXAMPLE CALCULATIONS

## Example 1 – Continuous Two Span CIP Box-Girder Stressed from One End:

Information given on contract plans:

- 270 ksi low relaxation prestressing strand.
- $E$ = 28,000 ksi.
- $P_{jack}$ = 202.5 ksi.

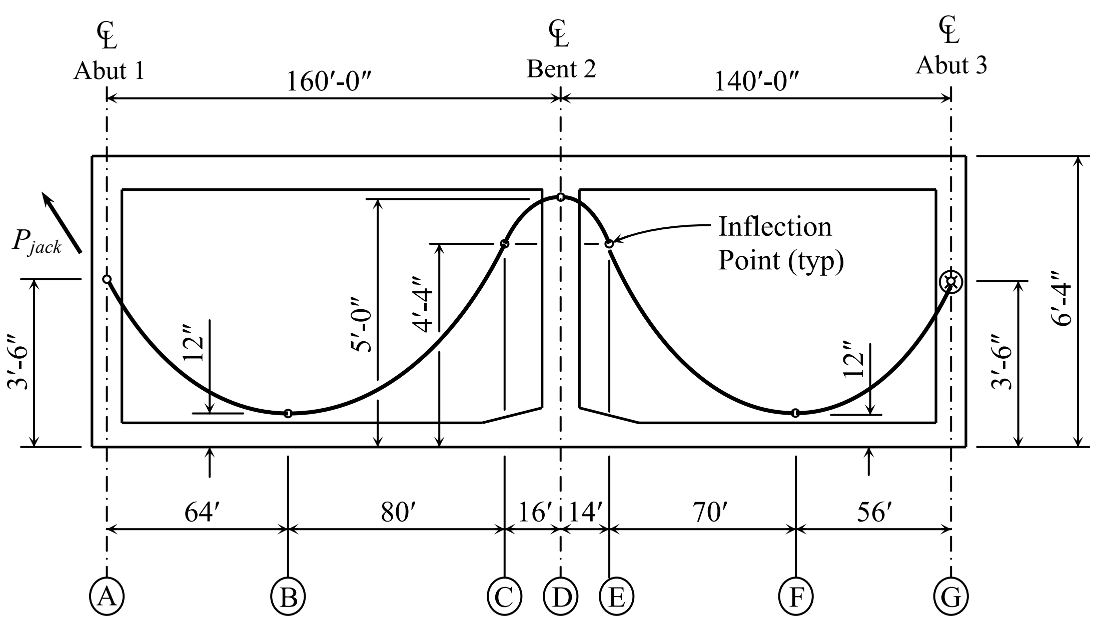



Figure 1 - Prestressing Cable Path.

The equation for stress in the prestressing steel at a distance $x$ from the jacking end of the frame is:

$$\boxed{T_x = T_o e^{-(\mu\alpha + KL)}}$$ **(Equation 1)**

Where: $\mu$ = 0.15 for frame lengths < 600 feet.

$K$ = 0.0002.

$\mu$ = Cumulative angle change at point of interest $x$ from jacking end.

$L$ = Distance to point of interest $x$ from jacking end.

Find the measurable elongation for the prestressing path in Figure 1:

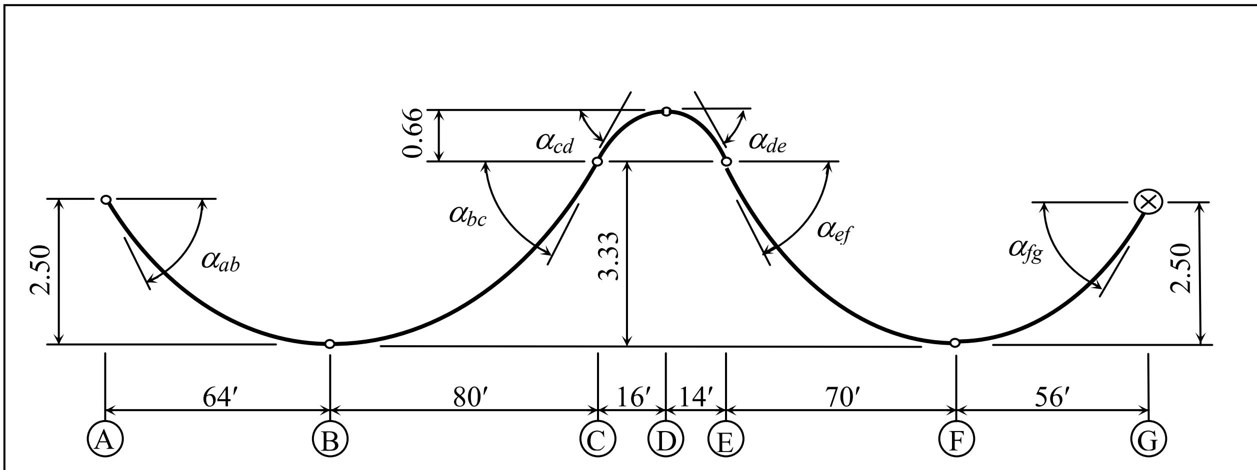



**Figure 2 – Vertical Angle Change.**

**Step 1**: Tendon elongations during the stressing operation are a function of both the average stress in the strands, and the length of the tendon. The stress in the strands vary along the tendon path due to angular friction between the tendon and the inside surface of the duct. Since there is no horizontal curvature given in this exercise, the angle changes are based on the vertical tendon profile only.

| Vertical Angle Change Calculations | | | |
|---|---|---|---|
| Segment | $y$ (feet) | $L$ (feet) | $\alpha = 2(y/L)$ (radians) |
| AB | 2.500 | 64 | 0.0781 |
| BC | 3.333 | 80 | 0.0833 |
| CD | 0.666 | 16 | 0.0833 |
| DE | 0.666 | 14 | 0.0952 |
| EF | 3.333 | 70 | 0.0952 |
| FG | 2.500 | 56 | 0.0893 |

**Step 2:** Now that the vertical angle change within each parabolic segment has been calculated, it is time to compute the initial friction coefficients. These coefficients represent a decimal percentage of the jacking stress at the end of each parabolic segment. Based on the results in the following table, there is slightly more than 87 percent of $P_{jack}$ in the strands at the dead end.

| Initial Friction Coefficient Calculations | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Segment | Friction μ | $\alpha = 2(y/L)$ (radians) | $\Sigma\,\alpha$ (radians) | Wobble K | $L$ (feet) | $\Sigma\,L$ (feet) | $(\mu\Sigma\alpha+K\Sigma L)$ | $e^{-(\mu\Sigma\alpha+K\Sigma L)}$ |
| AB | 0.15 | 0.0781 | 0.0781 | 0.0002 | 64 | 64 | 0.0245 | 0.976 |
| BC | 0.15 | 0.0833 | 0.1614 | 0.0002 | 80 | 144 | 0.0530 | 0.948 |
| CD | 0.15 | 0.0833 | 0.2447 | 0.0002 | 16 | 160 | 0.0687 | 0.934 |
| DE | 0.15 | 0.0952 | 0.3399 | 0.0002 | 14 | 174 | 0.0858 | 0.918 |
| EF | 0.15 | 0.0952 | 0.4351 | 0.0002 | 70 | 244 | 0.1141 | 0.892 |
| FG | 0.15 | 0.0893 | 0.5244 | 0.0002 | 56 | 300 | 0.1387 | 0.870 |

**Step 3:** With the initial friction coefficients in hand, it is now possible to compute the average stress in the strands in each segment. Knowing the stress distribution along the entire length of the frame, and assuming a Young's modulus for prestressing steel of $E$ = 28,000 ksi, the tendon elongation can be calculated using the following equation:

$$\boxed{\Delta = \frac{T_{avg}L}{E}}$$

| Elongation Calculations | | | | | | | |
|---|---|---|---|---|---|---|---|
| Segment | $e^{-(\mu\alpha+KL)}$ | $T_o$ (ksi) | $T_x=T_o e^{-(\mu\alpha+KL)}$ (ksi) | $T_{avg}$ (ksi) | $L$ (feet) | $L$ (in) | $\Delta_x=T_{avg}L/E$ (in) |
| AB | 0.976 | 202.5 | 197.6 | 200.1 | 64 | 768 | 5.49 |
| BC | 0.948 | 202.5 | 192.0 | 194.8 | 80 | 960 | 6.68 |
| CD | 0.934 | 202.5 | 189.1 | 190.6 | 16 | 192 | 1.31 |
| DE | 0.918 | 202.5 | 185.9 | 187.5 | 14 | 168 | 1.13 |
| EF | 0.892 | 202.5 | 180.6 | 183.3 | 70 | 840 | 5.50 |
| FG | 0.870 | 202.5 | 176.2 | 178.4 | 56 | 672 | 4.28 |
| | | | | Total Elongation | | | 24.39 |

Note that the length of the strand in the jack was not considered in the calculations. The total elongation calculated above must be reduced by 20 percent to account for take-up and reorienting of prestressing strand at the beginning of the stressing operation. The measurable elongation, $\Delta_{80\%}$, for this example problem is shown below:

$\Delta_{80\%} = 0.80\Delta_{100\%} = 0.80\ x\ 24.39 = 19.51$ $\boxed{\Delta_{80\%} = 19.51 \text{ in.}}$

## Example 2 – Anchor Set Calculation:

The contract plans usually identify an anchor set length of $^3/_8$ inch (10 mm). This length represents the distance the strand slips back into the anchor head during the seating process. Using the results from Example 1, what is the change in stress at the jacking end of the frame, and how far into the frame does anchor set loss affect the stress in the tendon?

Given:

- $E$ = 28,000 ksi
- $\Delta L$ = $^3/_8$ in.
- Friction loss in length L = 202.5 ksi – 192.0 ksi = 10.5 ksi = $d$

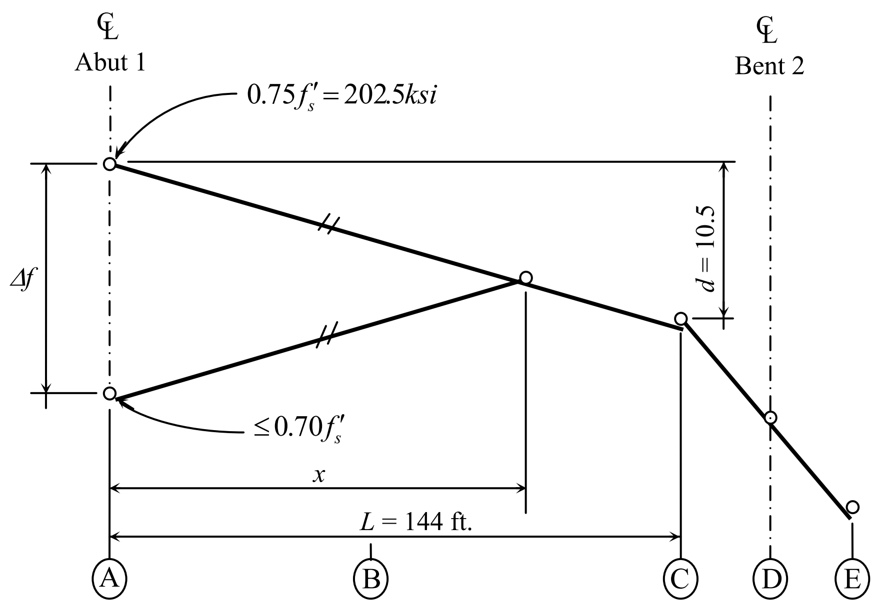



**Figure 3 – Anchor Set Loss Diagram**

$$x = \sqrt{\frac{E(\Delta L)L}{d}} = \sqrt{\frac{(28{,}000ksi)(0.375in)(144ft)}{(10.5ksi)(12in/ft)}} = 109.5ft$$

$$\Delta f = \frac{2dx}{L} = \frac{(2)(10.5ksi)(109.5ft)}{144ft} = 15.97ksi$$

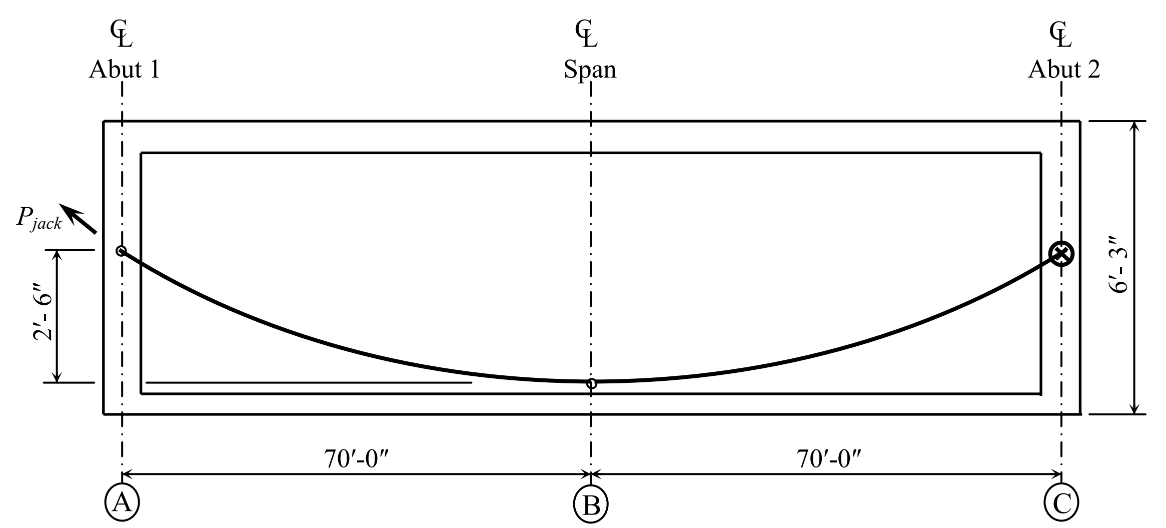
The stress at the anchorage after seating must be less than $0.70f'_s$:

{202.5 ksi – 15.97 ksi = 186.53 ksi} < {$0.70f'_s$ = 0.70 (270 ksi) = 189 ksi} $\therefore$ OK

## Example 3 – Simple Span Box Girder Stressed from One End:

Given:

- 140 ft long simply supported CIP P/S Box Girder = $L$.
- 270 ksi low relaxation strand.
- $P_{jack}$ = 12,600 kips.
- Area of 0.5 inch diameter strand = 0.153 in$^2$.
- Anchor set length = 0.375 in = $\Delta L$
- One end stressing.
- $\mu = 0.15$, $K = 0.0002$



**Center of Gravity of Prestressing Path**

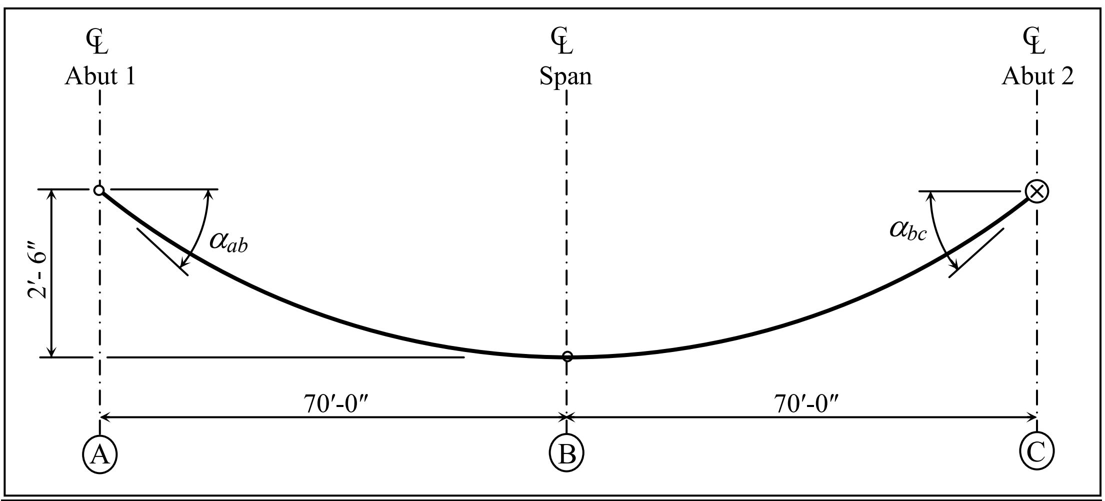
Find:

1. How many 0.5 inch diameter strands are required?
2. Find the initial and final stress distribution in the prestressing steel.
3. Find the final working force at the centerline of the span.
4. Find the theoretical and measurable elongation.

**Part 1**: Number of strands required:

$0.75 f'_s$ = The jacking stress on the contract plans = 0.75 (270 ksi) = 202.5 ksi
$A_{p/s} = P_{jack} / f_{jack}$ = 12,600 kips / 202.5 ksi = 62.22 in$^2$
$n_{p/s}$ = number of strands = $A_{p/s} / A_{strand}$ = 62.22 in$^2$ / 0.153 in$^2$ = 407 strands

**Part 2**: Initial and Final Stresses in Prestressing Steel:




Stress at dead end:

$\alpha_{ab} = \alpha_{bc}$ = 2y/L = 2 (2.5) / 70 = <u>0.0714</u>

At dead end, $\alpha_{ac}$ = 2 (0.0714) = <u>0.1428</u>

$T_x = T_o e^{-(\mu\alpha + KL)}$

At dead end, $T_c = 202.5\ e^{-[(0.15)(0.1428)+(0.0002)(140)]} = 202.5\ e^{-0.0494}$ = <u>192.73 ksi</u>

Effect of anchor set:

$$x = \sqrt{\frac{E(\Delta L)L}{d}} = \sqrt{\frac{(28{,}000ksi)(0.375in)(140ft)}{(9.77ksi)(12in/ft)}} = 112ft$$

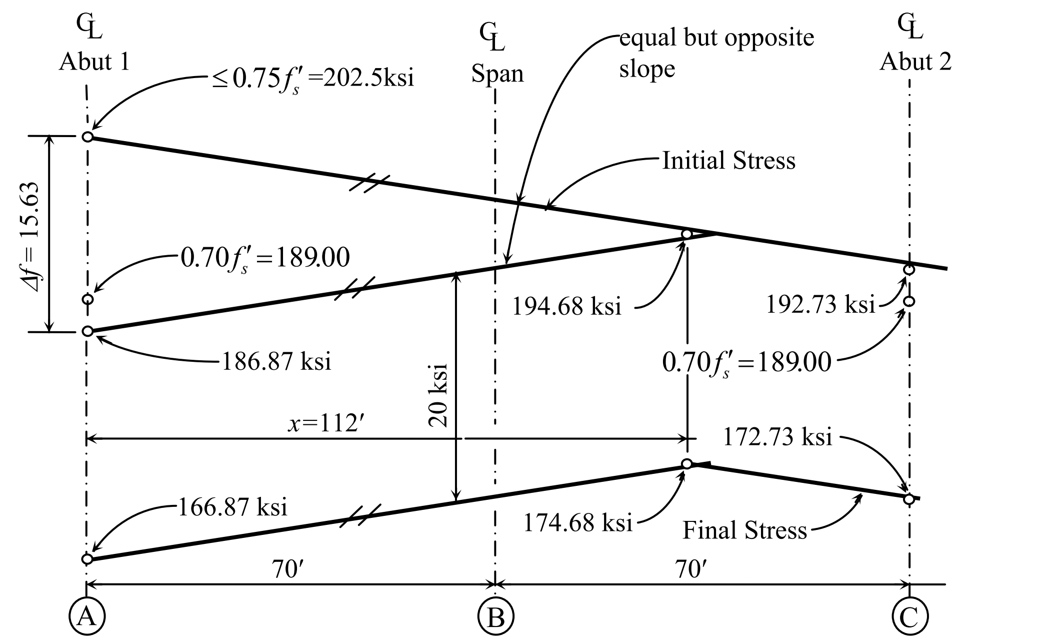
$$\Delta f = \frac{2dx}{L} = \frac{(2)(9.77ksi)(112ft)}{140ft} = 15.63ksi$$

Stress at jacking end:

$T_a = f_{jack} - \Delta f = 202.5$ ksi – 15.63 ksi = <u>186.87 ksi</u>

$\{T_a = 186.87 \text{ ksi}\} \leq \{0.70\, f'_s = 0.70\ (270 \text{ ksi}) = 189 \text{ ksi})\ \therefore$ OK

Assume long term losses = 20 ksi




**Initial and Final Stress Distribution in Prestressing Steel.**

**<u>Part 3</u>**: Final working force at the centerline of span.

$$\frac{f_b - 166.87}{70} = \frac{174.68 - 166.87}{112}$$

$$f_b - 166.87 = \frac{70(7.81)}{112}$$

$$f_b = 4.88 + 166.87 = 171.75 \quad ksi$$

**<u>Part 4:</u>** Theoretical and measurable elongation.

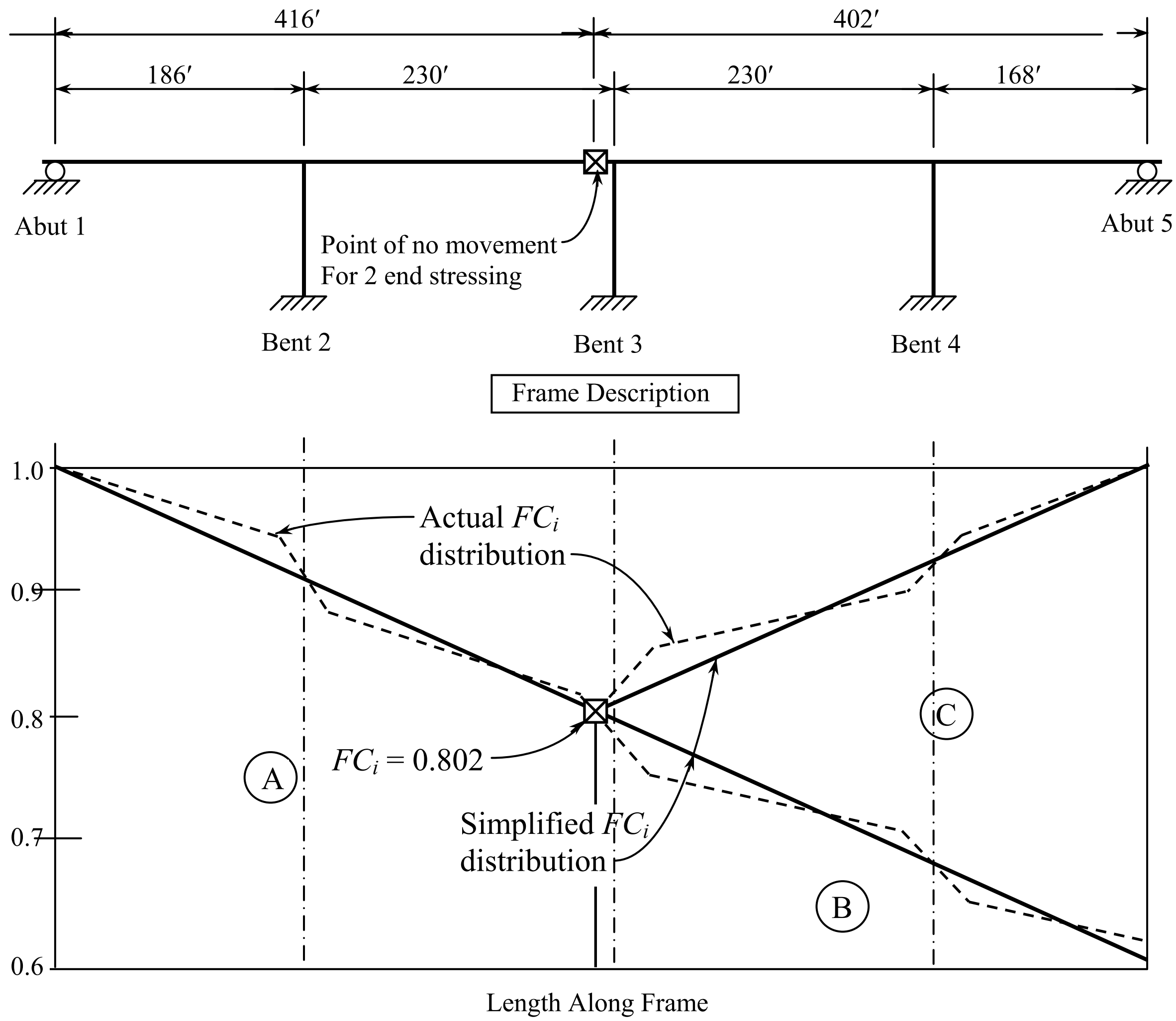
$$\Delta_{100\%} = \frac{\left[(202.5 + 192.73) ksi/2\right]\ (140\,ft)(12in/ft)}{28{,}000 ksi} = 11.86 \quad in.$$

$$\Delta_{80\%} = 0.80\ \ (11.86in) = 9.49 \quad in.$$

## Example 4 – Continuous Four Span CIP Box-Girder Stressed from Both Ends

Given:

- 818 ft long continuous 4 span CIP P/S box girder frame.
- Two end stressing, with first stage jacked from left end.
- 270 ksi Low Relaxation strand.
- Jacking stress = 202.5 ksi.
- Area of 0.5 inch diameter strand = 0.153 $in^2$.
- The initial force coefficient ($FC_i$) at the point of no movement = 0.802.
- $\mu$ = 0.20, $K$ = 0.0002 (informational only).




**Initial Force Coefficient Diagram.**

Find: What is the total theoretical (expected) 1st stage elongation?
1. What is the measurable 1st stage elongation?
2. What is the theoretical (expected) 2nd stage elongation?

**Part 1:** Total Theoretical (expected) 1st Stage Elongation:

When calculating the first stage elongation, it is common practice to break the force coefficient diagram into two parts, identified as areas A and B in the above diagram. The equation for calculating tendon elongations is shown as follows:

$$\boxed{\Delta = \frac{PL}{AE}}$$

When jacking to 202.5 *ksi*, and using a strand nominal area of 0.153 $in^2$ the jacking force per strand is calculated below:

$$P_{strand} = (202.5\ ksi)(0.153) = \underline{30.98\ kips/strand}$$

When calculating $\Delta_A$, it is important to include the length of tendon within the jack. The strand movement will be measured relative to the end of the ram, which generally results in 2½ to 3 feet of extra of strand within the length of the jack.

$$\Delta_A = \frac{(30.98\ kips)(1+0.802)}{2} x \frac{(416 ft + 3 ft)(12)}{(0.153\ in^2)(28{,}500\ ksi)} = (27.91)(1.153) = \underline{32.2\ inches}$$

In order to find the first stage elongation for area B, it is necessary to extrapolate the $FC_i$ out to the dead end of the first stage post tensioning:

$$FCi_{dead} = 1 - [(2)(1-0.802)] = \underline{0.604}$$

$$\Delta_B = \frac{(30.98\ kips)(0.802+0.604)}{2} x \frac{(402 ft)(12)}{(0.153\ in^2)(28{,}500\ ksi)} = (21.78)(1.106) = \underline{24.1\ inches}$$

$$\Delta_{A+B} = \Delta_{1st\ stage\ theo} = 32.2 + 24.1 = \underline{56.3\ inches}$$

**Part 2:** Measurable 1st Stage Elongation:

The total theoretical elongation does not have direct practical application because it does not take into account slack or strand reorientation in the tendon. The measurable elongation is determined to be 80% of the theoretical, as strands are marked with paint after being stressed to 20% of $P_{jack}$. In this case, after stressing the tendon to 20% of $P_{jack}$, the remaining 80% stressing should yield an elongation of:

$$\Delta_{1st\ stage\ meas} = (0.80)\Delta_{1st\ stage\ theo} = (0.80)(56.3\ inches) = \underline{45.0\ inches}$$

**Part 3:** Theoretical (expected) 2nd Stage Elongation:

Once the first stage stressing operation is complete, and the Engineer is satisfied with the physical measurements obtained, the second stage stressing operation can begin. Theoretical 2nd stage elongations must be calculated before stressing, to serve as a tool to guarantee that the proper amount of P/S force is being delivered to the structure. The second stage elongation equates to Area C in the force coefficient diagram. Again, the length of the tendon within the jack must be included in the calculation.

$$\Delta_C = \Delta_{2nd\ stage\ theo} = \frac{(30.98\ kips)(1-0.604)}{2}\ x\ \frac{(402\,ft + 3\,ft)(12)}{(0.153\ in^2)(28{,}500\ ksi)} = (6.13)(1.115) = \underline{6.8\ inches}$$