CHAPTER 5.2 POST-TENSIONED CONCRETE GIRDERS

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5.2.1 INTRODUCTION

5.2.2.1 General

Post-tensioned concrete box girders are widely used in highway bridges in California. Figure 5.2.1-1 shows the San Luis Rey River Bridge – a typical cast-in-place post-tensioned (CIP/PT) concrete box girder bridge.

Figure 5.2.1-1 San Luis Rey River Bridge: A Concrete Box CIP/PT Bridge

Basic concepts, definitions, and assumptions are first discussed in this Chapter. An example problem with the "longhand" solution is then worked through to illustrate a typical design procedure.

5.2.1.2 Basic Concepts

Post tensioning is one of the methods of prestressing concrete. The concrete members are cast first. Then after the concrete has gained sufficient strength, tendons (strands of high strength steel wire) are inserted into preformed ducts and tensioned to induce

compressive stresses in the expected tensile stress regions of the member. Concrete must be free to shorten under the precompression. The strands are then anchored and corrosion protection such as grout or grease is installed (Gerwick, 1997).

Before further discussing prestressing, we should compare it with conventionally reinforced concrete. Prior to gravity loading, the stress level in conventional reinforced concrete is zero. The reinforcing steel is only activated by the placement of the gravity load. The concrete and reinforcing steel act as a composite section. However, once the tensile capacity of the concrete surrounding the longitudinal reinforcement has been surpassed, the concrete cracks. Prestressed concrete activates the steel prior to gravity loading through prestressing the reinforcement. This prevents cracking at service loads in prestressed concrete.

Prestressed concrete utilizes high strength materials effectively. Concrete is strong in compression, but weak in tension. The high tensile strength of prestressing steel and high compressive strength of concrete can be utilized more efficiently by pre-tensioning high strength steel so that the concrete remains in compression under service loads activated while the surrounding concrete is compressed. The prestressing operation results in a self-equilibrating internal stress system that accomplishes tensile stress in the steel and compressive stress in the concrete that significantly improves the system response to induced service loads (Collins and Mitchell, 1997).

The primary objectives of using prestressing are to produce zero tension in the concrete under dead loads and to have service load stress less than the cracking strength of the concrete along the cross-section. Thus, the steel is in constant tension. Because of this, the concrete remains in compression under service loads throughout the life of the structure. Both materials are being activated and used to their maximum efficiency.

Figure 5.2.1-2 shows elastic stress distribution for a prestressed beam after initial prestressing.
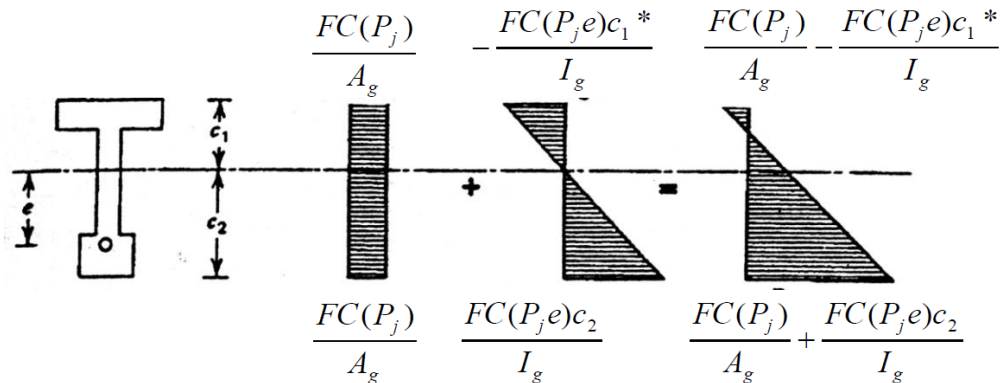
Note: *Component of Equation is negative because *c* is on the opposite side of the center of gravity from the tendon. Tension is denoted as negative (-), compression is denoted as positive (+)

Figure 5.2.1-2 Elastic Stresses in an Uncracked Prestressed Beam. Effects of Initial Prestress by Component (Nilson,1987)

The stress at any point of the cross-section can be expressed as:

$$f_{pe} = \frac{FC(P_j)}{A_g} \pm \left(\frac{FC(P_j e)y}{I_g} + \frac{MC_s(P_j)y}{I_g} \right)$$

(5.2.1-1)

Where:

A_g = gross area of section (in.^{2})

E = eccentricity of resultant of prestressing with respect to the centroid of the cross-section. Always taken as a positive (ft)

FC = force coefficient for loss

f_{pe} = effective stress in the prestressing steel after losses (ksi)

I_g = moment of inertia of the gross concrete section about the centroidal axis, neglecting reinforcement (in.^{4})

P_j = force in prestress strands before losses (ksi)

MC_s = secondary moment coefficient for loss (ft)

y = distance from the neutral axis to a point on member cross-section (in.)

The prestressing force effect is accomplished by two components of the general equation shown above as Equation 5.2.1-1. The first component is uniform compression stress due to the axial prestressing force. The second component is the bending stress caused by the eccentricity of the prestressing steel with respect to the center of gravity of the cross-section. This creates a linear change in stress throughout the beam cross-section (Figure 5.2.1-2). It is noted that the distance from the neutral axis to the fiber in question y, (y is the general term, c_1 and c_2 which are more specific terms shown in Figure 5.2.1-2), may result in a negative value for the bending part of the equation. The prestressing force may create tension across the center of gravity from the tendon, and therefore part of the beam section may be in tension prior to applying load.

The use of prestressed concrete has its advantages and limitations. Some limitations are its low superstructure ductility, the need for higher concrete compressive strengths, and larger member sizes to accommodate ducts inside the girders.

Post-tensioned box girder superstructures are commonly used due to their low costs, their performance throughout the life of the structure, and contractors' experience with the structure type. Post-tensioning also allows for thinner superstructures. A continuous superstructure increases the stiffness of the bridge frame in the longitudinal direction and gives the designer the option to fix the columns to the superstructure, reducing foundation costs.

5.2.2 MATERIAL PROPERTIES

At first glance, prestressed concrete and reinforced concrete make use of the same two core materials: concrete and steel. However, the behavior of the materials varies due to

usage. Conventional concrete structures use deformed bars for reinforcement. Most prestressed applications use tightly wrapped, low-relaxation (lo-lax) seven-wire strands. As shown in Figure 5.2.2-1, the stress-strain curves for those steel are quite different.

Table 5.2.2-1 shows the steel material properties for ASTM A706 Grade 60 and ASTM A416 PS Strand Grade 270. The mild reinforcement steel (ASTM A706 Grade 60) used for reinforced concrete has a much lower yield strength and tensile strength than the prestressing strands (ASTM A416 PS Strand Grade 270). Prestressing steel shall be high strength and possesses superior material properties. This enables a smaller quantity of steel to be used to support the bridge. Higher strength steel is also used because the ratio of effective prestress (prestress force after losses in force) to initial prestress (prestress force before losses in force) of high strength steel is much higher than that of mild steel (Figure 5.2.2-2). This is because losses, which will be discussed below, consume a large percentage of the strain in the elastic range of the mild steel, but a small portion of the prestressed steel.
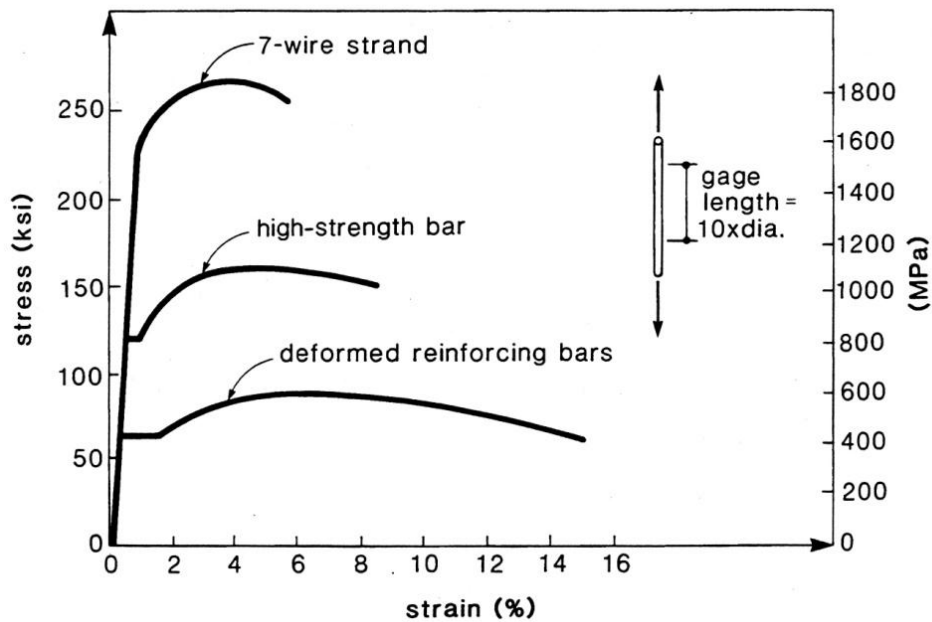
Figure 5.2.2-1 Stress-Strain Curves of Mild Steel (Deformed Reinforcing Bars) and Prestressing Steel (7-Wire Strand) (Collins and Mitchell, 1997)

Table 5.2.2-1 Steel Material Properties for Reinforced and Prestressed Concrete

ASTM A706 Grade 60		ASTM A416 Grade 270 Strand	
f_y	60 ksi	f_{py}	90% f_{pu}
f_u	80 ksi	f_{pu}	270 ksi
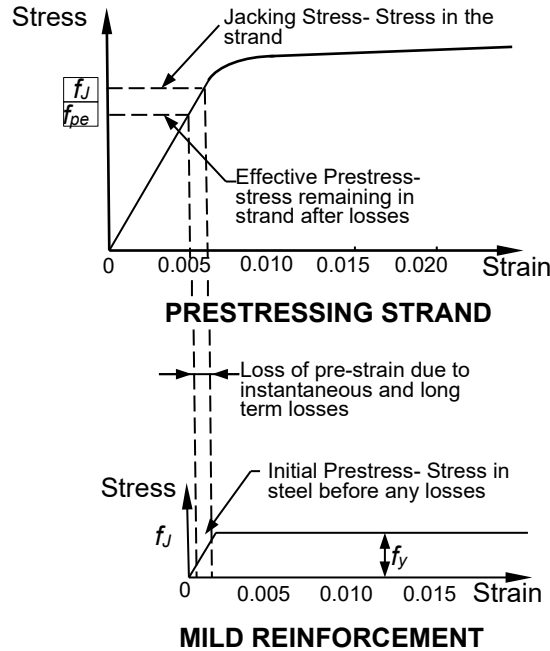
Figure 5.2.2-2 Loss Effects Comparison Between Prestressing Strand and Mild Reinforcement (Based in part on Portland Cement Association 2001 Bridge Professors' Seminar Chicago, IL)

5.2.3 GIRDER LAYOUT AND STRUCTURAL SECTION

Section design is a very important tenant of structure design. An efficient section maximizes the ability of a structure to carry applied loads while minimizing self-weight. Basic mechanics of materials theory shows that the further away the majority of the material lies from the centroid of the shape, the better that shape is at the resisting moment. A shape such as a basic "I" is perfect for maximizing flexural strength and minimizing weight. The placement of I-girders side by side results in a box; which is easier to construct and has seismic advantages over individual I-girders.

Determination of the typical section of a bridge has been made a simple process. The creation of a typical section begins with the calculation of structure depth for a given span length. Table 5.2.3-1 lists (AASHTO, 2017) the minimum structural depth for various structural spans.

Table 5.2.3-1 Traditional Minimum Depth for Constant Depth Superstructures (AASHTO Table 2.5.2.6.3-1, 2017)

Superstructure		Minimum Depth (Including Deck — When variable depth members are used, values may be adjusted to account for changes in relative stiffness of positive and negative moments sections.	
Material	Type	Simple Spans	Continuous Spans
Reinforced Concrete	Slab with main reinforcement parallel to traffic	$\dfrac{1.2(S+10)}{30}$	$\dfrac{S+10}{30} \geq 0.54$ ft.
	T-beams	0.070L	0.065L
	Box Beams	0.060L	0.055L
	Pedestrian Structure Beams	0.035L	0.033L
Prestressed Concrete	Slabs	0.030$L \geq 6.5$ in.	0.027$L \geq 6.5$ in.
	CIP Box Beams	0.045L	0.040L
	Precast I-beams	0.045L	0.040L
	Pedestrian Structure Beams	0.033L	0.030L
	Adjacent Box Beams	0.030L	0.025L
Steel	Overall Depth of Composite I-beam	0.040L	0.032L
	Depth of I-beam Portion of Composite I-beam	0.033L	0.027L
	Trusses	0.100L	0.100L

To use BDM 9.4 (Caltrans, 2021a) a simple rule of thumb is that the girder spacing for a prestressed box girder should not exceed twice the superstructure depth. A larger girder spacing may require a customized deck and soffit slab design and may result in a larger web thickness. BDM 9.4 (Caltrans, 2021a) provides the predetermined soffit and deck thickness based on the girder to girder spacing as well as the overhang length.

5.2.4 PRESTRESSING CABLE LAYOUT

To induce compressive stress along all locations of the bridge girder, the prestressing cable path must be raised and lowered along the length of the girder. A typical continuous girder is subjected to negative moments near fixed supports, and positive moments near mid-span. As Equation 5.2.1-1 shows, eccentricity determines the stress level at a given location on the cross-section. To meet the tension face criteria, the location of the prestressing cable path will be high (above the neutral axis) at fixed supports, low (below the neutral axis) at midspans, and at the centroid of the section near simply supported connections (Figure 5.2.4-1). The shape of the cable path is roughly the same as the opposite sign of the dead load moment diagram shown in Figure 5.2.4-2.
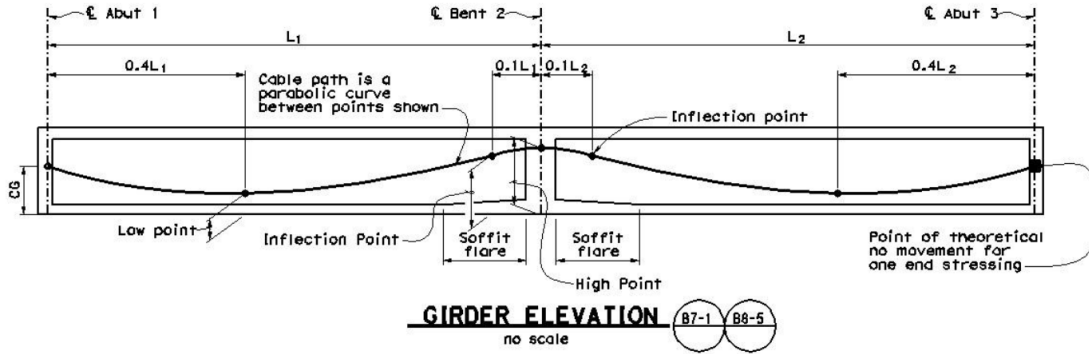
Figure 5.2.4-1 Typical Prestressing Path for a Two-Span Bridge
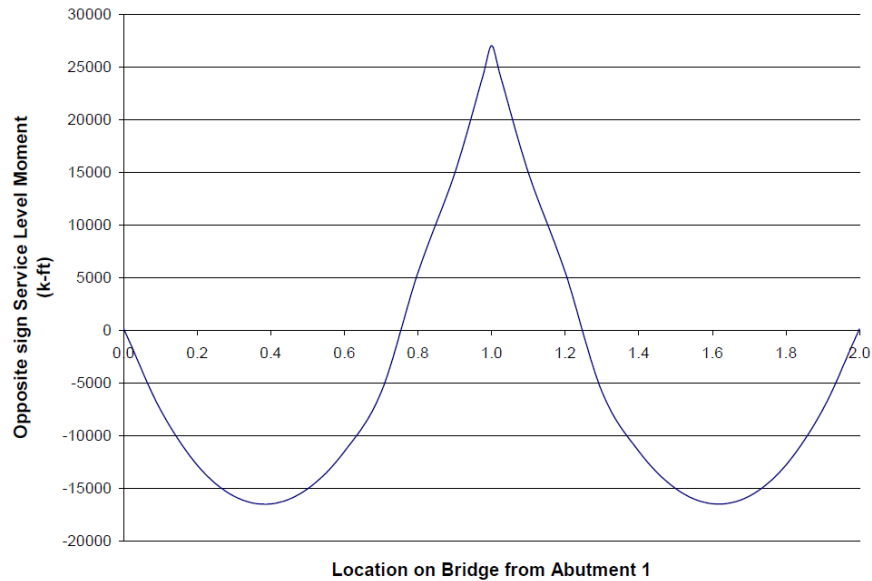
Figure 5.2.4-2 Opposite Sign of Dead Load Moment Diagram

Determining the most efficient possible final pre-stress cable layout is an iterative process. This is achieved by using the previously developed charts by Caltrans to determine a best "guess" initial prestressing force (P_{jack}) as a function of deck area, span length, and span configuration. The critical points along the cable path are those locations with larger moments. The highest point should occur at the locations of the highest negative moments, on our example bridge that would be at the bents. At its highest, the duct will fit in just below the bottom transverse mat of steel in the deck. The lowest point will occur near the mid-span and is limited by the location of the top mat of transverse soffit steel.

5.2.5 PRESTRESS LOSSES FOR POST-TENSIONING

Throughout the life of a prestressed concrete girder, the initial force applied to the pre-stress tendons significantly decreases. This decrease in the force is called loss. Loss of the stress in a girder can revert a location previously in the compression to the tension or increase the tension stress. This may be very dangerous when the stress in concrete is near its stress limit. Because of the significant impact on the structure of these losses, losses shall be quantified and accounted for in accordance with AASHTO-CA BDS-8.

In Caltrans practice, coefficients are usually used to estimate the reduction factor in the initial prestressing force to find a final prestressing force. These coefficients are called force (Equation 5.2.5-1) and moment coefficients (Equation 5.2.5-2). Both coefficients are used to determine a jacking force, as the most losses are both functions of and dependent of the jacking force. These force coefficients are given as the sum of the lost force of each component of loss divided by the allowable stress in the tendon (modification of AASHTO, 2017, Equations 5.9.3.1-1, 5.9.3.1-2).

$$FC_{pT} = \left(1 - \frac{\sum \Delta f_i}{f_{ps}}\right)$$
(5.2.5-1)

where:

FC_{pT} = force coefficient for loss

Δf_i = change in force in prestressing tendon due to an individual loss (ksi)

f_{ps} = average stress in prestressing steel at the time for which the nominal resistance is required (ksi) (AASHTO 5.6.3.1.1-1)

$$MC_p = \left(FC_{pT}\right)\left(e_x\right)$$
(5.2.5-2)

where:

MC_p = primary moment coefficient for loss (ft)

FC_{pT} = total force coefficient for loss

e_x = eccentricity as a function of x along parabolic segment (ft)

The force coefficient is defined as one at the jacking location and begins decreasing towards zero to the point of no movement. The point of no movement is a finite point of the strand that does not move when jacked and is defined as the location where internal strand forces are in equilibrium. For single-end post tensioning, the point of no movement is at the opposite anchorage from stressing. For two-end tensioning the location is where the movement in one direction is countered by movement from the other direction and is generally near the middle of the frame.

Force coefficients are determined at each critical point along the girder. The product of the force coefficients and strand eccentricities (e) is called moment coefficients. The

coefficients determined from the locked-in moments at fixed supports are used to convert initial strand moment resistant capacities into capacities after losses, or final capacities.

5.2.5.1 Instantaneous Losses

There are two types of losses: Instantaneous and long-term. The instantaneous losses are due to the anchor set, the friction, and the elastic shortening. Instantaneous losses are bridge-specific, yet still broad enough to be estimated in user-friendly equations. Therefore, a lump sum value is not used, and a bridge specific value is calculated. Given below are three different types of instantaneous losses.

5.2.5.1.1 Anchor Set Loss

The anchor set is caused by the movement of the tendon prior to the seating of the anchorage gripping device. This loss occurs prior to force transfer between the wedge (or jaws) and the anchor block. The anchor set loss is the reduction in the strand force through the loss in the stretched length of the strand. Once a force is applied to the strands, the wedges move against the anchor block until the wedges are "caught". Because of the elasticity of the strands, this movement will cause a loss in strain, stress, and force. This movement and the resulting loss of force prior to being "caught" is the anchor set loss. The force necessary to pull the movement out, will not be captured as the effective force. Even though the size of the slip is small, it still manifests as a reduction in prestressing force. AASHTO 5.9.3.2.1 suggests a common value for anchor set as 3/8 inch. This anchor set loss represents the amount of the slip in Caltrans approved anchorage systems. Equation 5.2.5.1.1-1 puts the anchor set into a more familiar change in force and the force coefficient form.

$$\Delta FC_{pA} = \frac{\Delta f_{pA}}{f_{pi}} = \frac{2(\Delta f_L)(x_{pA})}{L(f_{pi})}$$

(5.2.5.1.1-1)

$$x_{pA} = \sqrt{\frac{E_p(\Delta_{Aset})L}{12\Delta f_L}}$$

(5.2.5.1.1-2)

where:

FC_{pA} = force coefficient for loss from anchor set

x_{pA} = influence length of anchor set (ft)

E_p = modulus of elasticity of prestressing (ksi)

f_{pi} = prestressing steel stress immediately prior to transfer (ksi)

Δ_{Aset} = anchor set length (in.)

L = distance to a point of known stress loss (ft)

Δf_L = friction loss at the point of known stress loss (ksi)

Δf_{pA} = jacking stress lost in the P/S steel due to anchor set (ksi)

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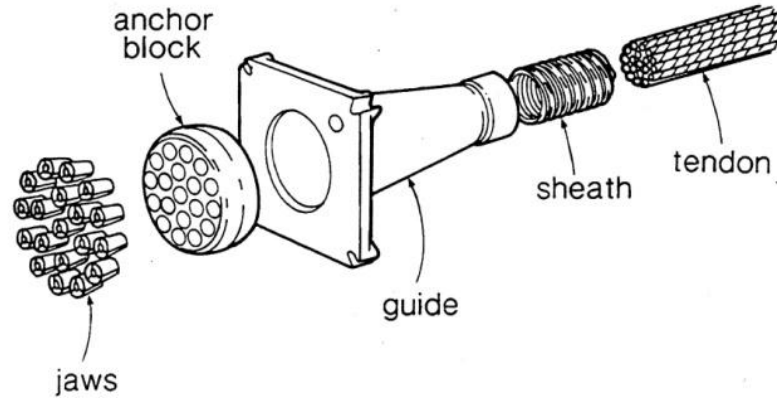
Figure 5.2.5.1-1 Anchorage System for Multi-Strand Tendon
(Collins and Mitchell, 1997)

5.2.5.1.2 Friction Loss

Friction loss is another type of instantaneous loss, which occurs when the prestressing tendons get physically caught on the ducts. This is a significant loss of force on non-linear prestressing paths because of the angle change of the ducts. The friction loss has two components: curvature and wobble frictional losses. Modified Equation AASHTO 5.9.3.2.2b-1 results in Equation 5.2.5.1.2-1, the equation used to obtain friction losses.

The curvature loss occurs when some fraction of the jacking force is used to maneuver a tendon around an angle change in a duct. An example would be: as a tendon is bending around a duct inflection point near a pier or a bent, the bottom of the tendon is touching (and scraping) the bottom of the duct. This scraping of the duct is a loss of force via friction.

$$FC_{pF} = \frac{\Delta f_{pF}}{f_{pj}} = e^{-(Kx+\mu\alpha)} \qquad (5.2.5.1.2\text{-}1)$$

where:

e	=	base of natural logarithms
FC_{pF}	=	force coefficient for loss from friction
f_{pj}	=	stress in the prestressing steel at jacking (ksi)
K	=	wobble friction coefficient (per ft of tendon)
x	=	length of a prestressing tendon from the jacking end to any point under consideration (ft)
α	=	sum of the absolute values of angular change of prestressing steel path from jacking end, or from the nearest jacking end if tensioning is done equally at both ends, to the point under investigation (rad.)
Δf_{pF}	=	loss due to friction (ksi)
μ	=	friction factor

Table 5.2.5-1 provides wobble friction coefficient and coefficient of friction as

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specified in the California Amendments (Caltrans, 2019).

Table 5.2.5-1 Friction Coefficient *K* and Coefficient of Friction μ

Type of Steel	Type of Duct	*K*	μ

Wire or strand	Rigid and semirigid galvanized metal sheathing
	Tendon Length:
	      < 600 ft
	  600 ft < 900 ft
	 900 ft < 1200 ft
	     > 1200 ft
		
		0.0002
		0.0002
		0.0002
		0.0002
			
			0.15
			0.20
			0.25
			>0.25

	Polyethylene	0.0002	0.23
	Rigid steel pipe deviators for external tendons	0.0002	0.25
High-strength bars	Galvanized metal sheathing	0.0002	0.30
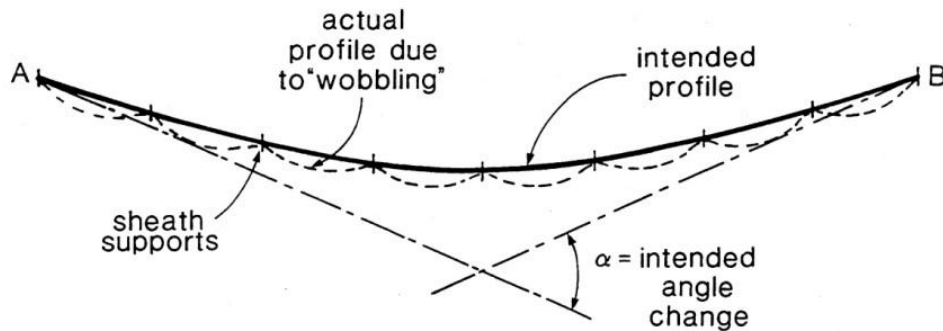
Figure 5.2.5.1-2 Wobble Friction Losses (Collins & Mitchell, 1997)

The wobble losses result from unintended angle changes of the tendon along the length of the cable path (Figure 5.2.5.1-2). These losses depend on the properties of the duct such as rigidity, diameter, support spacing, and type of the duct. Wobble losses are the accumulation of the wobble coefficient over the length of the cable path.
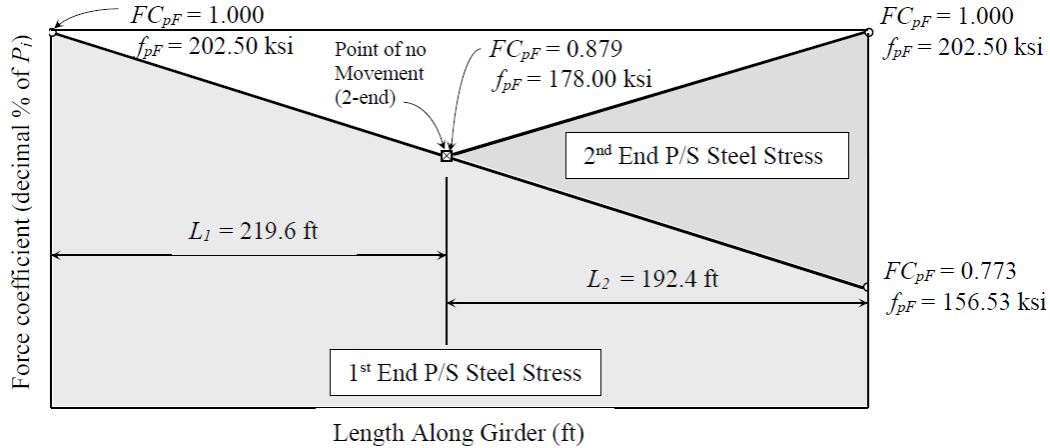
Figure 5.2.5.1-3 Two-End Stressing Versus One-End Stressing

Friction losses over a long girder begin to add up to a high percentage of the prestressing force. High friction losses can be counteracted by using two-end stressing. As stated above, two-end stressing moves the point of no movement from the anchored end to a point close to the middle of the frame. By jacking the end that was previously anchored, the friction stress that was building up in the tendons between the point of no movement and the second end is effectively pulled out thus reducing the friction losses between the second end and the point of no movement. Figure 5.2.5.1-3 shows the difference in stress when using two-end stressing instead of one for the design example in Section 5.2.12.

5.2.5.1.3 Elastic Shortening

When the pre-stressing force is applied to a concrete section, an elastic shortening of the concrete takes place simultaneously with the application of the pre-stressing force to the pre-stressing steel. It is caused by the compressive force from the tendons pulling both anchors of the concrete towards the center of the frame. Therefore, the distance between restraints has been decreased. Because of the elastic nature of the strand decreasing the distance between restraints after the pre-stressing force has been applied, thus it reduces the strain, stress, and force levels in the tendons.

The equations for the elastic shortening in pre-tensioned (such as precast elements) members are shown in AASHTO Equation 5.9.3.2.3a-1.

$$\Delta f_{pES} = \frac{E_p}{E_{ct}} f_{cgp}$$
(AASHTO 5.9.3.2.3a-1)

The equations for the elastic shortening in post-tensioned members other than slabs are shown in AASHTO Equation 5.9.3.2.3b-1.

$$\Delta f_{pES} = \frac{N-1}{2N} \frac{E_p}{E_{ct}} f_{cgp}$$
(AASHTO 5.9.3.2.3b-1)

where:

E_{ct} = modulus of elasticity of concrete at transfer or time of load application (ksi)

E_p = modulus of elasticity of prestressing tendons (ksi)

N = number of identical prestressing tendons

f_{cgp} = concrete stress at the center of gravity of prestressing tendons, that results from the prestressing force at either transfer or jacking and the self-weight of the member at sections maximum moment (ksi)

Δf_{pES} = sum of all losses or gains due to elastic shortening or extension at the time of application of prestress and/or external loads (ksi)

The California Amendments to the AASHTO LRFD specify that as the number of tendons increases, the first fractional term converges to 1/2, and the formula is simplified as follows:

$$\Delta f_{pES} = 0.5 \frac{E_p}{E_{ci}} f_{cgp}$$ (CA 5.9.3.2.3b-1)

$$f_{cgp} = f_g + f_{ps} = \frac{M_{DL} e}{I_g} - \left(\frac{P_j}{A_g} + \frac{P_j e^2}{I_g} \right)$$ (5.2.5.1.3-1)

where:

A_g = gross area of section (in.^{2})

E_{ci} = modulus of elasticity of concrete at transfer (ksi)

e = eccentricity of the anchorage device or group of devices with respect to the centroid of the cross section. Always taken as a positive (in.)

f_g = stress in the member from dead load (ksi)

f_{ps} = average stress in prestressing steel at the time for which the nominal resistance is required (ksi)

I_g = moment of inertia of the gross concrete section about the centroidal axis, neglecting reinforcement (in.4)

M_{DL} = dead load moment of structure (kip-in.)

P_j = force in prestress strands before losses (ksi)

Turning this into a force coefficient results in:

$$FC_{pES} = \frac{\Delta f_{pES}}{f_{pi}} = 0.5 \frac{E_p}{E_{ct}} \frac{f_{cgp}}{f_{pi}}$$ (5.2.5.1.3-2)

where:

FC_{pES} = force coefficient for loss from elastic shortening

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5.2.5.2 Long-Term Losses

Long-term time-dependent losses are the losses of prestress force in the tendon during the life of the structure. When using long-term losses on post-tensioned members it is acceptable to use a lump sum value (CA Amendments to AASHTO 5.9.3.3) in lieu of a detailed analysis. In the CA Amendments to AASHTO 2019, this value is 20 ksi. When completing a detailed analysis, long-term losses are the combination of the following three losses.

5.2.5.2.1 Shrinkage of Concrete

The evaporation of free water in concrete causing the concrete to lose volume is a process known as shrinkage. The amount of shrinkage, and therefore the amount of loss caused by shrinkage, is dependent on the composition of the concrete and the curing process (Libby, 1990). Empirical equations for calculating shrinkage have been developed which rely on concrete strength, and relative humidity of the region where the bridge will be placed. Combining and modifying AASHTO Equations 5.9.3.3-1, 5.9.3.3-2, and 5.9.3.3-3 results in Equation 5.2.5.2.1-1.

$$\Delta f_{pSR} = 12.0(1.7 - 0.01H)\frac{5}{\left(1+f'_{ci}\right)} \tag{5.2.5.2.1-1}$$

where:

Δf_{pSR} = prestress loss due to shrinkage of girder concrete between the transfer and deck placement (ksi)

H = average annual ambient mean relative humidity (percent)

f'_{ci} = specified compressive strength of concrete at the time of the initial loading or prestressing (ksi); nominal concrete strength at the time of application of tendon force (ksi)

5.2.5.2.2 Creep

Creep is a phenomenon of gradual increase of the deformation of concrete under sustained load. There are two types of creep, drying creep and basic creep. Drying creep is affected by moisture loss of the curing concrete and is similar to shrinkage, as it can be controlled by humidity during the curing process. Basic creep is the constant stress of the post-tensioning steel straining the concrete. Creep is determined by relative humidity at the bridge site, concrete strengths, gross area of concrete, area of prestressing steel, and stress in prestressing steel. Combining and modifying AASHTO Equations 5.9.3.3-1, 5.9.3.3-2, and 5.9.3.3-3 results in Equation 5.2.5.2.2-1.

$$\Delta f_{pCR} = 10.0\frac{f_{pi}A_{ps}}{A_g}(1.7 - 0.01H)\frac{5}{\left(1+f'_{ci}\right)} \tag{5.2.5.2.2-1}$$

where:

Δf_{pCR} = prestress loss due to creep of girder concrete between transfer and deck placement (ksi)

f_{pi} = stress in prestressing steel immediately prior to transfer (ksi)

A_{ps} = area of prestressing steel (in.^{2})

A_g = gross area of section (in.^{2})

H = average annual ambient mean relative humidity (percent)

5.2.5.2.3 Relaxation of Steel

The relaxation of steel is a phenomenon of gradual decrease of stress when the strain is held constant over time. As time goes by, the force is decreasing in the elongated steel. Relaxation losses are dependent on how the steel was manufactured (Figure 5.2.5.2.3-1). The manufacturing processes used to create prestressing strands result in significant residual stresses in the strands.

The steel can be manufactured to reduce relaxation as much as possible; this steel is called low relaxation (lo-lax). Lo-lax is generally the type of prestressing steel used in Caltrans post-tensioned girder bridges. A lo-lax strand goes through the production of high strength steel (patenting, cold drawing, stranding) and is then heated and cooled under tension. This process removes residual stresses and reduces the time-dependent losses due to the relaxation of the strand. Article 5.9.3.3 allows for the use of lump-sum values. These are given as 2.4 ksi for lo-lax and 10.0 ksi for stress-relieved steel.
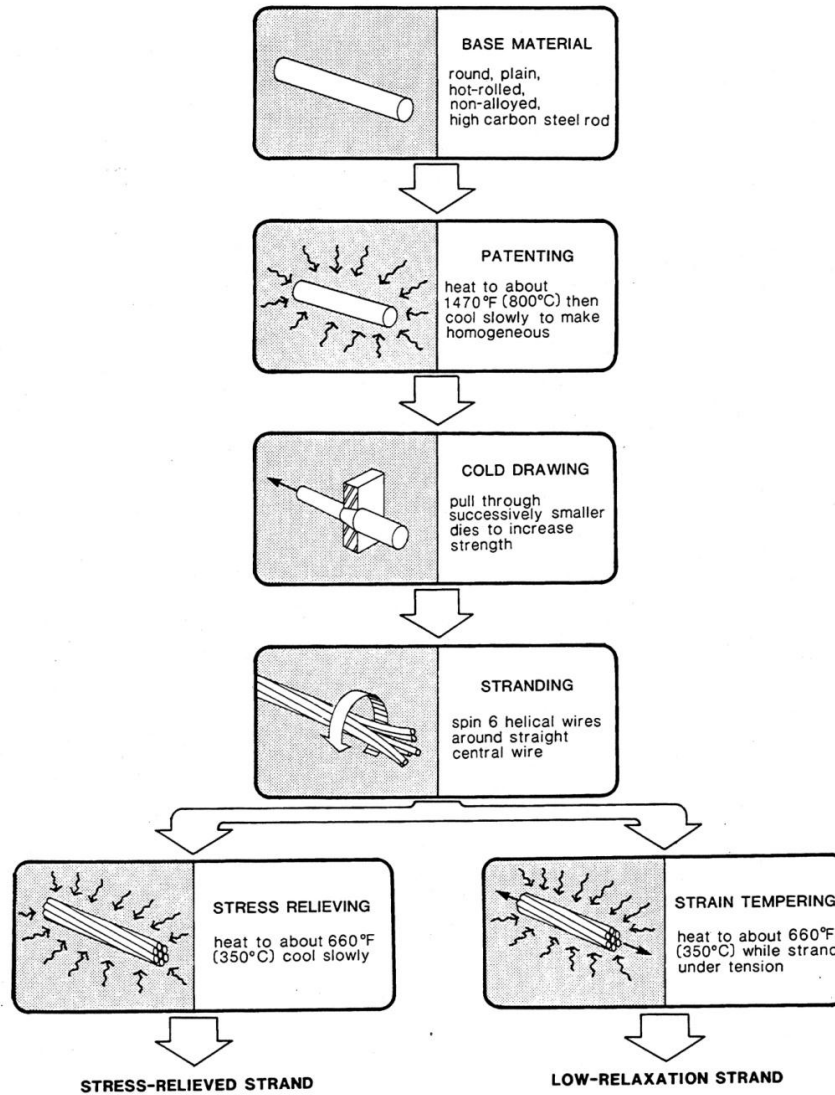
Figure 5.2.5.2.3-1 7- Wire Strand Production Method (Collins & Mitchell, 1997)

5.2.6 SECONDARY MOMENTS AND RESULTING PRESTRESS LOSS

Another type of loss exists based on the frame configuration and support boundary conditions. A continuous prestressed flexural member which is free to deform (i.e. unrestrained by its supports), will deform axially and deflect from its original shape (Libby, 1990). If the prestress reactions are restrained by the supports, moments and shear forces are created as a result of the restraint. Distortions due to primary prestress moments generate fixed end moments at rigid supports. These fixed end moments are always positive, due to the geometry of the cable path, and always enhance the effects of prestressing. These locked-in secondary moments decrease the effect of prestressing by lowering the effective prestressing force.

For statically indeterminate concrete flexural members, the loss of prestress can be tabulated by using the moment distribution method; accounting for eccentricities, and curvature of tendons. Secondary moments vary linearly between supports.

5.2.7 STRESS LIMITATIONS

5.2.7.1 Prestressing Tendons

The tensile stress is limited to a portion of the ultimate strength to provide a margin of safety against the tendon fracture or end-anchorage failures. Stress limits are also used to avoid the inelastic tendon deformation, and to limit relaxation losses. Table 5.2.7.1-1 provides the stress limitations for prestressing tendons as specified in the California Amendments (Caltrans, 2019). Those limitations can be increased if necessary, in long bridges where losses are high.

**Table 5.2.7.1-1 Stress Limitations for Prestressing Tendons CA Amendments
Table CA5.9.2.2-1, (Caltrans, 2019)**

Condition	Tendon Type		
	Plain High-Strength Bars	Low Relaxation Strand	Deformed High-Strength Bars
Pretensioning			
Prior to Seating --- short-term (f_{pbt})	0.90 f_{py}	0.90 f_{py}	0.90 f_{py}
Immediately prior to transfer (f_{pbt})	0.70 f_{pu}	0.75 f_{pu}	
At service limit state after all losses (f_{pe})	0.80 f_{py}	0.80 f_{py}	0.80 f_{py}
Post-Tensioning			
Maximum Jacking Stress– short term (f_{pbt})	0.75 f_{pu}	0.75 f_{pu}	0.75 f_{pu}
At anchorages and couplers immediately after anchor set	0.70 f_{pu}	0.70 f_{pu}	0.70 f_{pu}
Elsewhere along length of member away from anchorages and couplers immediately after anchor set	0.70 f_{pu}	0.74 f_{pu}	0.70 f_{pu}
At service limit state after losses (f_{pe})	0.80 f_{py}	0.80 f_{py}	0.80 f_{py}

5.2.7.2 Concrete

The stress in concrete varies at discrete stages within the life of an element. These discreet stages vary based on how the element is loaded, and how much prestress loss the element has experienced. The stages to be examined are the Initial Stage: Temporary Stresses Before Losses, and the Final Stage: Service Limit State after Losses, as defined by AASHTO-CA BDS-8. The prestress force is designed as the minimum force required to meet the stress limitations in the concrete as specified in Article 5.9.2.3 (AASHTO, 2017).

During the time period right after stressing, the concrete in tension is especially susceptible to cracking. This is before losses occur when prestress force is the highest and the concrete is still young. At this point, the concrete has not completely gained strength. Caltrans project plans should show an initial strength of concrete that must be met before the stressing operation can begin. This is done to indicate a strength required to resist the post tensioning during the concrete's vulnerable state. During this initial temporary state, Table 5.2.7.2-1 (AASHTO, 2017) allows for a higher tensile stress limit, and the concrete is allowed to crack. The concrete is allowed to crack because as the losses reduce the high tension stress and the young concrete strengthens, the crack widths will reduce. Then the high axial force from prestressing will pull the cracks closed.

Table 5.2.7.2-1 Temporary Tensile Stress Limits in Prestressed Concrete Before Losses in Non-Segmental Bridges (AASHTO Table 5.9.2.3.1b-1, 2017)

Location	Stress Limit
• In precompressed tensile zone without bonded reinforcement	N/A
• In area other than the precompressed tensile zone and without bonded reinforcement	$0.0948\lambda\sqrt{f'_{ci}} \leq 0.2$ (ksi)
• In areas with bonded reinforcement (reinforcing bars or prestressing steel) sufficient to resist the tensile force in the concrete computed assuming an uncracked section, where reinforcement is proportioned using a stress of 0.5 f_y, not to excessed 30.0 ksi.	$0.24\lambda\sqrt{f'_{ci}}$ (ksi)
• For handling stresses in prestressed piles	$0.158\lambda\sqrt{f'_{ci}}$ (ksi)

The final stage of a bridge's lifespan is known as the "in place" condition. Stresses resisted by concrete and prestressing steel in this condition are from gravity loads. At the service limit, the bridge superstructure concrete should not crack. The code provides for this by setting the stress limit (Table 5.2.7.2-2) to be less than the tensile strength of the concrete. Under permanent loads, the tension is not allowed in any concrete fiber (Caltrans, 2019).

Table 5.2.7.2-2 Tensile Stress Limits on Prestressed Concrete at Service Limit State, After Losses, Fully Prestressed Components (CA Amendments Table 5.9.2.3.2b-1, 2019)

Tension in the Precompressed Tensile Zone, Assuming Uncracked Sections	Stress Limit
• For components with bonded prestressing tendons or reinforcement that are subjected to not worse than moderate corrosion conditions, and are located in Caltrans Environmental "non-freeze-thaw area".	$0.19\lambda\sqrt{f_c'} \leq 0.6$ (ksi)
• For components with bonded prestressing tendons or reinforcement that are subjected to severe corrosive conditions, or are located in Caltrans Environmental "freeze-thaw area".	$0.0948\lambda\sqrt{f_c'} \leq 0.3$ (ksi)
• For components with unbonded prestressing tendons	No tension

Because concrete is strong in compression, the code-defined compression limits are much higher than the corresponding tension limits. Compression limits as shown in Table 5.2.7.2-3 are in place to prevent the concrete from crushing. The code establishes a portion of the concrete strength to resist both gravity loads and compression from the prestressing tendons. Because compression limits are dependent on stage, like tension limits, careful consideration should not only be taken for the force level in the steel but the loading conditions as well.

Table 5.2.7.2-3 Compressive Stress Limits in Prestressed Concrete at Service Limit State, Fully Prestressed Components (AASHTO Table 5.9.2.3.2a-1, 2017)

Location	Stress Limit
• Due to the sum of effective prestress and permanent loads	$0.45f_c'$ (ksi)
• Due to the sum of effective prestress, permanent loads, and transient loads as well as during shipping and handling	$0.60\phi f_c'$ (ksi)

5.2.8 STRENGTH DESIGN

Prestressing force and concrete strength are usually determined for the Service Limit States, while mild steel is determined for Strength Limit States. Flexural and shear design are discussed in detail in Chapter 5.1.

5.2.9 DEFLECTION AND CAMBER

Deflection is a term that is used to describe the degree to which a structural element is displaced under a load. The California Amendments to the AASHTO 2017 specifications define camber as the deflection built into a member, other than prestressing, to achieve

the desired grade. Camber is the physical manifestation of removing deflection from a bridge by building that deformation into the initial shape. This is done in the long term to give the superstructure a straight appearance, which is more pleasing to the public and also for drainage purposes.

There are two types of deflections. Instantaneous deflections consider the appropriate combinations of dead load, live load, prestressing forces, erection loads, as well as instantaneous prestress losses. All deflections are based on the stiffness of the structure versus the stiffness of the supports. This stiffness is a function of the moment of inertia (I_g or I_e) and modulus of elasticity (E). AASHTO Equation 5.6.3.5.2-1 defines I_e based on I_g. With I_e obtained an instantaneous deflection can be determined by using a method such as virtual work, or design software such as CT-BRIDGE.

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right]I_{cr} \leq I_g \qquad \text{(AASHTO 5.6.3.5.2-1)}$$

where:

I_e = effective moment of inertia (in.^{4})

M_{cr} = cracking moment (kip-in.)

M_a = maximum moment in a member at the stage in which the deformation is computed (kip-in.)

I_g = moment of inertia of the gross concrete section about the centroidal axis, neglecting reinforcement (in.^{4})

I_{cr} = moment of inertia of the cracked section, transformed to concrete (in.^{4})

$$M_{cr} = f_y \frac{I_g}{y_t} \qquad \text{(AASHTO 5.6.3.5.2-2)}$$

where:

f_y = specified minimum yield strength of reinforcement ((ksi)

y_t = distance from the neutral axis to the extreme tension fiber (in.)

The primary function of calculating long-term deflections is to provide a camber value. Permanent loads will deflect the superstructure down and give the bridge a sagging appearance. Prestressing force and eccentricity cause upward camber in superstructures. When calculating long-term deflections of a bridge, creep, shrinkage and relaxation of the steel should be considered. This is done by multiplying the instantaneous deflection (deflection caused by DC and PS case) by code defined factor (CA 5.6.3.5.2). This product of instantaneous deflection and the long-term coefficient is the long term deflection of the bridge. The opposite sign of these deflections is what is placed on the project plans of new structures as camber. Generally, negative cambers (upward deflection) are ignored.

(a) Uniform loading

$$\Delta = \frac{5}{384} \frac{w\ell^4}{EI}$$

(b) Constant eccentricity

$$\Delta = \frac{1}{8} \frac{Pe\ell^2}{EI}$$

(c) Single harping point

$$\Delta = \frac{2e_c + e_e}{24} \frac{P\ell^2}{EI}$$

(d) Double harping point

$$\Delta = \left[\frac{e_c}{8} - \frac{\beta^2}{6}(e_c - e_e) \right] \frac{P\ell^2}{EI}$$

(e) Parabolic profile

$$\Delta = \frac{5}{48} \frac{Pe\ell^2}{EI}$$
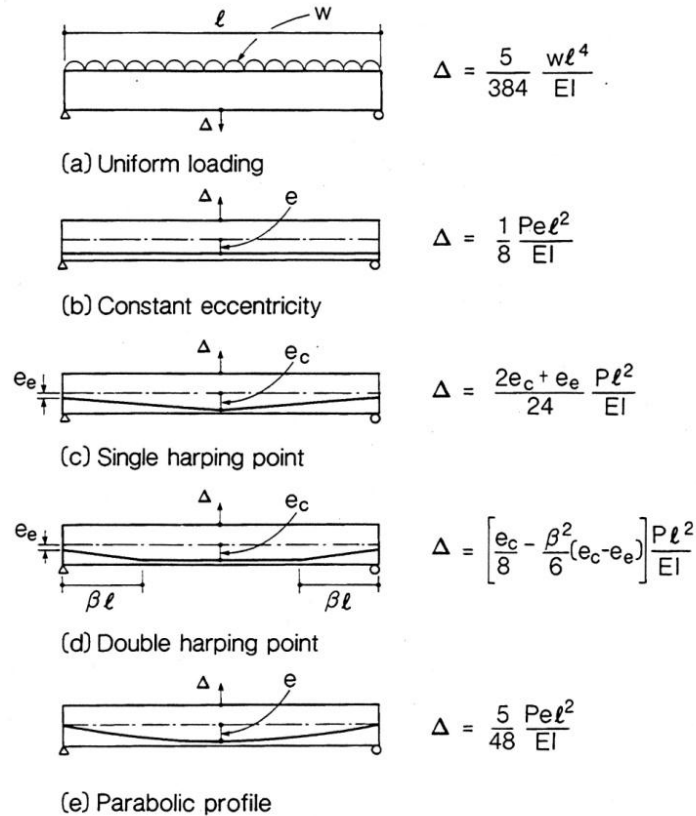
Figure 5.2.9-1 Expressions for Deflections Due to Uniform Load and Camber for Hand Checks (Collins and Mitchell, 1997)

5.2.10 POST-TENSIONING ANCHOR DESIGN

The abrupt termination of high force strands within the girder generates a large stress ahead of the anchorage. Immediately ahead of the girder are bursting stresses, and surrounding the anchorages are spalling stresses. The code specifies that for ease of design, the anchorage zone shall consist of two zones (Figure 5.2.10-1). One is a "Local Zone" consisting of high compression stresses which lead to spalling. Also, there is a General Zone consisting of high tensile stresses which lead to bursting as addressed in Article 5.9.5.6.1.
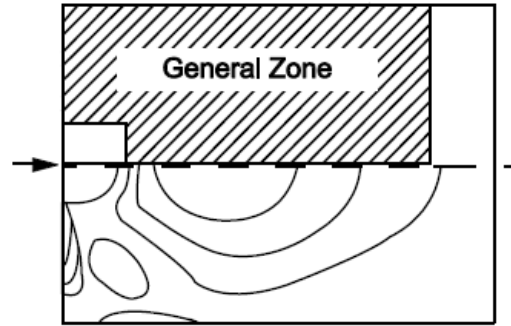
a) Principal Tensile Stresses and the General Zone
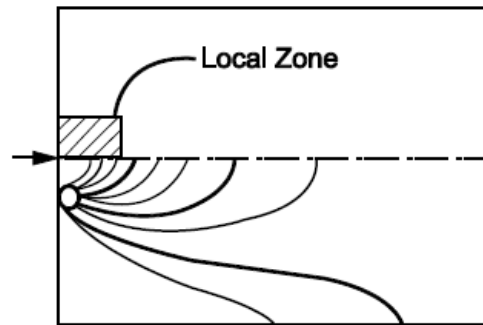
b) Principal Compressive Stresses and the Local Zone

Figure 5.2.10-1 General and Local Zone (AASHTO, 2017)

The local zone of the anchorage system is dependent on the nearby crushing demand. Compression reinforcement is used within the local zone to keep concrete from spalling and eventually crushing. The local zone is more influenced by the characteristics of the anchorage device and its anchorage characteristics than by loading and geometry. Anchorage reinforcement is usually designed by the prestressing contractor and reviewed/approved by the design engineer during the shop drawing process.

The general zone is defined by tensile stresses due to the spreading of the tendon force into the structure. These areas of large tension stresses occur just ahead of the anchorage and slowly dissipate from there. Tension reinforcement is used in the general zone as a means to manage cracking and bursting. The specifications permit the general anchorage zone to be designed using:

- the Finite Element Method
- the Approximate Method contained in the specifications
- the Strut and Tie method, which is the preferred method

The result of this design yields additional stirrups and transverse bars into and near the end diaphragm.

5.2.11   DESIGN PROCEDURE

Figure 5.2.11-1 shows flowcharts for typical design procedure for post-tensioned concrete box girders.

Start

CIP/PT bridge
superstructure
selected

Span lengths and bridge
widths are determined
by need and existing
surroundings

Span Lengths (L)
Bridge Width (W)

Material Properties

Superstructure concrete:
- f'_c = 4.0 ksi min, and 10 ksi max (Article 5.4.2.1)
- f'_{ci} = 3.5 ksi
- Normal weight concrete w_c = 0.15 kcf
- $0.75 \leq \lambda = 7.5\, w_c \leq 1.0$ (AASHTO 5.4.2.8-1)
- E_c = 120,000 $K_1\, w_c^{2.0}\, f'_c{}^{0.33}$
   = $(120{,}000)(1)(0.15)^{2.0}(4)^{0.33}$ = 4,266 ksi
   (AASHTO 5.4.2.4-1)

Prestressing Steel:
- f_{pu} = 270 ksi, f_{py} = 0.9 f_{pu} = 243 ksi
   (AASHTO Table 5.4.4.1-1)
- Maximum jacking stress, f_{pj} = 0.75 f_{pu} = 202.5 ksi
   (CA Amendments Table 5.9.2.2-1)
- E_p = 28,500 ksi (Article 5.4.4.2)

Mild Steel
- A706 bar reinforcing steel:
- f_y = 60 ksi
- E_s = 29,000 ksi (Article 5.4.3.2)

Span Lengths (*L*)
Bridge Width (*W*)
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)

Span Lengths (*L*)
Bridge Width (*W*)
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Structure Depth (D_s)
Girder Spacing (*S*, S_{ch})
Deck and Soffit Thickness (*t*, T_D)

Select Typical Section
- D/S Ratio = 0.40 Continuous, 0.45 Simple Span (AASHTO Table 2.5.2.6.3-1)
- Deck and Soffit Size, Overhang, Typical Section Rebar BDM 9.4 (Caltrans 2021a)

Seismic Design
(see other chapters)

Detail Typical
Section

Span Lengths (*L*)
Bridge Width (*W*)
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Structure Depth (D_s)
Girder Spacing (*S*, S_{ch})
Deck and Soffit Thickness (*t*, T_D)
Typical Section Rebar

Loads

DC = Dead load of structural components and non-structural attachments (Article 3.3.2)
- Unit weight of concrete (w_c) (AASHTO Table 3.5.1-1)
- Includes the weight of the box girder structural section
- Barrier rail where appropriate

DW = Dead load of wearing surfaces and utilities (Article 3.3.2)
- A future wearing surface load of 35 psf of roadway (Assume 3 in. of 0.140 kcf) = 0.035 ksf (AASHTO Table 3.5.1-1)

HL93, which includes the design truck plus the design lane load (Article 3.6.1.2)

California long-deck *P15* (CA Amendments Article 3.6.1.8)

(1)

(2)

①

Calculate Section Properties (I_g, A_g, y_b, y_b)

Span Lengths (L)
Bridge Width (W)
Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_b, y_t)

②

M_{DC}, V_{DC}, $M_{DC \, w/o \, Barrier}$
M_{DW}, V_{DW}
$M+_{HL93}$, $M-_{HL93}$,
assoc $V+_{HL93}$, $V-_{HL93}$
$V+_{HL93}$, $V-_{HL93}$,
assoc $M+_{HL93}$, $M-_{HL93}$
$M+_{P-15}$, $M-_{P-15}$,
assoc $V+_{P-15}$, $V-_{P-15}$
$V+_{P-15}$, $V-_{P-15}$,
assoc $M+_{P-15}$, $M-_{P-15}$

**Estimate P_j by using Figure 5.2.12.6-2
and Equation 5.2.12.6-1**

Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_b, y_t)
Estimated Prestress Force ($P_{j(est)}$)

Prestressing Path
• Minimum distance to soffit and deck
  STP 5.2 & BDM 5.22
• One-end versus two-end stressing

Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_b, y_t)
Estimated Prestress Force ($P_{j(est)}$)
Eccentricities (e)

Calculate Losses
Instantaneous Losses
• Anchor Set Loss (Article AASHTO 5.9.3.2.1)
• Friction Losses (CA Amendments Table
  5.9.3.2.2b-1)
• Elastic Shortening (AASHTO 5.9.3.2.3)

Long Term Losses
• Shrinkage of Concrete (AASHTO 5.3.5.3)
• Creep (AASHTO 5.9.3.3)
• Relaxation of Steel (AASHTO 5.9.3.3)

④ ⑤

③

Chapter 5.2 – Post-Tensioned Concrete Girders

Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_b, y_t)
Estimated Prestress Force ($P_{j(est)}$)
Eccentricities (e)
Anchor Set Loss (Δf_{pA})
Friction Loss (Δf_{pF})
Elastic Shortening Loss (Δf_{pES})
Shrinkage Loss (Δf_{pSH})
Creep (Δf_{pCR})
Relaxation of Steel (Δf_{pR})

Secondary Moments
- Moment Distribution

Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_b, y_t)
Estimated Prestress Force ($P_{j(est)}$)
Eccentricities (e)
Anchor Set Loss (Δf_{pA})
Friction Loss (Δf_{pF})
Elastic Shortening Loss (Δf_{pES})
Shrinkage Loss (Δf_{pSH})
Creep (Δf_{pCR})
Relaxation of Steel (Δf_{pR})
Secondary Moment Coefficient (MC_s)

Coefficients
- Force Coefficients
- Moment Coefficients

Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_b, y_t)
Estimated Prestress Force ($P_{j(est)}$)
Eccentricities (e)
Friction Force Coefficient (FC_{pF})
Final Force Coefficient (FC_{pT})
Final Moment Coefficient (MC_{pT})

Environmental Area

Non-Freeze-Thaw   Freeze-Thaw

④ ⑤ ⑥ ⑦ ②

Calculate P_j – Service III
Components with unbonded prestressing tendons
(CA Amendments Table 5.9.2.3.2b-1)

$$\frac{M_{DC+DW}\,y}{I} + \frac{FC_{pT}\left(P_j\right)}{A} + \frac{MC_{pT}\left(P_j\right)y}{I} = 0$$

Components with bonded prestressing tendons
or reinforcement (CA Amendments Table
5.9.2.3.2b-1) ≤ 0.6 (ksi)

$$\frac{M_{DC+DW}\,y}{I} + \frac{FC_{pT}\left(P_j\right)}{A} + \frac{MC_{pT}\left(P_j\right)y}{I} = 0.19\lambda\sqrt{f'_c}$$

Calculate P_j – Service III
Components with unbonded prestressing tendons
(CA Amendments Table 5.9.2.3.2b-1)

$$\frac{M_{DC+DW}\,y}{I} + \frac{FC_{pT}\left(P_j\right)}{A} + \frac{MC_{pT}\left(P_j\right)y}{I} = 0$$

Components with bonded prestressing tendons
or reinforcement (CA Amendments Table
5.9.2.3.2b-1) ≤ 0.3 (ksi)

$$\frac{M_{DC+DW}\,y}{I} + \frac{FC_{pT}\left(P_j\right)}{A} + \frac{MC_{pT}\left(P_j\right)y}{I} = 0.0948\lambda\sqrt{f'_c}$$

Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_b, y_t)
Estimated Prestress Force ($P_{j(est)}$)
Eccentricities (e)
Prestress Force (P_j)
Friction Force Coefficient (FC_{pF})
Final Force Coefficient (FC_{pT})Final
Moment Coefficient (MC_{pT})

No ⟵ ◇ Was P_j closely approximated?
i.e. is $P_j \approx P_{j(est)}$

Yes

⑤ ⑧

⑤　　　　　　　　　　　⑧　　　　　　　　　　　　　　　②

Do Elastic Shortening Losses converge on assumed?
Now that P_j is known, does the assumed Elastic Shortening value approximate the actual?

No

Yes

Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_b, y_t)
Prestress Force (P_j)
Eccentricities (e)
Friction Force Coefficient (FC_{pF})
Final Force Coefficient (FC_{pT})
Final Moment Coefficient (MC_{pT})

Calculate f_{pi} and f_{pe}
Effective stress in concrete after initial losses but before long term losses:

$$f_{pi} = \frac{P_j FC_F}{A_g} + \frac{P_j(e)(FC_F)y_b}{I_g}$$

Effective stress in steel after all losses:

$$f_{pe} = \frac{P_j FC_{PT}}{A_g} + \frac{P_j(e)(FC_{PT})y_b}{I_g}$$

Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_b, y_t)
Prestress Force (P_j)
Eccentricities (e)
Friction Force Coefficient (FC_{pF})
Final Force Coefficient (FC_{pT})
Final Moment Coefficient (MC_{pT})
Initial effective steel stress (f_{pi})
Final effective steel stress (f_{pe})

Design f'_c and f'_{ci} (AASHTO Table 5.9.2.3.2a-1 & Article 5.9.2.3.1a)

Dead Load Only

$$f'_c \geq \frac{f_{pe} + f_{DC+DW}}{0.45}$$

Live Load + Dead Load

$$f'_c \geq \frac{f_{pe} + f_{DC+DW} + f_{HL93}}{0.60\phi_w}$$

Dead Load w/o barrier

$$f'_{ci} \geq \frac{f_{pe} + f_{DC+DW\ w/o\ b}}{0.65}$$

f_{DC+DW}
f_{HL93}

Convert Moments into Stresses

Dead Load Only

$f = My / I$

⑨　　　　　　　　　　　　　　　　　　　　　　②

(9) (2)

Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_b, y_t)
Prestress Force (P_i)
Eccentricities (e)
Friction Force Coefficient (FC_{pF})
Final Force Coefficient (FC_{pT})
Final Moment Coefficient (MC_{pT})
Compressive Strength of Conrete (f'_c)
Initial Compressive Strength of Conrete (f'_{ci})

Flexural Reinforcement Design – Strength I and II
AASHTO 5.6.3.1.1-1

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$

AASHTO 5.6.3.1.1-4 (for rectangular section, see AASHTO for T-sections):

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{\alpha_1 f'_c \beta_1 b + k A_{ps}\dfrac{f_{pu}}{d_p}}$$

AASHTO 5.6.3.2.2-1

$$M_n = \underbrace{A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right)}_{\substack{\text{Presstressed}\\\text{Steel Only}}} + \underbrace{A_s f_s\left(d_s - \frac{a}{2}\right)}_{\substack{\text{Additional}\\\text{Flexural Steel}}} - \underbrace{A'_s f'_s\left(d'_s - \frac{a}{2}\right)}_{\substack{\text{Compression}\\\text{Steel}}} + \underbrace{\alpha_1 f'_c (b - b_w)h_f\left(\frac{a}{2} - \frac{h_f}{2}\right)}_{\substack{\text{Flanged Section Component}\\ a = \beta_1 c \le h_f}}$$

Minimum Requirement M_n / ϕ shall be larger than or equal to the lesser of:

- 1.33 M_u as defined in Article 5.6.3.3 or AASHTO 5.6.3.3-1

$$M_{cr} = \gamma_3\left[\left(\gamma_1 f_r + \gamma_2 f_{cpe}\right)S_c - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right)\right]$$

Modified AASHTO 5.6.3.1.1-4 (for rectangular section, see AASHTO for T-sections)

$$a = \frac{\dfrac{P_i}{0.75f_{pu}}f_{pu} + A_s f_s - A'_s f'_s}{\alpha_1 f'_c b + \dfrac{k}{\beta_1}\dfrac{P_i}{0.75f_{pu}}\dfrac{f_{pu}}{d_p}}$$

Use AASHTO 5.6.3.2.2-1 to solve for A_s once a is known.

(2)

Chapter 5.2 – Post-Tensioned Concrete Girders

Material Properties (f_{pu}, f_y, E_p, E_s, E_c)
Section Properties (I_g, A_g, y_b, y_t)
Prestress Force (P_j)
Eccentricities (e)
Friction Force Coefficient (FC_{pF})
Final Force Coefficient (FC_{pT})
Final Moment Coefficient (MC_{pT})
Compressive Strength of Conrete (f'_c)
Initial Compressive Strength of Conrete (f'_{ci})

Area of Additional Mild Steel (A_s)

(2)

Shear Reinforcement Design – Strength I and II
See AASHTO Figure CB5.2-5

Prestress Force (P_j)
Area of Additional Mild Steel (A_s)
Area of Shear Stirrup (A_v)
Shear Stirrup Spacing (s)
Final Force Coefficient (FC_{pT})
Compressive Strength of Conrete (f'_c)
Initial Compressive Strength of Conrete (f'_{ci})

End

Figure 5.2.11-1 Design Procedure for Post-Tensioned Concrete Box Girders

5.2.12 DESIGN EXAMPLE

5.2.12.1 Prestressed Concrete Girder Bridge Data
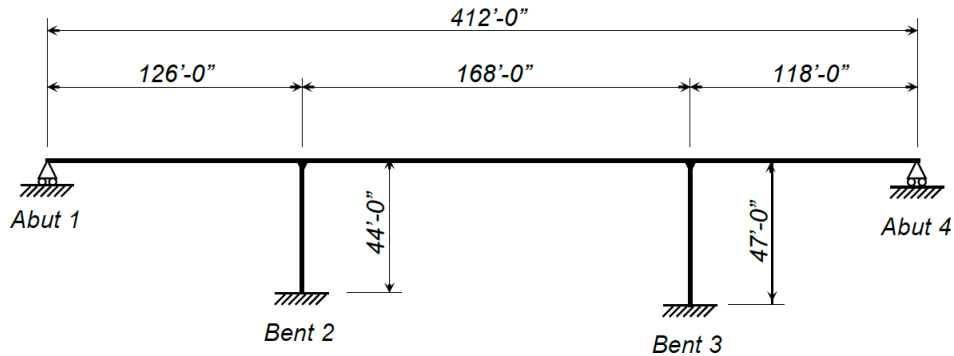
Figure 5.2.12.1-1 Elevation View of the Example Bridge

It is important, in a two-span configuration, to try and achieve equal, or nearly equal spans. In frames consisting of three or more spans, the designer should strive for 75% end spans, with nearly equal interior spans whenever possible.

The overall width of the bridge in this example is based on the following traffic requirements:

1. 3 – 12-foot lanes with traffic flow in the same direction
2. 2 – 10-foot shoulders
3. 2 – Type 842 barriers supporting Type 7 chain link railing
4. Overall bridge width, W = 59 ft – 6 in. (See Figure 5.2.12.3-1)

Materials

Superstructure Concrete:

f'_c =   4.0 ksi min, and 10.0 ksi max (Article 5.4.2.1)

$f'_{c\ min}$ =   3.5 ksi

Normal weight concrete w_c = 0.15 kcf

$$E_c = 120,000K_1 w_c^{2.0} f'^{0.33}_c$$
$$= (120,000)(1)(0.15)^{2.0}(4)^{0.33} = 4,266 \text{ ksi}$$

(AASHTO 5.4.2.4-1)

Article 5.4.2.8 defines the concrete density modification factor, λ, as follows:

- Where the splitting tensile strength, f_{ct}, is specified:

$$\lambda = 4.7 \frac{f_{ct}}{\sqrt{f'_c}} \leq 1.0$$ (AASHTO 5.4.2.8-1)

- Where f_{ct} is not specified:

$$0.75 \leq \lambda = 7.5 w_c \leq 1.0$$ (AASHTO 5.4.2.8-2)

- Where normal weight concrete is used, λ shall be taken as 1.0.

Since normal weight concrete is used, λ = 1.0.

Prestressing Steel:

f_{pu} = 270 ksi, f_{py} = 0.9 f_{pu} = 243 ksi (AASHTO Table 5.4.4.1-1)

Maximum jacking stress, f_{pj} = 0.75 f_{pu} = 202.5 ksi (CA Table 5.9.2.2-1)

E_p = 28,500 ksi (AASHTO Article 5.4.4.2)

Mild Steel:

A706 bar reinforcing steel

f_y = 60 ksi, E_s = 29,000 ksi

5.2.12.2 Design Requirements

Perform the following design portions for the box girder in accordance with the AASHTO-CA BDS-8 (AASHTO, 2017; Caltrans, 2019).

5.2.12.3 Select Girder Layout and Section

AASHTO Table 2.5.2.6.3-1 states that the traditional minimum depth for a continuous CIP box girder shall be calculated using 0.040 L, where L is the length of the longest span within the frame.

Structure depth, d ~ (0.040)(168) = 6.72 ft   use: d = 6.75 ft = 81 in.

Assuming an overhang width that is about 40 – 50% of the clear spacing between girders, and that the maximum girder spacing, S_{max}, should not exceed (2)(d), where d = structure depth, find the number and spacing of the girders. Overhang width should be limited to 6′-0″ max. When span lengths are of similar length on the same structure, it's generally a good idea to use the same depth for the entire frame.

Maximum girder spacing, S_{max} = (2)(6.75) = 13.50 ft

- Try 4 girders: As an estimate, assuming the combined width of the overhangs is approximately equal to a bay width, *S~W/4*. Therefore S_4 = 59.5/4 = 14.88 ft. Since S_4 = 14.88 > S_{max} = 13.50 ft, an extra girder should be added to the typical section.
- Try 5 girders: S_5 = 59.5/5 = 11.9 ft. Since S_5 = 11.9 ft < S_{max} = 13.50 ft, 5 girders should be used to develop the typical section. Using 5 girders will improve shear resistance, provide one more girder stem for placing *P/S* ducts, and keep the overhang width less than 6 feet. With 5 girders use an exterior girder spacing of 11 ft -11 in. and an interior girder spacing of 12 feet.
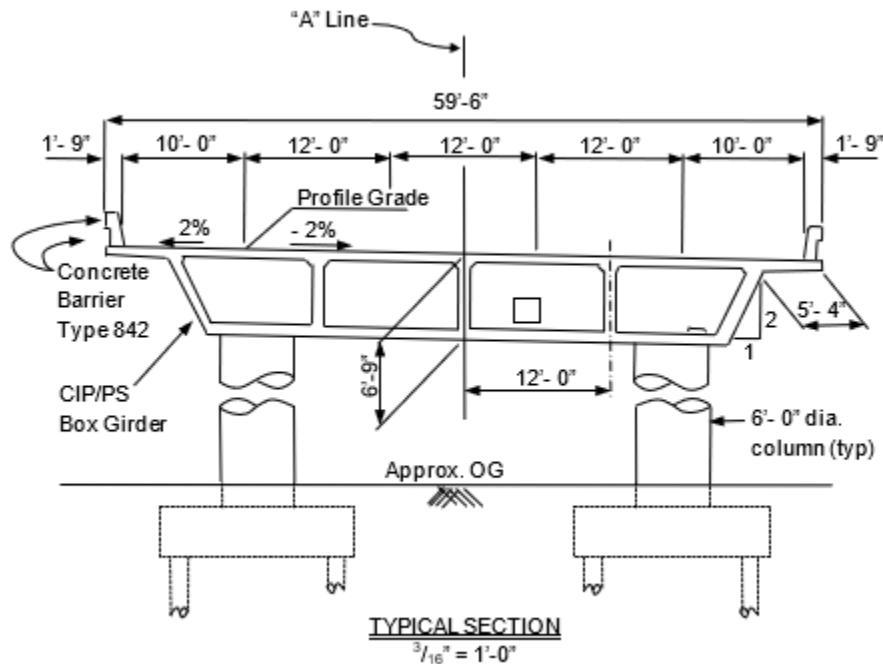
Figure 5.2.12.3-1 Typical Section View of Example Bridge

- The column diameter D_c = 6 ft
- 2:1 Sloped exterior girders used for aesthetic purposes
- Assuming no other issues, the distance as measured perpendicular to the "*A*" line from the "*A* " line to the centerline of the column is 16 ft-7 in.
- Bent cap width is D_c + 2 ft = 8 ft
- Overhang thickness varies from 12 in. at the exterior girder to 8 in. at the Edge of Deck (*EOD*)
- Girders are 12 in. wide to accommodate concrete vibration
- Exterior girders flared to 18 in. minimum at abutment diaphragms to accommodate prestressing hardware
- Soffit flares to 12 in. at the face of bent caps for seismic detailing and to optimize prestress design. Length of flares approximately 1/10 Span Length
- All supports are skewed 20° relative to the centerline of the bridge. Use the

abutment diaphragm thickness of 3 ft -3 in.

- Four-inch fillets are to be located between perpendicular surfaces except for those adjoining the soffit

5.2.12.4 Determine Basic Design Data

Section Properties

Prismatic Section (midspan):

- Area (A_g) = 103 ft^2
- Moment of Inertia (I_g) = 729 ft^4
- Bottom fiber to C.G. (y_b) = 3.80 ft

Flared Section (bent):

- Area (A_g) = 115 ft^2
- Moment of Inertia (I_g) = 824 ft^4
- Bottom fiber to C.G. (y_b) = 3.50 ft

Loads

DC = Dead load of structural components and non-structural attachments (AASHTO Article 3.3.2)

- Unit weight of concrete (w_c) = 0.15 kcf (Article 3.5.1)
- Includes the weight of the box girder structural section
- Type 842 Barrier rail on both sides (0.64 klf each)

DW = Dead load of wearing surfaces and utilities (AASHTO Article 3.3.2)

- A future wearing surface load of 35 psf of roadway (CA Article 3.5.1)
- HL93, which includes the design truck plus the design lane load (Article 3.6.1.2)
- California P15 (CA Article 3.6.1.8, Caltrans, 2019)

Slab design: The design is based on the approximate method of analysis – strip method requirements from Article 4.6.2.1 (AASHTO, 2017), and the slab is designed for strength, service, and extreme event Limit States as specified in Article 9.5. Caltrans BDM 9.4 (Caltrans, 2021a) provides deck thickness and reinforcement.

5.2.12.5 Design Deck Slab and Soffit

Deck Slab: Refer to STP 9.4 and BDM 9.4. Enter the centerline girder spacing into the design chart and read the required slab thickness and steel requirements. In this example, the centerline spacing of girders is a maximum of 12 feet. Using the chart, the deck slab thickness required is 9 inches

Table 5.2.12.5-1 LRFD Deck Design Chart, taken from BDM 9.4 (Caltrans, 2021a)

Table 9.4.5.2 Deck Design for Cast-in-Place Prestressed Box Girders, Precast "I"-Girders, and Steel Girders with flange widths 24 inches or greater

NOTE: The negative moment design section for transverse reinforcing is 6 inches from the girder centerline.

Girder CL to CL Spacing (feet)	Deck Slab Thickness, T (inches)	Transverse Bar Size and Maximum Bar Spacing, S (inches)		F dimension (inches)	#5 "D" Bars per Bay (quantity)	#4 "G" Bars per Bay (quantity)
		Bar Size	S			
11.75	8.875	#6	11	17	23	5
12.0	9.0	#6	11	17	23	5
12.25	9.125	#6	11	18	23	5

Soffit Slab: Refer to Caltrans BDM 9.4. Enter the effective girder spacing into the design chart. Read the required slab thickness and steel requirements. In this example, the effective spacings for interior and exterior bays are 11 ft and 8 ft - 4 in., respectively.

Use a constant soffit thickness of 8.25 in. and "*E*" bar spacing based on 11 ft, and "*H*" bar spacing based on individual bay widths.

Table 5.2.12.5-2 LRFD Soffit Design Chart, taken from BDM 9.4 (Caltrans, 2021a)

Girder Clear Span (feet)	Minimum Slab Thickness (inches)	"E" Bar Size and Maximum Spacing (inches)	"H" Bar Total Quantity per Bay and Size
10.75	8.125	#5 at 15	8 - #7
11.0	8.25	#5 at 15	8 - #7
11.25	8.50	#5 at 14	8 - #7
11.50	8.625	#5 at 14	8 - #7

5.2.12.6 Select Prestressing Cable Path

In general, the maximum eccentricities (vertical distance between the C.G.s of the superstructure concrete and the *P/S* steel) should occur at the points of the maximum gravity moment. These points usually occur at the maximum negative moment near the bent cap, or the maximum positive moment regions near the midspan. To define the prestressing path for this frame, we need to get an estimate of the jacking force, P_j. An estimate of P_j will aid us in determining how much vertical room is needed to physically fit the strands/ducts in the girders and will help optimize the prestress design.

Estimate pounds of *P/S* steel per square ft of deck area using the chart found on the next page.
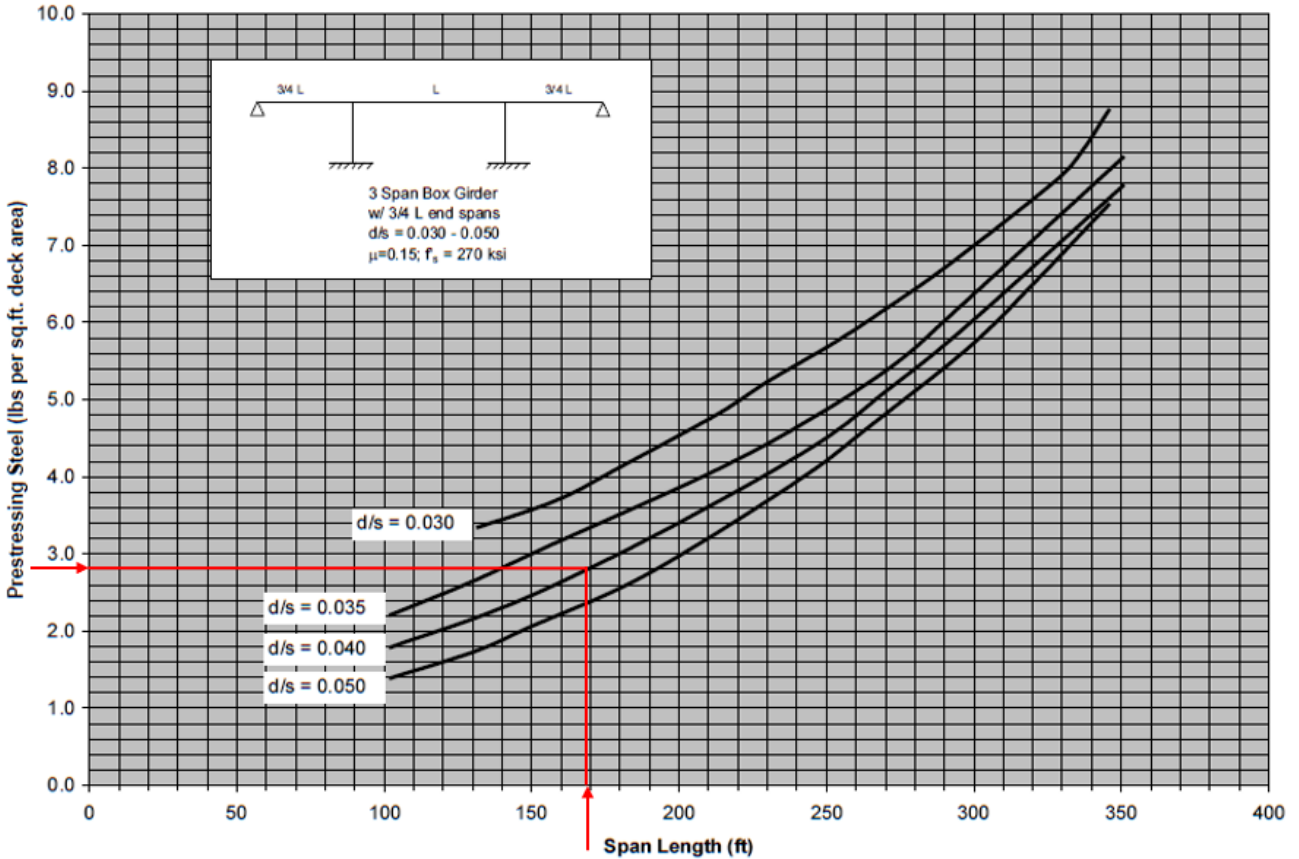
Figure 5.2.12.6-1 Estimate of Required P_j from the Modified for use with English Units

With a length of span = 168 ft, and a depth of span ratio of 0.040, read 2.8 lb per square ft deck area off the chart shown below:

Estimate P_j using the following equation developed by Caltrans for English units

$$P_j = \frac{W \times 202.5}{L_{frame} \times 3.4}$$

(5.2.12.6-1)

where:

P_j =   force in prestress strands before losses (ksi)

L_{frame} =   length of frame to be post-tensioned (ft)

W =   weight of prestressing steel obtained from Figure 5.2.12.6-1

Total weight of prestressing steel

$$= \left(2.80 \text{ lb/ft}^2\right)\left(\text{deck area}\right)$$

$$= \left(2.80 \text{ lb/ft}^2\right)\left(412.0 \text{ ft}\right)\left(59.5 ft\right) = 68,639 \text{ lb}$$

$$P_j = \frac{68,639 \times 202.5}{412 \times 3.4} = 9,922 \text{ kips}$$

Develop preliminary maximum eccentricities at midspan and bent cap using STP 5.2 and Figure 5.2.12.6-2.

Determine the "D" value based on an estimate of P_j:

- P_j/girder = 9,922 kips/5 girders = 1,984 kips/girder.
- Enter the "D" chart for cast-in-place girders, and record a value of "D" as 5 inches. This chart accounts for the "Z" factor, which considers the vertical shift of the tendon within the duct, depending on whether you are at midspan, or the centerline of bent. The "D" values produced in this chart are conservative, and the designer may choose to optimize the prestressing path by using an actual shop drawing to compute a "D" value.
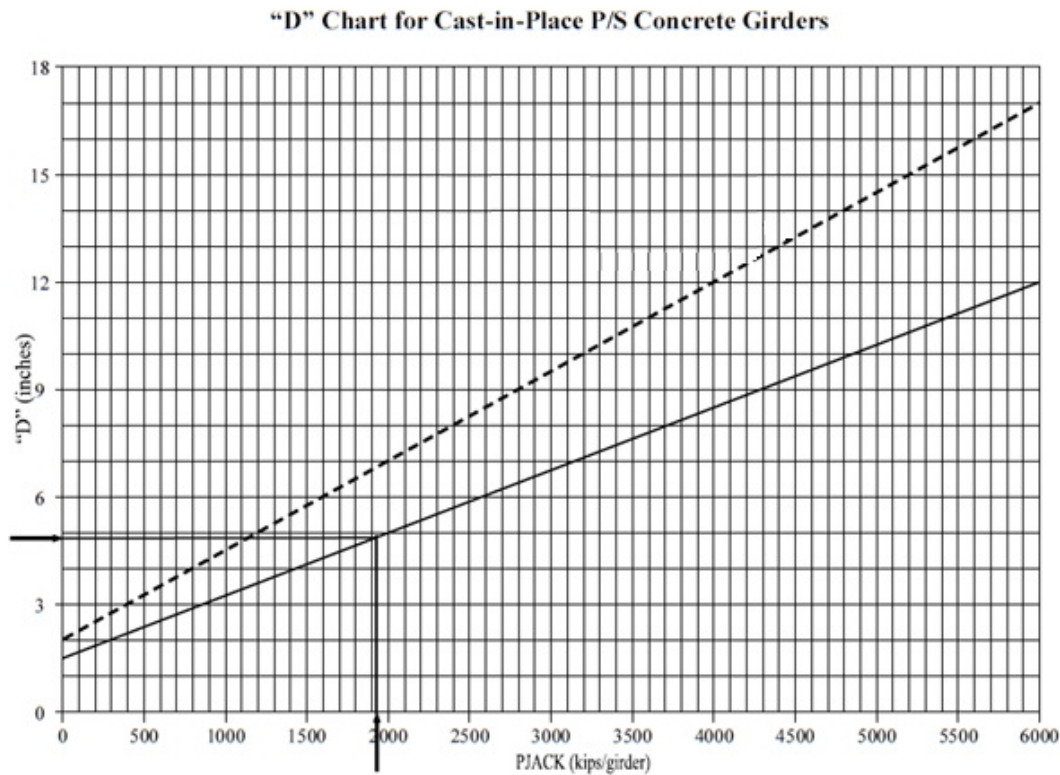
Figure 5.12.6-2 "D" Chart used to Optimize Prestressing Tendon Profile

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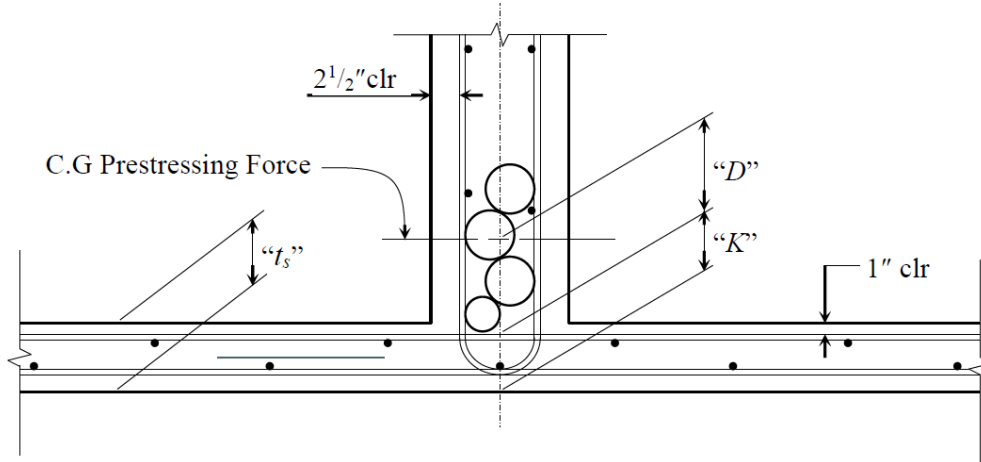
Figure 5.2.12.6-3 Tendon Configuration at Low Point of Tendon Profile

The value *K* for a bridge with a skew of less than 20° is the distance from the top mat of steel in the soffit to the bottom mat of steel in the soffit. Therefore, for this bridge, *K* is:

$$K = t - clr_{int} \tag{5.2.12.6-2}$$

where:

K = distance to the closest duct from the bottom of the soffit or top of the deck (in.)

t = thickness of the soffit or deck (in.)

clr_{int} = clearance from the interior face of the bay to the first mat of steel in the soffit or deck (usually taken as 1 in.) (in.)

$$\delta_{lp} = t_s - 1 + "D" \tag{5.2.12.6-3}$$

where:

δ_{lp} = offset from soffit to centroid of duct (in.)

t_s = thickness of soffit (in.)

Using Figure 5.2.12.6-3 and Equation 5.2.12.6-3, determine the offset from the bottom fiber to the *C.G.* of the *P/S* path at the low point:

$\delta_{lp} = t_{soffit} - 1 + "D" = 8.25 - 1 + 5 = 12.25$ in.
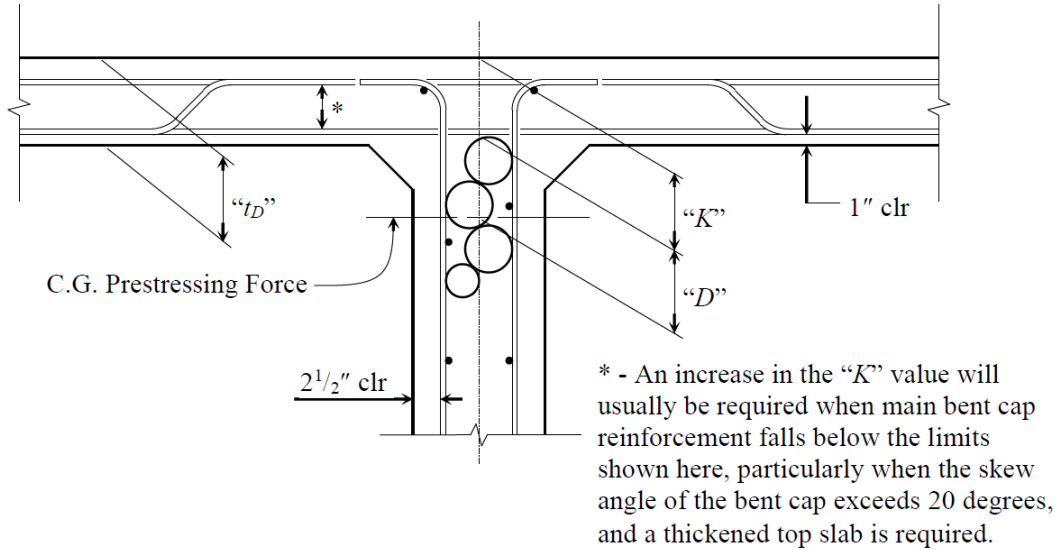
Figure 5.2.12.6-4 Tendon Configuration at High Point of Tendon Profile

$$\delta_{hp} = t_D - 1 + "D" \tag{5.2.12.6-4}$$

where:

δ_{hp} = offset from deck to centroid of duct (in.)

t_D = thickness of deck (in.)

Using Figure 5.2.12.6-2-4 and Equation 5.2.12.6-4, determine the offset from the bottom fiber to the *C.G.* of the *P/S* path at the low point:

$$\delta_{hp} = t_D - 1 + "D" = 9 - 1 + 5 = 13 \text{ in.}$$

Another method to optimize the prestressing path is to use an actual set of *P/S* shop drawings to find "*D*". Use 27 tendon - 0.6 in. diameter strand systems, with a maximum capacity of 27 strands @ 44 kips/strand = 1,188 kips. The calculation of "*D*" is as follows:

The equation 5.2.12.6-1 for P_j gave us an estimate of 1,984 kips/girder. Assuming 0.6 in. diameter strands, with P_j per strand = 44 kips, the number of strands per girder is as given in Equation 5.2.12.6-5:

$$\frac{\text{strands}}{\text{girder}} = \frac{\text{force per girder}}{\text{force per strand}} \tag{5.2.12.6-5}$$

$$\frac{\text{strands}}{\text{girder}} = \frac{\text{force per girder}}{\text{forcer per strand}} = \frac{1,984 \text{ kips per girder}}{44 \text{ kips per strand}} = 45.09$$

Use: 46 Strands

- Assume 23 strands in duct *A*, and 23 strands in duct *B*.
- Find *"D"* based on Equation 5.2.12.6-6
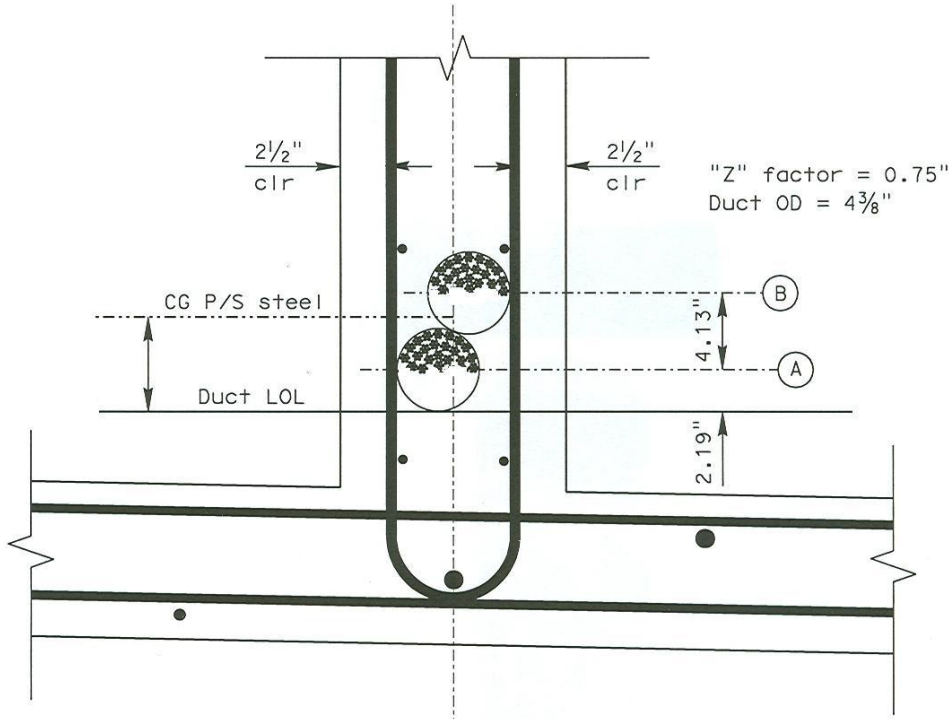
**Figure 5.2.12.6-5 Example of Sub-contractor Tendon Layout
for a Two-Duct per Girder Configuration**

$$"D" = \frac{\sum_{i=1}^{n}\left(n_i \times d_i\right)}{\sum_{i=1}^{n} n_i} + Z \qquad\qquad (5.2.12.6\text{-}6)$$

where:

n_i = number of strands in the ith duct

d_i = distance between C.G. of the ith duct and the ith duct LOL (See Figure 5.2.12.6-5) (in.)

Z = C.G. tendon shift within the duct (in.)

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$$"D" = \frac{\sum_{i=1}^{n}(n_i \times d_i)}{\sum_{i=1}^{n}n_i} + Z = \frac{(n_a \times d_a) + (n_b \times d_b)}{n_a + n_b} + Z$$

$$= \frac{(23 \times 2.19) + \left[23 \times (2.19 + 4.13)\right]}{23 + 23} + 0.75 \text{ in.}$$

$$= 5.0 \text{ in.}$$

Final design offsets using Equations 5.2.12.6-3 and 5.2.12.6-4:

Values from Tendon Layout:

$$\delta_{lp} = t_s - 1 + \text{"}D\text{"} = 8.25 - 1 + 5 = 12.25 \text{ in.}$$

$$\delta_{hp} = t_D - 1 + \text{"}D\text{"} = 9 - 1 + 5 = 13 \text{ in.}$$

Values using "D" are as follows:

δ_{lp} = 12.25 in.

δ_{hp} = 13 in.

It is noted that for each change in P_{jack} there is an accompanying change in the "D" value. Changes in the "D" values result in rerunning a model and can change other portions of the design. Because of this, it is recommended to use a conservative value that will result in stable "D" values. However, this is an area where P_j can be decreased through iteration, if necessary. In this case, these values are equal, use a minimum value of 12.5 in. at the soffit and of 13.5 in. at the deck.

Once the high and low points of the tendon path are established, the locations of the inflection points can be obtained. Locating the inflection points at the 10% span length locations on either side of the bent cap not only delivers adequate moment resistance to this zone but provides a smooth path that allows for easy tendon installation. The vertical position of these inflection points lies on a straight line between the high and low points of the tendon path, at the 0.1L locations. Similar triangles can be used to find the vertical location of the inflection point: (See Figures 5.2.12.6-6 and 5.2.12.6-7)

Spans 1 and 3:

- y_{BD} = 81 – 13.5 – 12.5 = 55 in.
- Similar triangles: $\dfrac{55}{0.5 + 0.1} = \dfrac{y_{BC}}{0.5}$
- Rearranging yields: $y_{BC} = \dfrac{55 \text{ in.} \times 0.5}{0.6} = 45.83 \text{ in.}$
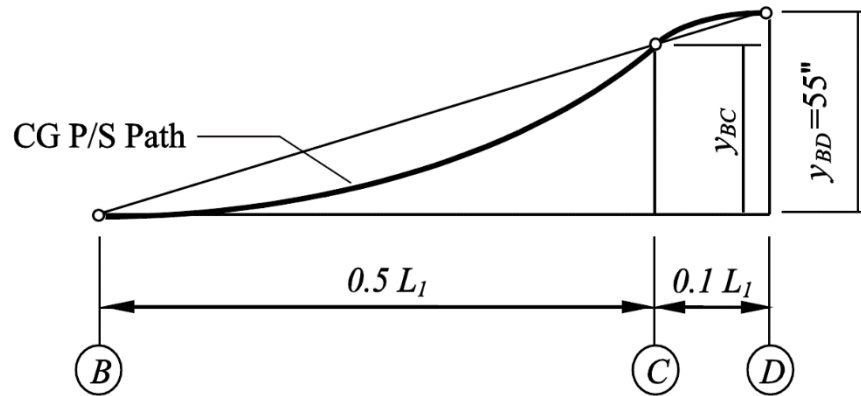
Figure 5.2.12.6-6 Spans 1 and 3 Inflection Point Sketch

Span 2:

- $Y_{FH}= 81 – 13.5 – 12.5 = 55$ in.
- Similar triangles: $\dfrac{55}{0.4+0.1}=\dfrac{y_{FG}}{0.4}$
- Rearranging yields: $y_{FG}=\dfrac{55\ \text{in.}\times 0.4}{0.5}=44$ in.
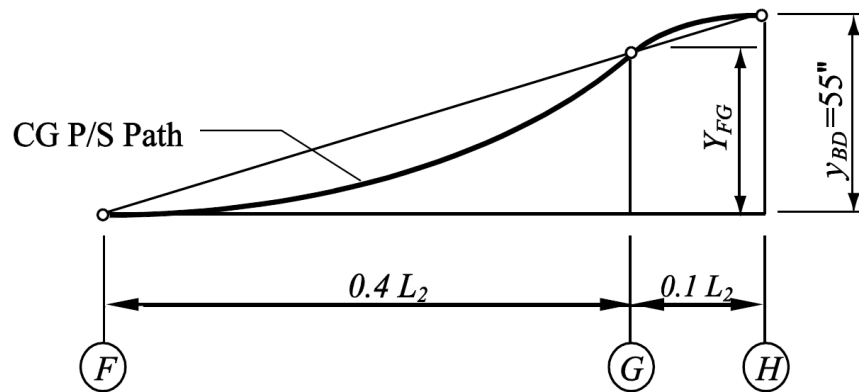

Figure 5.2.12.6- 7 Span 2 Inflection Point Sketch. Middle Spans of Frames Typically have Inflection Points at Mid-Span

The final cable path used for design is shown in Figure 5.2.12.6-8. The y_b = 45.75 in. the value at the abutment diaphragms is the distance to the C.G. of the Concrete Box Section, with 6 in. of tolerance (up or down) to allow for constructability issues.
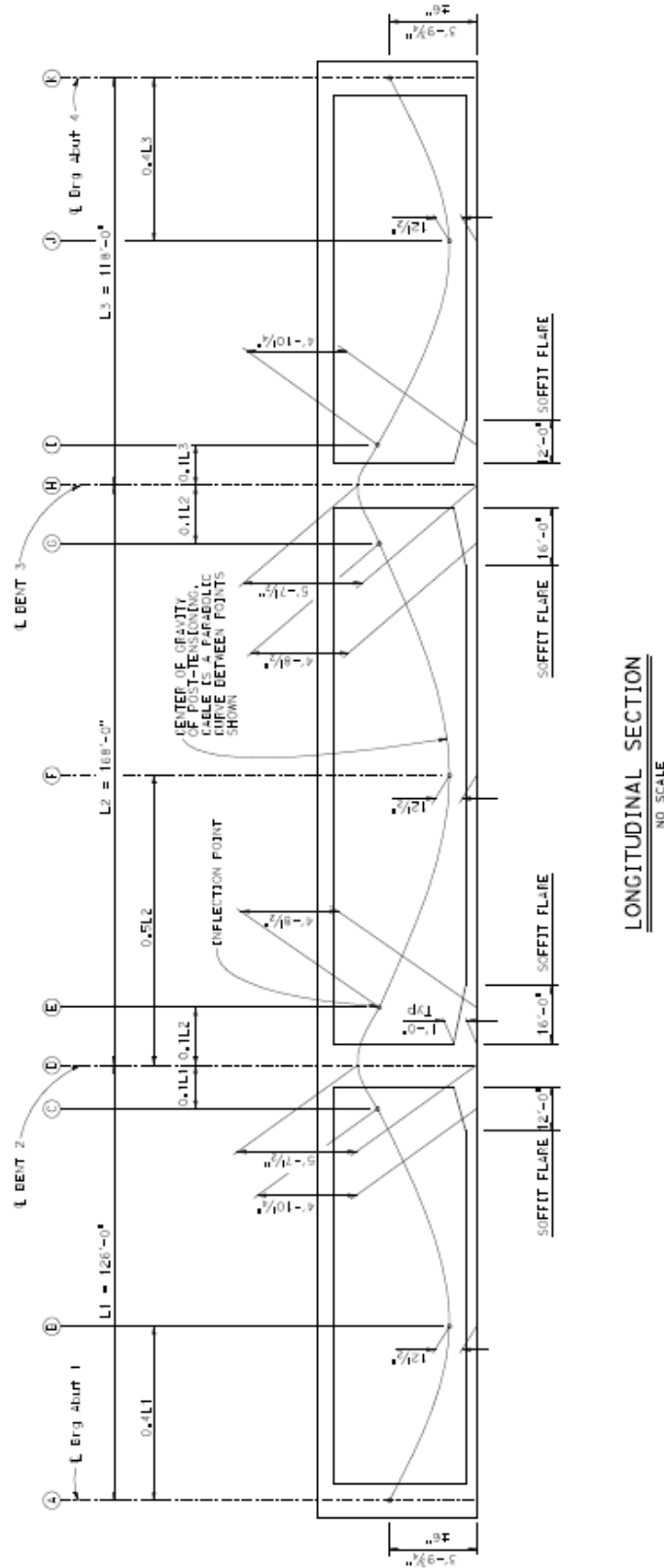
Figure 5.2.12.6-8 Final Cable Path as it would Appear on the Plans

One vs. Two-End Stressing: One-end stressing is considered economical when the increase in P_j does not exceed 3% when compared to two-end stressing. An increase in P_j corresponds to an equivalent increase in materials, and 3% is the breakeven point between the cost of materials vs. the cost of time and labor in moving the stressing operation to the opposite end of the frame. Two-end stressing controls the design of most 3+ span frames, and we will assume it controls in this example problem. The two-end string is used as a means to control the friction loss.

5.2.12.7 Calculate Post-Tensioning Losses

5.2.12.7.1 Friction Loss

An angle change of *P/S* path: The cumulative angle change of the *P/S* path is needed to determine the friction loss. Each parabola (10 in. this example) should be isolated so that angle change (α_j) can be calculated for each segment. Friction losses have a cumulative effect and increase as you get further away from the jacking end. Use the following formula to solve for angle change in each parabolic segment:

$$\alpha_{ij} = \frac{2y_{ij}}{l_{ij}}(rad) \tag{5.2.12.7.1-1}$$

where:

y_{ij} = height of individual parabola (in.)
l_{ij} = length of individual parabola (in.)

Find the angle change in segment *BC* in span 1:

$$y_{BC} = 4.854 - 1.042 = 3.812 \text{ ft}$$

$$l_{BC} = 0.5L_1 = (0.5)(126) = 63 \text{ ft}$$

$$\alpha_{ij} = \frac{2y_{BC}}{l_{BC}} = \frac{2 \times 3.812}{63} = 0.121(rad)$$

Table 5.2.12.7.1-1 summarizes values that will be used to calculate initial friction losses:

Note in Table 5.2.12.7.1-1 that the prestressing cable is located at the exact neutral axis of the member (3.813 in.). However, for all future equations the rounded number 3.8 in. will be used.

Table 5.2.12.7.1-1 Summary of *P/S* Path Angle Changes used in Friction Loss Calculations

Segment	y_{ij} Calculation (ft)	y_{ij} (ft)	x_{ij} Calculation (ft)	x_{ij} (ft)	Left End Σx_{AK} (ft)	Right End Σx_{KA} (ft)	α_{ij} (rad)	Left End $\Sigma \alpha_{AK}$ (rad)	Right End $\Sigma \alpha_{KA}$ (rad)
AB	3.813 - 1.042	2.77	(0.4)(126)	50.4	50.4	412.0	0.110	0.110	1.166
BC	4.854 - 1.042	3.81	(0.5)(126)	63.0	113.4	361.6	0.121	0.231	1.056
CD	5.625 - 4.854	0.76	(0.1)(126)	12.6	126.0	298.6	0.121	0.352	0.934
DE	5.625 - 4.708	0.92	(0.1)(168)	16.8	142.8	286.0	0.109	0.462	0.813
EF	4.708 - 1.042	3.67	(0.4)(168)	67.2	210.0	269.2	0.109	0.571	0.704
FG	4.708 - 1.042	3.67	(0.4)(168)	67.2	277.2	202.0	0.109	0.680	0.595
GH	5.625 - 4.708	0.92	(0.1)(168)	16.8	294.0	134.8	0.109	0.789	0.486
HI	5.625 - 4.854	0.76	(0.1)(118)	11.8	305.8	118.0	0.130	0.919	0.376
IJ	4.854 - 1.042	3.81	(0.5)(118)	59.0	364.8	106.2	0.130	1.048	0.247
JK	3.813 - 1.042	2.77	(0.4)(118)	47.2	412.0	47.2	0.117	1.166	0.117

Sum of x_{ij} = 412.0
Sum of α_{ij} = 1.165

Initial Friction Coefficient: The percent in the decimal form of the jacking stress remaining in the *P/S* steel after losses due to friction, FC_{pF}, can now be calculated based on the cumulative l_{ij} and α_{ij} computed above. Using Equation 5.2.5.1.2-1, modified Equation 5.9.3.2.2b-1 (AASHTO, 2017) from Article 5.2.5.1.2 above:

$$FC_{pF} = \frac{\Delta f_{pF}}{f_{pj}} = e^{-(Kx+\mu\alpha)} \qquad (5.2.5.1.2\text{-}1)$$

Inputting values from CA Amendments Table 5.9.3.2.2b-1 (Caltrans, 2019)

- K = 0.0002 per ft
- μ = 0.15 (L_{frame} < 600 ft)

Find the percent in the decimal form of P_j remaining after the effects of friction loss at the 0.1 point in span 2.

- At pt. E: $FC_{pF} = e^{-(Kx+\mu\alpha)}$
- x_E (left-end) = 142.8 ft
- α_E (left-end) = 0.4616 rad

- $FC_{pF}(@E) = e^{-[(0.0002)(142.8) + (0.15)(0.462)]} = 0.907$

Table 5.2.12.7.1-2 includes a summary of values of initial friction losses:

Table 5.2.12.7.1-2 Summary of Cumulative Angle Change for Two-End Stressing

	Left-end Stressing			Right-end Stressing		
	Σx_{AK}	$\Sigma \alpha_{AK}$	$e^{-(kx_{AK}+\mu\alpha_{AK})} = FC_{pF}$	Σx_{KA}	$\Sigma \alpha_{KA}$	$e^{-(kx_{AK}+\mu\alpha_{AK})} = FC_{pF}$
Location	(ft)	(rad)	(decimal %)	(ft)	(rad)	(decimal %)
A	0.0	0.000	1.000	412.0	1.166	0.773
B	50.4	0.110	0.974	361.6	1.056	0.794
C	113.4	0.231	0.944	298.6	0.934	0.819
D	126.0	0.352	0.925	286.0	0.813	0.836
E	142.8	0.462	0.907	269.2	0.704	0.853
F	210.0	0.571	0.880	202.0	0.594	0.878
G	277.2	0.680	0.854	134.8	0.486	0.905
H	294.0	0.789	0.837	118.0	0.376	0.923
I	305.8	0.919	0.819	106.2	0.247	0.943
J	364.8	1.048	0.794	47.2	0.117	0.973
K	412.0	1.166	0.773	0.0	0.000	1.000

5.2.12.7.2 Anchor Set Losses

The method used for determining losses due to the anchor set is based on "similar triangles". The assumption is that the effects of friction are the same whether a tendon is being stressed or released back into the duct to seat the wedges. Most of the time, the end of the influence length of the anchor set, x_{pA}, lies between the high and low inflection points from the jacking end in a multi-span frame. The equations that solve for ΔFC_{pA} and x_{pA} are shown as follows:
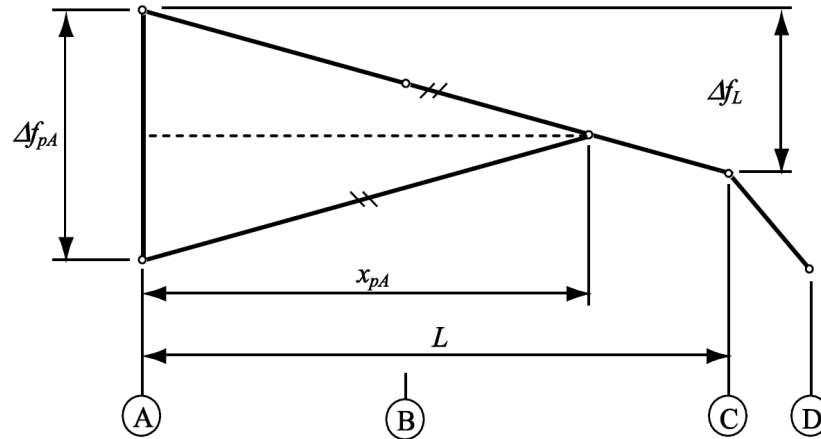
Figure 5.2.12.7.2-1 Typical Anchor Set Loss

$$FC_{pA} = \frac{\Delta f_{pA}}{f_{ps}} = \frac{2(\Delta f_L)(x_{pA})}{L(f_{ps})} \qquad (5.2.5.1.1\text{-}1)$$

$$x_{pA} = \sqrt{\frac{E_p(\Delta_{Aset})L}{12\Delta f_L}} \qquad (5.2.5.1.1\text{-}2)$$

Define the anchor set loss diagram by finding x_{pA} and ΔFC_{pA} in span 1 due to the left-end stressing operation:

Given:

E_p = 28,500 ksi

A_{set} = 0.375 in.

L = Distance from point A to point C

= 0.9 L_1 = (0.9) (126 ft)

= 113.4 ft

f_{pu} = 270 ksi

f_{pj} = (0.75)(270 ksi)

= 202.5 ksi (Table 5.9.2.2-1 CA Amendments, 2019)

Δf_L = [1 - ΔFC_{pF} (@ point C)](202.5 ksi) = [1 − 0.944](202.5) = 11.34 ksi

Solving for x_{pA} and ΔFC_{pA} at the intersection of the initial losses and the anchor set (the location x_{pA} away from the anchor).

$$x_{pA} = \sqrt{\frac{E_p\left(\Delta_{Aset}\right)L}{12\Delta f_L}} = \sqrt{\frac{28,500\left(0.375\right)113.4}{12\left(11.34\right)}} = 94.37 \text{ ft}$$

$$FC_{pA} = \frac{\Delta f_{pA}}{f_{ps}} = \frac{2\left(\Delta f_L\right)\left(x_{pA}\right)}{L\left(f_{ps}\right)} = \frac{2\left(11.34\right)\left(94.37\right)}{113.4\left(202.5\right)} = 0.093$$
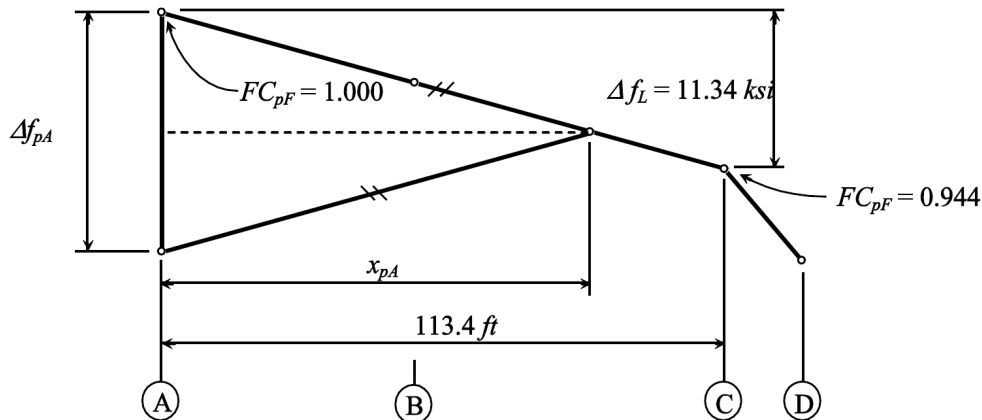
Figure 5.2.12.7.2-2 Bridge Specific Anchor Set Loss

The anchor set loss diagram is found in a similar manner in span 3 due to the second (right) end stressing operation.

Given:

    E_p = 28,500 ksi

    A_{set} = 0.375 in.

    L = Distance from point K to point I

      = 0.9 L_3 = (0.9) (118 ft) = 106.2 ft

    f_{pu} = 270 ksi

    f_{pj} = (0.75)(270 ksi)

      = 202.5 ksi (Table 5.9.2.2-1 CA Amendments, 2019)

    Δf_L = [1 - ΔFC_{pF} (@ point I)](202.5 ksi)

      = [1 – 0.943](202.5) = 11.54 ksi

Solving for x_{pA} and ΔFC_{pA} at the intersection of the initial losses and the anchor set (the location x_{pA} away from the anchor).

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$$x_{pA} = \sqrt{\frac{E_p\left(\Delta_{Aset}\right)L}{12\Delta f_L}} = \sqrt{\frac{28{,}500\left(0.375\right)106.2}{12\left(11.54\right)}} = 90.53 \text{ ft}$$

$$FC_{pA} = \frac{\Delta f_{pA}}{f_{ps}} = \frac{2\left(\Delta f_L\right)\left(x_{pA}\right)}{L\left(f_{ps}\right)} = \frac{2\left(11.54\right)\left(90.53\right)}{106.2\left(202.5\right)} = 0.097$$

5.2.12.7.3 Elastic Shortening

Losses due to elastic shortening are usually assumed at the beginning and then checked once more for convergent numbers when M_{DL}, P_j, and e have been found. Therefore, based on experience, we will assume a realistic and typically conservative value of Δf_{pES} = 3 ksi for this practice problem.

To turn this into a Force Coefficient:

$$FC_{pES} = \frac{\Delta f_{pES}}{f_{ps}} = \frac{3 \text{ ksi}}{202.5 \text{ ksi}} = 0.015$$

5.2.12.7.4 Approximate Estimate of Time-Dependent Long-Term Loss

The long-term loss in prestressing steel stress due to creep of concrete, shrinkage of concrete, and relaxation of *P/S* steel occurs over time and begins immediately after stressing.

$$\Delta f_{pSR} = 12.0\left(1.7 - 0.01H\right)\frac{5}{\left(1 + f'_{ci}\right)} \qquad (5.2.5.2.1\text{-}1)$$

$$\Delta f_{pCR} = 10.0\frac{f_{pi}A_{ps}}{A_g}\left(1.7 - 0.01H\right)\frac{5}{\left(1 + f'_{ci}\right)} \qquad (5.2.5.2.2\text{-}1)$$

Δf_{pR} = 2.4 ksi for lo-lax strands $\qquad$ (AASHTO 5.9.3.3-1)

Most of the research done on time-dependent losses considered precast concrete girders, without much consideration given to continuous, CIP PT box girder structures. Ongoing research indicates time-dependent losses as high as 30 ksi may be appropriate for cast-in-place, post-tensioned structures. This portion of the code has undergone revision from 2008 to 2014.

Δf_{pLT} = 20 ksi $\qquad$ (CA Amendments 5.9.3.3)

For this example, let's use 25 ksi. This is a reasonable value and was the value used in the 2008 CA Amendments.

Converting to a Force Coefficient:

$$FC_{pLT} = \frac{\Delta f_{pLT}}{f_{ps}} = \frac{25\ \text{ksi}}{202.5\ \text{ksi}} = 0.123$$

5.2.12.7.5 Total Loss of Prestress Force

As stated in section 5.2.5 the loss of force in the prestressing steel is cumulative.

In lieu of a more detailed analysis, prestress losses in members constructed and prestressed in a single stage, relative to the stress immediately before the transfer, may be taken as:

$$FC_{pT} = \left(1 - \frac{\sum \Delta f_i}{f_{ps}}\right) \tag{5.2.5-1}$$

A summary of our calculated immediate and total prestress stress remaining along the prestressing path is included in Table 5.2.12.7.5-1.

Table 5.2.12.7.5-1 Summary of Cumulative Prestress Loss

Percentage of Cumulative Prestress Stresses Remaining After Losses

Location	Left-end Stressing				Right-end Stressing			
	FC_{pF}	FC_{pA}	FC_{pES}	FC_{pLT}	FC_{pF}	FC_{pA}	FC_{pES}	FC_{pLT}
A	1.000	0.907	0.893	0.770	0.773	0.773	0.758	0.635
B	0.974	0.933	0.917	0.794	0.794	0.794	0.779	0.656
C	0.944	0.944	0.929	0.806	0.819	0.819	0.804	0.681
D	0.925	0.925	0.910	0.787	0.836	0.836	0.821	0.698
E	0.907	0.907	0.892	0.769	0.853	0.853	0.838	0.715
F	0.880	0.880	0.865	0.742	0.878	0.878	0.863	0.740
G	0.854	0.854	0.839	0.716	0.905	0.905	0.890	0.767
H	0.837	0.837	0.822	0.699	0.923	0.923	0.908	0.785
I	0.819	0.819	0.804	0.681	0.943	0.943	0.928	0.805
J	0.794	0.794	0.779	0.656	0.973	0.930	0.915	0.792
K	0.773	0.773	0.758 FC_{pl}	0.635 FC_{pT}	1.000	0.903	0.890 FC_{pl}	0.767 FC_{pT}
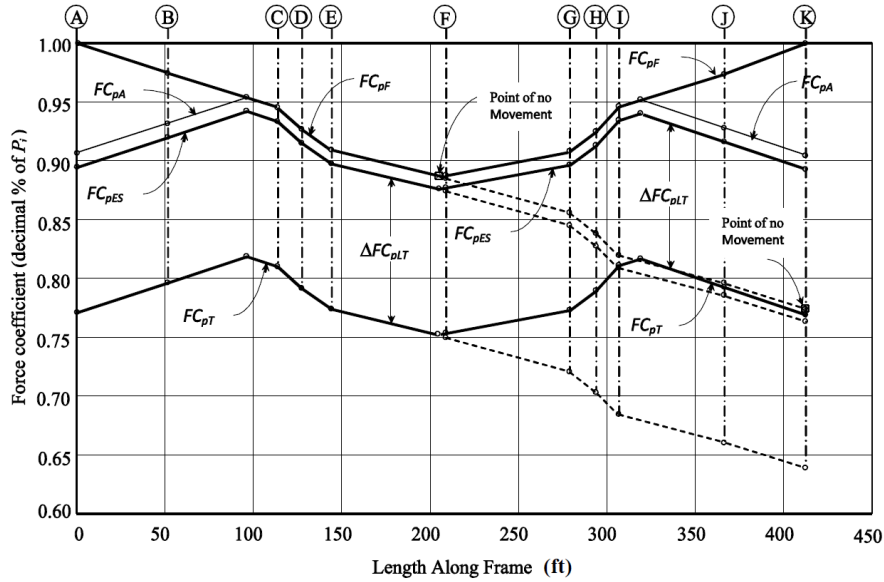
Figure 5.2.12.7.5-1 Summary of Prestress Losses for Two-End Stressing

5.2.12.8 Calculate Cable Path Eccentricities

To design the jacking force, and track stresses in both top and bottom fibers, cable path eccentricities must be calculated at the 10th points of each span in the frame. Derived from the equation of a parabola, the diagram and formula shown below can be used to compute eccentricities for parabolic *P/S* paths:

$$e_x = -\left[y_{ij} \left(1 - \frac{x^2}{x_{ij}^2} \right) \right] + c \qquad\qquad (5.2.12.8\text{-}1)$$

where:

e_x =   eccentricity as a function of x along parabolic segment (ft)

y_{ij} =   height of the individual parabola (ft)

x =   location along parabolic segment where eccentricity is calculated (percent of span L)

x_{ij} =   length of parabolic segment under consideration (must originate at vertex) (percent of span L)

c =   shifting term to adjust eccentricities when y_{ij} does not coincide with the C.G. of concrete

Chapter 5.2 – Post-Tensioned Concrete Girders   5.2-53
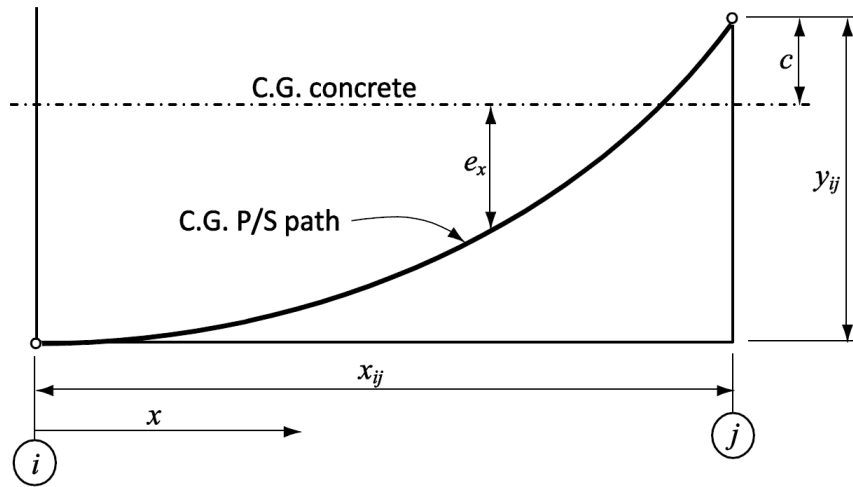
Figure 5.2.12.8-1 Cable Path Calculation Diagram

Find the cable path eccentricities in Span 1:
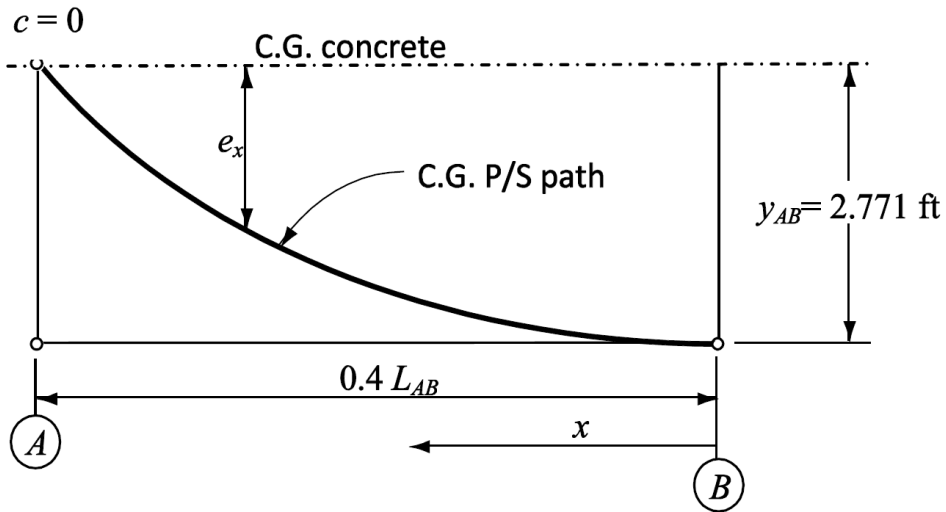
Figure 5.2.12.8-2 Cable Path Calculation Diagram from Abut 1 to the Low Point of Span 1

$$e_{0.0L1} = -\left[2.771\left(1-\frac{0.4^2}{0.4^2}\right)\right]+0 = 0 \text{ ft}$$

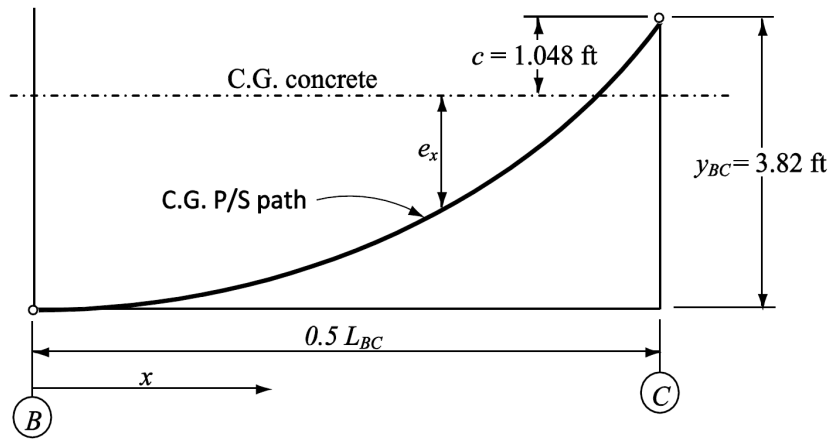$$e_{0.1L1} = -\left[2.771\left(1-\frac{0.3^2}{0.4^2}\right)\right]+0 = -1.212 \text{ ft}$$

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$$e_{0.2L1} = -\left[2.771\left(1-\frac{0.2^2}{0.4^2}\right)\right]+0 = -2.078 \text{ ft}$$

$$e_{0.3L1} = -\left[2.771\left(1-\frac{0.1^2}{0.4^2}\right)\right]+0 = -2.598 \text{ ft}$$

$$e_{0.4L1} = -\left[2.771\left(1-\frac{0.0^2}{0.4^2}\right)\right]+0 = -2.771 \text{ ft}$$

Figure 5.2.12.8-3 Cable Path Calculation Diagram from the Low Point of Span 1 to the Inflection Point Near Bent 2

$$e_{0.5L1} = -\left[3.819\left(1-\frac{0.1^2}{0.5^2}\right)\right]+1.048 = -2.618 \text{ ft}$$

$$e_{0.6L1} = -\left[3.819\left(1-\frac{0.2^2}{0.5^2}\right)\right]+1.048 = -2.160 \text{ ft}$$

$$e_{0.7L1} = -\left[3.819\left(1-\frac{0.3^2}{0.5^2}\right)\right]+1.048 = -1.396 \text{ ft}$$

$$e_{0.8L1} = -\left[3.819\left(1-\frac{0.4^2}{0.5^2}\right)\right]+1.048 = -0.327 \text{ ft}$$

$$e_{0.9L1} = -\left[3.819\left(1 - \frac{0.5^2}{0.5^2}\right)\right] + 1.048 = 1.048 \text{ ft}$$

The eccentricity at the centerline of Bent 2 must be calculated using the section properties that include the soffit flare:

$$e_{1.0L1} = y_{CD} + c + \Delta_{yb} = 0.764 + 1.048 + 0.310 = 2.122 \text{ ft}$$
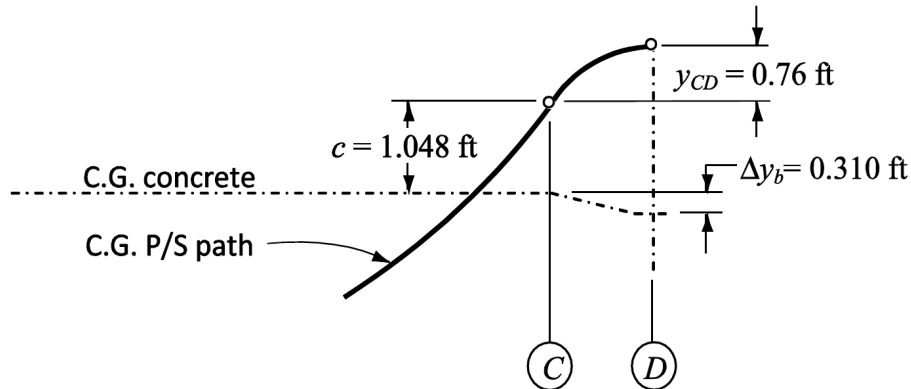
Figure 5.2.12.8-4 Cable Path Calculation Diagram from the Inflection Point Near Bent 2 to CL of Bent 2

Cable path eccentricities for all three spans are summarized in Figure 5.2.12.8-5 below:
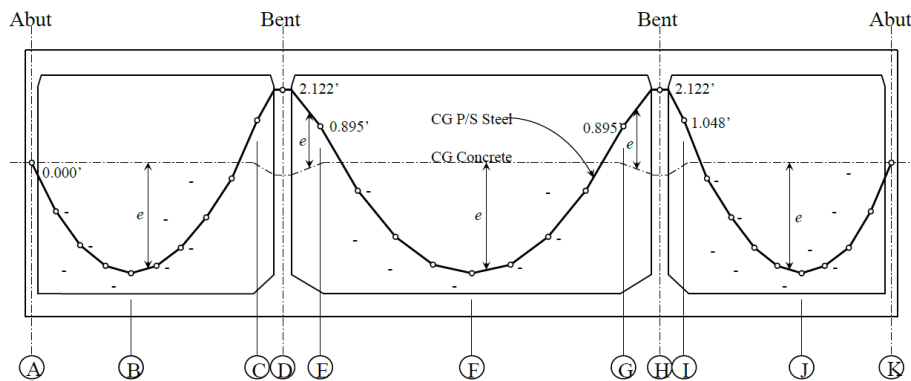
Figure 5.2.12.8-5 Cable Path Eccentricities

5.2.12.9 Calculate Moment Coefficients

Now that we've identified both the eccentricity and the percentage of prestressing force present at each tenth point along each span in the frame, we can now find the moment coefficients. The moment coefficients will help us solve for P_j, as well as compute flexural

stresses in the concrete for determining f'_c and $f'_{ci\,i}$. The total moment coefficient consists of two parts; primary and secondary moment coefficients.

Primary Moment Coefficient: The primary moment coefficient at any location along the frame is simply defined as the total force coefficient (FC_{pT}) multiplied by the eccentricity (e_x).

Find the primary moment coefficients at each tenth point in Span 1:

$$MC_p = \left(FC_{pT}\right)\left(e_x\right) \tag{5.2.12.9-1}$$

where:

MC_p = primary moment force coefficient for loss (ft)

FC_{pT} = total force coefficient for loss

e_x = eccentricity as a function of x along parabolic segment (ft)

@ 0.0 L_1: (0.770) (0) = 0 ft

@ 0.1 L_1: (0.776) (-1.212) = -0.941 ft

@ 0.2 L_1: (0.782) (-2.078) = -1.625 ft

@ 0.3 L_1: (0.788) (-2.598) = -2.047 ft

@ 0.4 L_1: (0.794) (-2.771) = -2.200 ft

@ 0.5 L_1: (0.800) (-2.618) = -2.094 ft

@ 0.6 L_1: (0.806) (-2.160) = -1.741 ft

@ 0.7 L_1: (0.812) (-1.396) = -1.134 ft

@ 0.8 L_1: (0.812) (-0.327) = -0.266 ft

@ 0.9 L_1: (0.806) (1.048) = 0.845 ft

@ 1.0 L_1: (0.787) (2.122) = 1.670 ft

Secondary Moment Coefficient: Prestress secondary moments occur in multi-span post-tensioned concrete frames where the superstructure is "fixed" to the column. The result of this fixity is an indeterminate structure. Prestress secondary moments are made up of two components:

- Distortions due to the primary prestress moments, $MC_P = (\Delta FC_{pT})(e_x)$, generate fixed-end moments at rigid column supports. These fixed-end moments are always positive due to the geometry of the cable path and always enhance the effects of prestressing at the bent caps. On the other hand, this component of prestress secondary moment always reduces the flexural effects of the prestressing force near midspan.
- Prestress shortening of superstructure between rigid supports generates moments in the columns, which result in fixed-end moments in the superstructure. This

- component of secondary prestress moment occurs in frames with three or more spans. Long frames with short columns result in larger secondary prestress moments, which can be a significant factor in the design of the superstructure.
- There are several analysis methods a designer can use to find the prestress secondary moments for a given frame. In stiffness based frame analysis software packages, the forces generated by prestressing the concrete are replicated with a series of uniform and point loads. In other words, the primary "internally applied" moments and axial loads are converted into "externally applied" loads. The drawback of this method is that it is extremely difficult to do by hand, especially in multi-span frames.
- Each span within the frame is transformed into a simple span so that the ends can rotate freely.
- Create an MC_P / EI diagram, as the applied prestress moments are simply the prestress force times eccentricity.
- Using conjugate beam theory, sum moments about one end of the beam to solve for the rotation at the opposite end. The moment needed to rotate the end of the beam back to zero is the "fixed-end" secondary moment due to ΔMC_P distortion.
- When a frame is three spans or longer, secondary prestress deflections are generated in the column supports. The resulting column moments are a result of prestress shortening of the superstructure between the interior spans of the frame.
- The fixed-end moments of the two components of prestress secondary moments are then combined; with the use of moment distribution, these fixed-end moments are distributed to both the superstructure and columns based on the relative stiffness of each member.
- Because P_j is still unknown, the prestress secondary moments must be solved in terms of coefficient (MC_S).

The secondary prestress moment coefficients used in this example problem are a result of the conjugate beam and moment distribution methods of analysis. The primary (MC_P) and secondary (MC_S) are then added together algebraically resulting in the total moment coefficient (MC_{PT}). A summary of MC_P, MC_S, and MC_{PT} are summarized in the following diagram:
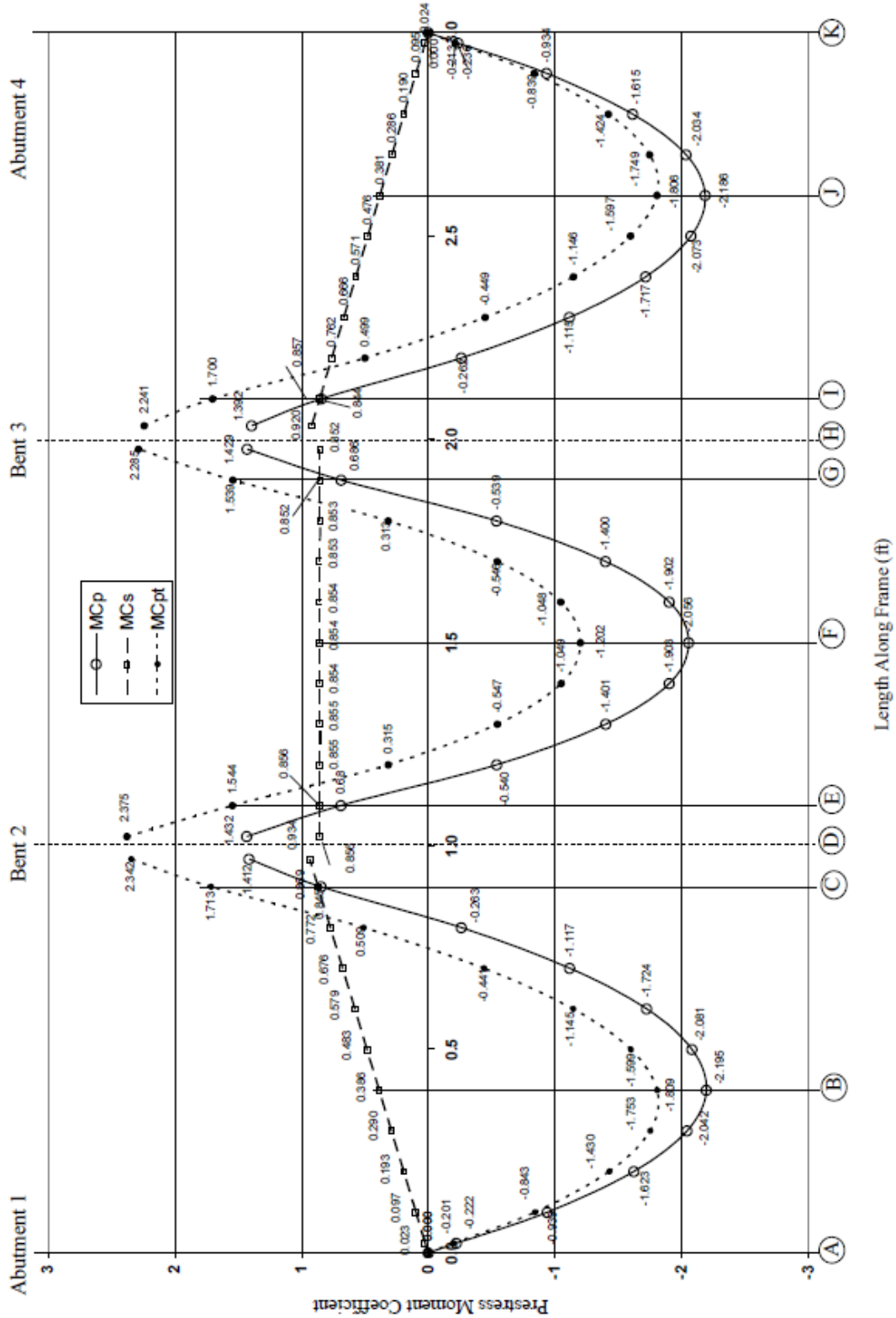
Figure 5.2.12.9-1 Moment Coefficients

5.2.12.10 Calculate Gravity Loads

DC = Dead load of structural components and nonstructural attachments.

- Includes the self-weight of the box section itself, assuming a unit weight of reinforced concrete (w_c) = 0.15 (kcf) (AASHTO Table 3.5.1-1)
- Type 842 barrier rail: (2 barriers) (0.64 klf ea.) = 1.28 klf

DW = Dead load of wearing surface and utilities.

- Future wearing surface (56 ft) (0.035 ksf) = 1.96 klf

Vehicular Live load (LL): The application of vehicular live loads on the superstructure shall be calculated separately for flexure and shear design. In each case, live load distribution factors shall be calculated for an interior girder, then multiplied by the total number of girders in the cross-section. Treating all girders as the interior is justifiable because exterior girders become the interior when bridges are widened.

Distribution of live load per lane for moments in interior beams, with two or more design lanes loaded:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25}$$
(AASHTO Table 4.6.2.2.2b-1)

where:

N_c = number of cells in a concrete box girder ($N_c \geq 3$)

S = spacing of beams or webs (ft) ($7.0 \leq S \leq 13.0$)

L = individual span length (ft) ($60 \leq L \leq 240$)

    (Note: if L varies from span to span in a multi-span frame, so will the distribution factors)

Distribution of live load per lane for shear in interior beams, with two or more design lanes loaded:

$$\left(\frac{S}{7.3}\right)^{0.9}\left(\frac{d}{12.0L}\right)^{0.1}$$
(AASHTO Table 4.6.2.2.3a-1)

where:

S = spacing of beams or webs (ft) ($6.0 \leq S \leq 13.0$)

L = individual span length (ft) ($20 \leq L \leq 240$)

d = depth of member (in.) ($35 \leq d \leq 110$)

    (Note: if L varies from span to span in a multi-span frame, so will the distribution factors)

Calculate the number of live load lanes for both moment and shear design for Span 1:

5.2.12.10.1 Live Load Lanes for Moment

Using an equation from AASHTO Table 4.6.2.2.2b-1:

$$\left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25} = \left(\frac{13}{4}\right)^{0.3}\left(\frac{12}{5.8}\right)\left(\frac{1}{126}\right)^{0.25} = 0.879 \text{ lanes/girder}$$

Number of live load moment lanes (Span 1)   = (0.879 lanes/girder) x (5 girders)

= 4.395 lanes

5.2.12.10.2 Live Load Lanes for Shear

Using an equation from Table 4.6.2.2.3a-1:

$$\left(\frac{S}{7.3}\right)^{0.9}\left(\frac{d}{12.0L}\right)^{0.1} = \left(\frac{12}{7.3}\right)^{0.9}\left(\frac{81}{(12.0)(126)}\right)^{0.1} = 1.167 \text{ lanes/girder}$$

Number of live load shear lanes (Span 1)   = (1.167 lanes/girder) x (5 girders)

= 5.835 lanes

Notes:

- Assuming a constant structure width, the only term that will probably vary within a frame is the span length, *L*. Therefore, the number of live load lanes for both moment and shear will vary from span to span. Article 4.6.2.2.1 (Caltrans, 2019).
- In negative moment regions, near interior supports, between points of DC flexural contraflexure, the "*L*" used to calculate negative moment is the average length of the two adjacent spans.
- The Dynamic Load Allowance Factor (*IM*) (in the LFD code known as Impact) is applied to the design and permit trucks only, not the design lane load. Table 3.6.2.1-1 (Caltrans, 2019) summarizes the values of *IM* for various components and load cases.
- The results of a gravity load analysis, including the unfactored moment envelopes of the (*DC*), (*DC* + *DW*) and (*DC* + *DW*+ *HL93*) load cases, are summarized in the diagram on the next page:
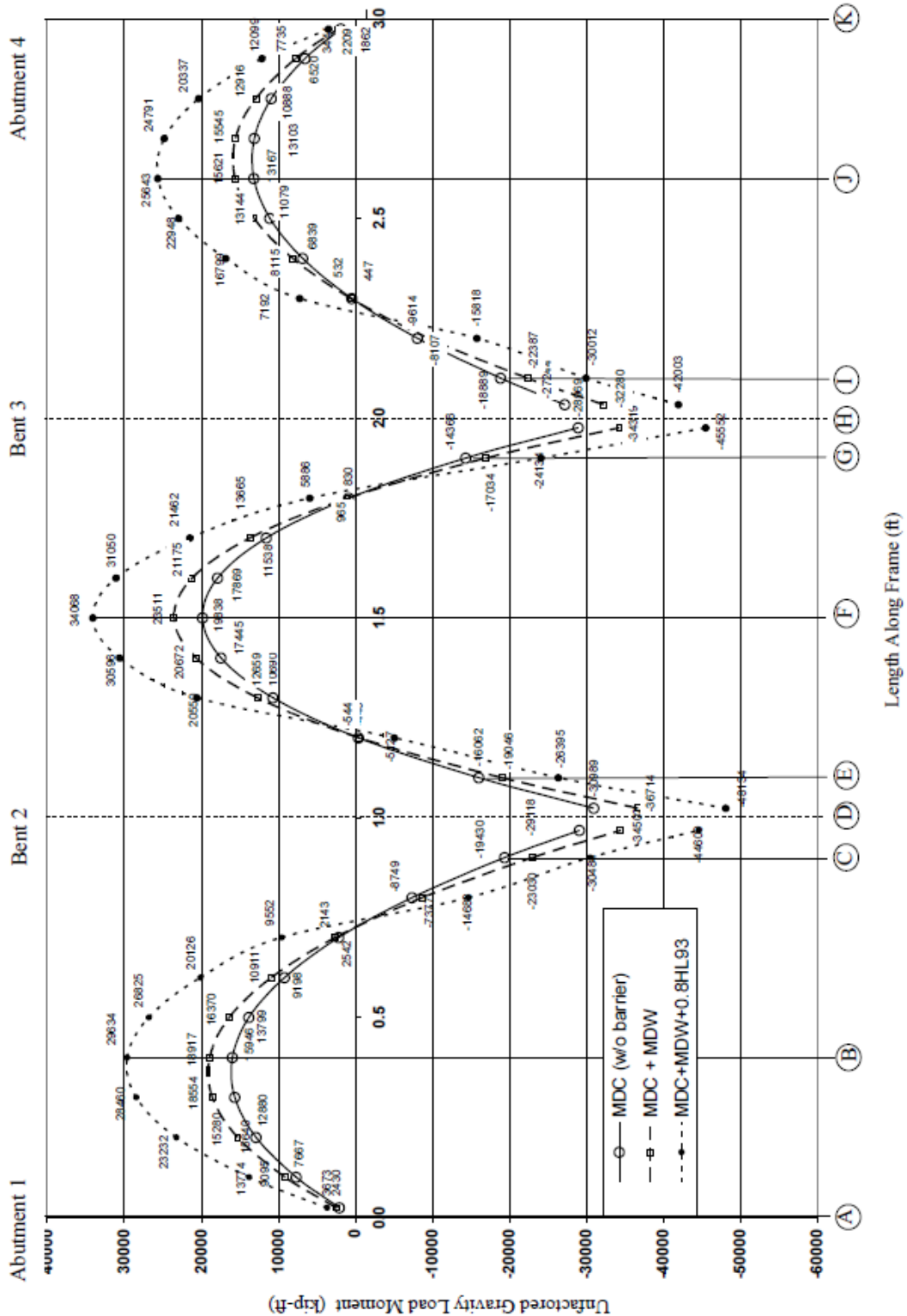
Figure 5.2.12.10-1 Gravity and Service Load Moment Envelopes for Design of Prestressing

5.2.12.11 Determine the Prestressing Force

The design of the prestressing force, P_j, is based on the Service III Limit States. The Service III Limit States is defined as the "Load combination for longitudinal analysis relating to tension in prestressed concrete superstructures with the objective of crack control…" Article 3.4.1 (AASHTO, 2017). The following loads, and corresponding load factors, shall be considered in the design of P_j:

> DC = Dead load of structural components and non-structural attachments, (γ = 1.0) Article 3.3.2 (AASHTO, 2017)

> DW = Dead load of wearing surface and utilities, (γ = 1.0) Article 3.3.2 (AASHTO, 2017)

> $HL93$ = Service live load, (γ = 0.8) Section 3.6.1.2.1 (AASHTO, 2017)

Service III Limit States load cases: CA Amendments Table 5.9.2.3.2b-1 (Caltrans, 2019).

Case 1: No tension allowed for components with bonded prestressing tendons or reinforcement, subjected to permanent loads (DC, DW) only.

$$\frac{M_{DC+DW}y}{I} + \frac{FC_{pT}\left(P_j\right)}{A} + \frac{MC_{pT}\left(P_j\right)y}{I} = 0 \qquad \text{(5.2.12.11-1, modified 5.2.1-1)}$$

Case 2: Allowable tension $0.19\lambda\sqrt{f'_c} \leq 0.6$ ksi are for components subjected to the Service III Limit States (*DC*, *DW*, (0.8) *HL93*), and subjected to no worse than moderate corrosion conditions, located in Caltrans Environmental "non-freeze-thaw area". Allowable tension $0.0948\lambda\sqrt{f'_c} \leq 0.3$ ksi are for components subjected to severe corrosion conditions located in Caltrans Environmental "freeze-thaw area".

$$\frac{M_{DC+DW+0.8HL93}y_{t(bent)}}{I} + \frac{FC_{pT}\left(P_j\right)}{A} + \frac{MC_{pT}\left(P_j\right)y}{I} = 0.19\lambda\sqrt{f'_c} \leq 0.6$$
$$\text{or } 0.0948\lambda\sqrt{f'_c} \leq 0.3$$
$$\text{(5.2.12.11-2, modified 5.2.1-1)}$$

For this example:

$$0.19\lambda\sqrt{f'_c} = 0.19(1)\sqrt{4} = 0.38 < 0.6$$
$$0.0948\lambda\sqrt{f'_c} = 0.0948(1)\sqrt{4} = 0.19 < 0.3$$

The design of the jacking force usually controls at locations with the highest demand moments within a given frame. Upon inspection of the demand moment diagram plotted earlier in this example, the design of P_j will control at one of two locations:

- The right face of the cap at Bent 2 (top fiber)
- Mid-span of Span 2 (bottom fiber)

Load cases 1 and 2 must be applied at both the right face of the cap at Bent 2, and at midspan of Span 2, with the overall largest P_j controlling the design of the entire frame.

Solve for the jacking force based on two-end stressing data gathered earlier in the example problem:

Right face of the cap at Bent 2 (top fiber):
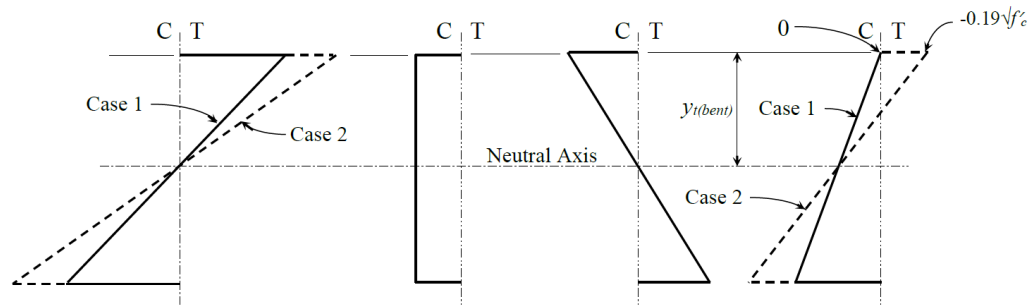
Case 1: $\dfrac{M_{DC+DW} y_{t(\text{bent face})}}{I_{(\text{bent face})}} + \dfrac{FC_{pT}\left(P_j\right)}{A_{(\text{bent face})}} + \dfrac{MC_{pT}\left(P_j\right) y_{t(\text{bent face})}}{I_{(\text{bent face})}} = 0$

Case 2: $\dfrac{M_{DC+DW+0.8HL93} y_{t(\text{bent face})}}{I_{(\text{bent face})}} + \dfrac{FC_{pT}\left(P_j\right)}{A_{(\text{bent face})}} + \dfrac{MC_{pT}\left(P_j\right) y_{t(\text{bent face})}}{I_{(\text{bent face})}} = -0.19\lambda\sqrt{f'_c}$

Figure 5.2.12.11-1 Elastic Stresses in an Uncracked Prestress Beam. Effects of Prestress by Component at Top of the Beam

Reading Figure 5.2.12.10-1

M_{DC+DW} = -36,714 kip-ft

$M_{DC+DW+0.8HL93}$ = -48,134 kip-ft

Interpolating the Force Coefficient from Table 5.2.12.7.5-1 between points D and E

$FC_{@pt\ D}$ = 0.787

$FC_{@pt\ E}$ = 0.769

Span 2 Length = 168 ft

Distance from CL of column to face of cap (pt D) = 4 ft

Distance from CL of column to location of first inflection point of Span 2 (pt E) = 16.8 ft

$$\left(\frac{0.787-0.769}{0-16.8}\right)(4-0)+0.787=0.783$$

Reading Figure 5.12.9-1 for MC_{PT} @ the face of cap at Bent 2:

MC_{PT} = 2.375

Rearranging terms of 5.2.12.11-1 and 5.2.12.11-2 to solve for P_j:

Case 1: $P_j = -\dfrac{\dfrac{M_{DC+DW}\, y_{t(\text{bent face})}}{I_{(\text{bent face})}} + 0}{\dfrac{FC_{pT}}{A_{(\text{bent face})}} + \dfrac{\left(MC_{pT}\right) y_{t(\text{bent face})}}{I_{(\text{bent face})}}}$

$$P_j = -\dfrac{\dfrac{(-36,714)(3.25)}{824}+0}{\dfrac{0.783}{115}+\dfrac{(2.375)(3.25)}{824}} = 8,984 \text{ kips}$$

Case 2: $P_j = -\dfrac{\dfrac{M_{DC+DW+0.8HL93}\, y_{t(\text{bent face})}}{I_{(\text{bent face})}} + (0.19)\lambda\sqrt{f'_c}\,(144)}{\dfrac{FC_{pT}}{A_{(\text{bent face})}} + \dfrac{\left(MC_{pT}\right) y_{t(\text{bent face})}}{I_{(\text{bent face})}}}$

$$P_j = -\dfrac{\dfrac{(-48,134)(3.25)}{824}+(0.19)(1)\sqrt{4}\,(144)}{\dfrac{0.783}{115}+\dfrac{(2.375)(3.25)}{824}} = 8,354 \text{ kips}$$

Mid-span of Span 2 (bottom fiber):
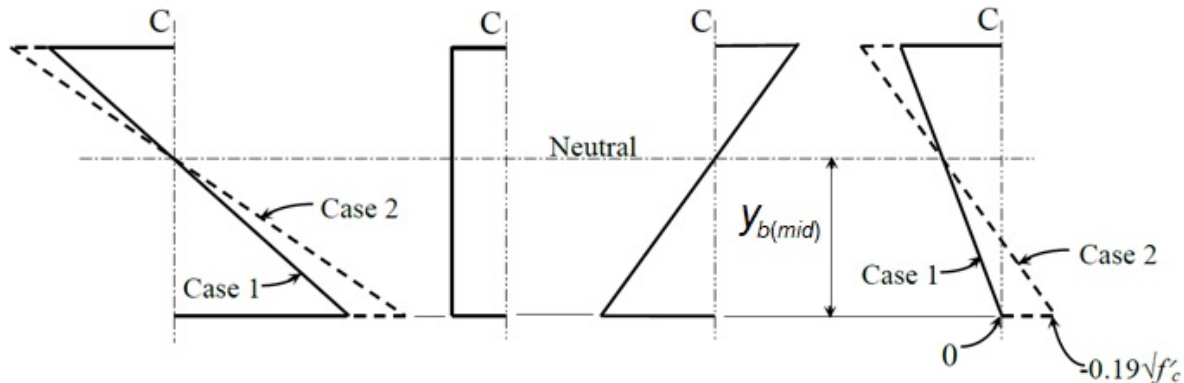
Case 1: $\dfrac{M_{DC+DW}\,y_{b(mid)}}{I_{(mid)}}+\dfrac{FC_{pT}\left(P_j\right)}{A_{(mid)}}+\dfrac{MC_{pT}\left(P_j\right)y_{b(mid)}}{I_{(mid)}}=0$

Case 2: $\dfrac{M_{DC+DW+0.8HL93}\,y_{b(mid)}}{I_{(mid)}}+\dfrac{FC_{pT}\left(P_j\right)}{A_{(mid)}}+\dfrac{MC_{pT}\left(P_j\right)y_{b(mid)}}{I_{(mid)}}=-0.19\lambda\sqrt{f'_c}$

Figure 5.2.12.11-2 Elastic Stresses in an Uncracked Prestress Beam. Effects of Prestress by Component at the Bottom of the Beam

Reading the Force Coefficient from Table 5.2.12.7.5-1 at point F

$FC_{@pt\,F}$ = 0.742

Reading Figure 5.2.12.9-1 for MC_{PT} @ the CL of Span 2: MC_{PT} = -1.202

Reading Figure 5.2.12.10-1

M_{DC+DW} = 23,511 kip-ft

$M_{DC+DW+0.8HL93}$ = 34,068 kip-ft

Case 1: $P_j=-\dfrac{\dfrac{M_{DC+DW}\,y_{b(mid)}}{I_{(mid)}}+0}{\dfrac{FC_{pT}}{A_{(mid)}}+\dfrac{\left(MC_{pT}\right)y_{t(mid)}}{I_{(mid)}}}$

$$P_j = -\frac{\dfrac{(23{,}511)(-3.8)}{729} + 0}{\dfrac{0.742}{103} + \dfrac{(-1.202)(-3.8)}{729}} = 9{,}100 \text{ kips}$$

Case 2: $P_j = -\dfrac{\dfrac{M_{DC+DW+0.8HL93}\, y_{t\text{(bent face)}}}{I_{\text{(bent face)}}} + (0.19)\lambda\sqrt{f'_c}\,(144)}{\dfrac{FC_{pT}}{A_{\text{(bent face)}}} + \dfrac{(MC_{pT})\, y_{t\text{(bent face)}}}{I_{\text{(bent face)}}}}$

$$P_j = -\frac{\dfrac{(34{,}068)(-3.8)}{729} + (0.19)(1)\sqrt{4}\,(144)}{\dfrac{0.742}{103} + \dfrac{(-1.202)(-3.8)}{729}} = 9{,}122 \text{ kips}$$

Therefore, P_j = 9,122 kips, round to the nearest 10 kips, P_j = 9,120 kips

> Notes: The overall largest P_j was calculated at the midspan of Span 2 under the Case 2 load condition. Two observations can be made:

Now that we have a P_j we can check our elastic shortening assumption using CA Amendments 5.9.3.2.3b-1 and Equation 5.2.5.1.3-1:

$$f_{cgp} = f_g + f_{ps} = \frac{M_{DL}e}{I_g} - \left(\frac{P_j}{A_g} + \frac{P_j e^2}{I_g} \right) \qquad\qquad (5.2.5.1.3\text{-}1)$$

$$\Delta f_{pES} = 0.5\frac{E_p}{E_{ci}} f_{cgp} \qquad\qquad (\text{CA Amendments } 5.9.3.2.3b\text{-}1)$$

Table 5.2.12.11-1 Values of f_{cgp} and Δf_{pES}

Loc	f_{cgp}	Δf_{pES}	Loc	f_{cgp}	Δf_{pES}	Loc	f_{cgp}	Δf_{pES}
0.024	-0.615	-2.285	1.024	-1.546	-5.745	2.034	-1.424	-5.292
0.100	-0.770	-2.863	1.100	-0.847	-3.147	2.100	-0.934	-3.470
0.200	-1.170	-4.348	1.200	-0.655	-2.433	2.200	-0.594	-2.208
0.300	-1.579	-5.870	1.300	-1.137	-4.227	2.300	-0.791	-2.940
0.400	-1.772	-6.585	1.400	-1.677	-6.232	2.400	-1.187	-4.412
0.500	-1.682	-6.252	1.500	-1.903	-7.071	2.500	-1.538	-5.716
0.600	-1.357	-5.043	1.600	-1.689	-6.277	2.600	-1.694	-6.297
0.700	-0.929	-3.453	1.700	-1.155	-4.293	2.700	-1.586	-5.894
0.800	-0.632	-2.349	1.800	-0.665	-2.471	2.800	-1.246	-4.629
0.900	-0.797	-2.964	1.900	-0.830	-3.083	2.900	-0.832	-3.091
0.968	-1.281	-4.761	1.976	-1.504	-5.590	2.975	-0.615	-2.285

Average Δf_{pES} = -4.351

At this point, we would rerun our numbers using the 4.4 ksi value for Δf_{pES}. However, for this example we will choose not to rerun the numbers. The 1.4 ksi difference between assumed and calculated result in about a 3% increase in P_j.

5.2.12.12   Determine the Required Concrete Strength

Now that the jacking force has been calculated for this structure, we can determine the stresses in the concrete due to prestressing. Prestress stresses need to be computed in order to determine the initial (f'_{ci}) and final (f'_c) concrete strengths required. The design of f'_{ci} is based on concrete stresses present at the time of jacking, which includes the initial prestress stress, f_{pi}. The initial prestress stress considers losses due to friction (ΔFC_{pF}) only. The design of f'_c is based on service level concrete stresses that occur after a period of time, which includes the effective prestress stress, f_{pe}. The effective (total) prestress stress considers the effects of all prestress losses (ΔFC_{pT}). The equations for concrete stresses due to prestressing are as follows:

$$f_{pi} = \frac{P_j FC_F}{A_g} + \frac{P_j(e)(FC_F)y_b}{I_g}$$

(5.2.12.12-1)

$$f_{pe} = \frac{P_j FC_{PT}}{A_g} + \frac{P_j(MC_{PT})y_b}{I_g}$$

(5.2.12.12-2)

where:

A_g =   gross area of section (in.^{2})

e =   eccentricity of the anchorage device or group of devices with respect to the centroid of the cross section. Always taken as a positive (ft)

FC_F =   force coefficient for loss

FC_{pT} =   force coefficient for loss from friction

f_{pe} =   effective stress in the prestressing steel after losses (ksi)

f_{pi} =   initial stress in the prestressing steel after losses, considering only the effects of friction loss. No other *P/S* losses have occurred (ksi)

I_g =   moment of inertia of the gross concrete section about the centroidal axis, neglecting reinforcement (in.^{4})

MC_{PT} =   total moment force coefficient for loss (ft)

P_j =   force in prestress strands before losses (kip)

Both initial and final stresses for the concrete top and bottom fibers due to the effects of prestressing have been calculated and are shown on the following two diagrams.
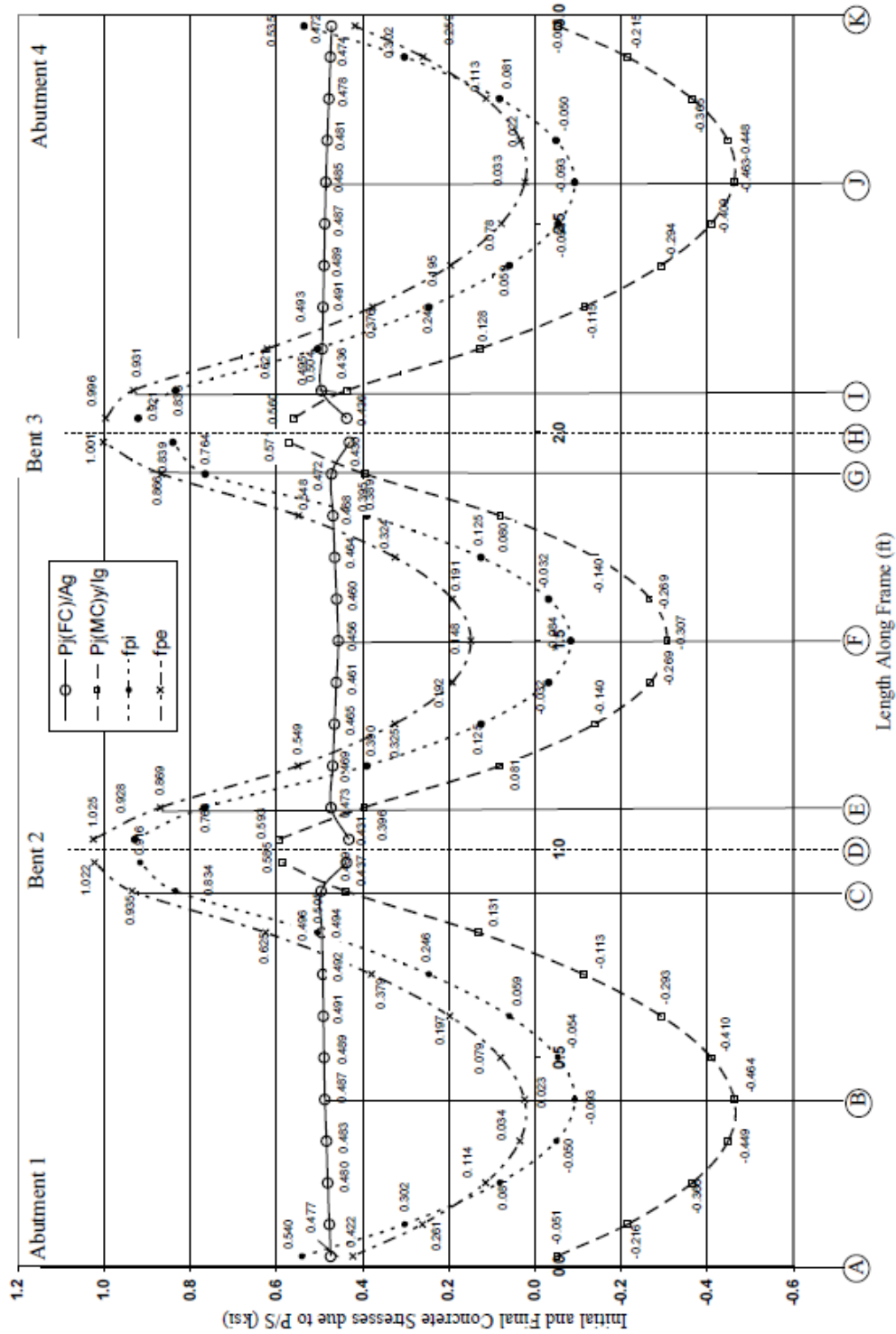
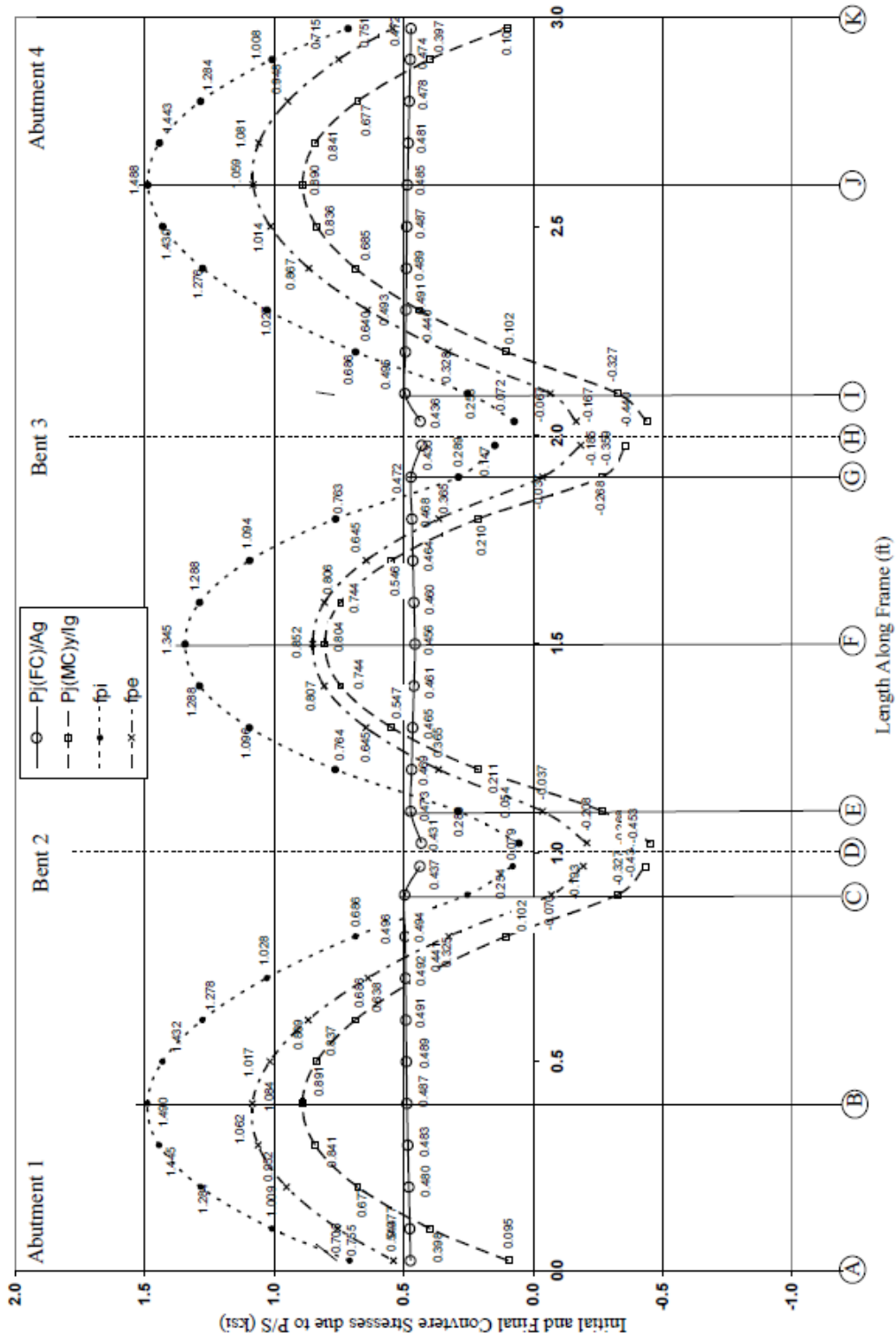**Figure 5.2.12.12-1 Top Fiber Concrete Stresses Due to Prestressing**

Figure 5.2.12.12-2 Bottom Fiber Concrete Stresses Due to Prestressing

Design of both initial and final concrete strengths required is governed by the Service I load case. This load case is defined as a load combination relating to the normal operational use of the bridge with a 70 mph wind and all loads taken at their nominal value (γ = 1.0) Article 3.4.1(AASHTO, 2017).

5.2.12.12.1 Design of f_c'

f_c' is defined as "the specified strength concrete for use in design" in Article 5.3 (AASHTO, 2017). Article 5.4.2.1 (AASHTO, 2017) specifies the compressive strength for prestressed concrete and decks shall not be less than 4.0 ksi.

Additionally, Table 5.9.2.3.2a-1 (AASHTO, 2017) lists compressive stress limits for prestressed components at the service Limit States after all losses as:

- Due to the sum of the effective prestress and permanent loads, the concrete has a compressive stress limit of 0.45 f'_c (ksi).

$$f_c' \geq \frac{f_{pe} + f_{DC+DW}}{0.45} \qquad (5.2.12.12-3)$$

  where:

  f_{DC+DW} =   stress in bridge from DC and DW load cases (ksi)

- Due to the sum of effective prestress, permanent loads and transient loads, the concrete has a compressive stress limit of 0.60 f'_c (ksi).

$$f_c' \geq \frac{f_{pe} + f_{DC+DW} + f_{HL93}}{0.60} \qquad (5.2.12.12-4)$$

Solving for f_c' at face of support at Bent 2 (0.0 L2 point).

Using Equations 5.2.12.12-2 and 5.2.12.12-3 for only the DL case.

At the cap face for Bent 2 the deck is in tension under service loads, therefore the prestressing steel is close to the deck to pull the section together, and resist tension. The controlling concrete strength demand will be opposite of the tension where the service loads act in compression, and the prestressing force acts in tension.

$$
\begin{aligned}
f_{pe} &= \frac{P_j FC_{PT}}{A_g} + \frac{P_j (MC_{PT}) y_b}{I_g} \\
&= \frac{9,120(0.783)}{115} + \frac{(9,120)(2.375)(-3.5)}{824} = -30.0 \text{ ksf}
\end{aligned}
$$

Notice that the stress due to the prestressing steel is negative. This means that the prestressing steel is pulling together the face of the concrete that is in tension, causing

the side opposite side of the prestressing steel to be in tension.

$$f_{pe} = \text{-30.0 ksf} \frac{1}{144} = \text{-0.21 ksi}$$

$$f_{DC+DW} = \frac{M_{DC+DW}(y_b)}{I_g} = \frac{-36,714(-3.5)}{824} = 156.0 \text{ ksf}$$

$$f_{DC+DW} = 156.0 \text{ ksf} \frac{1}{144} = 1.08 \text{ ksi}$$

$$f_c' \geq \frac{f_{pe} + f_{DC+DW}}{0.45} \text{ (ksi)} = \frac{-0.21+1.08}{0.45} = 1.9 \text{ ksi}$$

Using Equation 5.2.12.12-4 for only the *LL* case.

$$f_{HL93} = \frac{M_{HL93}(y_b)}{I_g} = \frac{-14,275(-3.5)}{824} = 60.6 \text{ ksf}$$

$$f_{HL93} = 60.6 \text{ ksf} \frac{1}{144} = 0.42 \text{ ksi}$$

$$f_c' \geq \frac{f_{pe} + f_{DC+DW} + f_{HL93}}{0.60} \text{ (ksi)} = 2.16 \text{ ksi}$$

Therefore, the minimum f_c' controls from Article 5.4.2.1 (AASHTO, 2017):
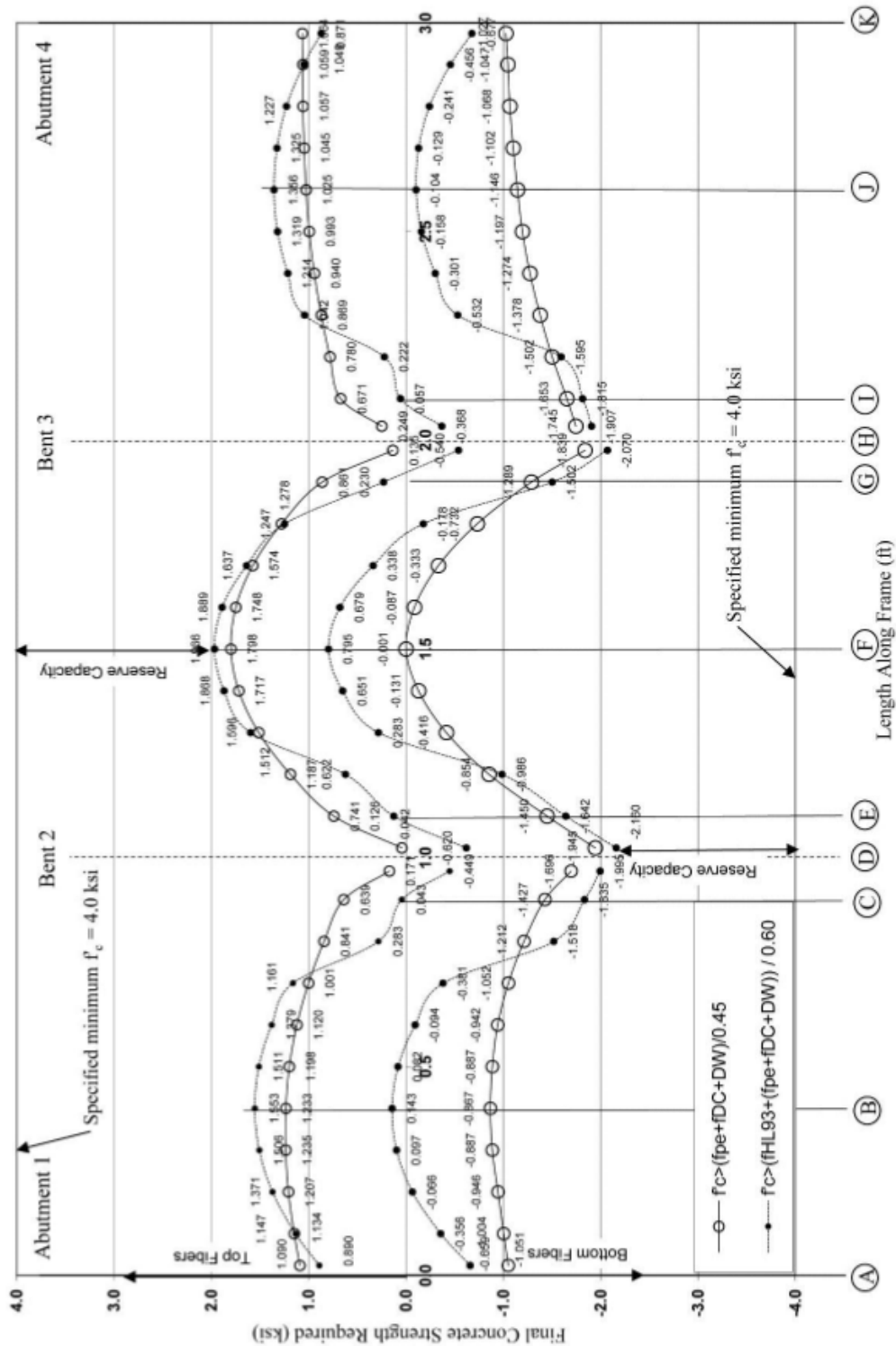
$$f_c' = 4.0 \text{ ksi}$$

Figure 5.2.12.12-3 Determining Final Concrete Strength Required

5.2.12.12.2 Design of f'_{ci}

f'_{ci} is defined as "design concrete compressive strength at time of prestressing for pretensioned members and at time of initial loading for nonprestressed members" in Article 5.4.2.3.2. There are two criteria used to design f'_{ci}, and they are as follows:

The compressive stress limit for pretensioned and post-tensioned concrete components, including segmentally constructed bridges, shall be 0.65 f'_{ci} (ksi). Only the stress components present during the time of prestressing shall be considered (Article 5.9.2.3.1a).

$$f'_{ci} \geq \frac{f_{pi} + f_{DC \; w/o \; b}}{0.60} \hspace{4cm} (5.2.12.12\text{-}5)$$

where:

$f_{DC \; w/o \; b}$ = stress in concrete due to the Dead Load of the structural section only (ksi)

The specified initial compressive strength of prestressed concrete should not be less than 3.5 ksi.

Solving for f'_{ci} at the cap face Bent 2 (0.02 L_2 point).

Interpolating the Force Coefficient due to Friction from Table 5.2.12.7.5-1 between points *D* and *E*

$FC_{pF@pt \; D}$ = 0.925

$FC_{pF@pt \; E}$ = 0.907

Span 2 Length = 168 ft

Distance from CL of the column to the face of the cap (pt *D*) = 4 ft

Distance from CL of the column to the location of the first inflection point of Span 2 (pt *E*) = 16.8 ft

$$\left(\frac{0.925 - 0.907}{0 - 16.8} \right)(4 - 0) + 0.925 = 0.921$$

Interpolating the eccentricities from Figure 5.2.12.8-5 between points *D* and *E*

$e_{@pt \; D}$ = 2.122 ft

$e_{@pt \; E}$ = 0.895

Span 2 Length = 168 ft

Distance from CL of the column to the face of the cap (pt D) = 4 ft

Distance from CL of the column to the location of the first inflection point of Span 2 (pt E) = 16.8 ft

$$\left(\frac{2.122-0.896}{0-16.8}\right)(4-0)+2.122=1.830$$

Using Equation 5.2.12.12-1

$$f_{pi} = \frac{P_j FC_F}{A_g} + \frac{P_j(e)(FC_F)y_b}{I_g}$$

$$= \frac{9,120(0.921)}{115} + \frac{9,120(1.830)(0.921)(-3.5)}{824} = 7.75 \text{ ksf}$$

$$f_{pi} = 7.75 \text{ ksf} \frac{1}{144} = 0.054 \text{ ksi}$$

$$f_{DC \text{ w/o b}} = \frac{M_{DC \text{ w/o b}}(y_b)}{I_g} = \frac{-30,989(-3.5)}{824} = 131.63 \text{ ksf}$$

$$f_{DC \text{ w/o b}} = 131.63 \text{ ksf} \frac{1}{144} = 0.914 \text{ ksi}$$

Using Equation 5.2.12.12-5

$$f'_{ci} \geq \frac{f_{pi} + f_{DC \text{ w/o b}}}{0.60} = \frac{0.054 + 0.914}{0.6} = 1.63 \text{ ksi}$$

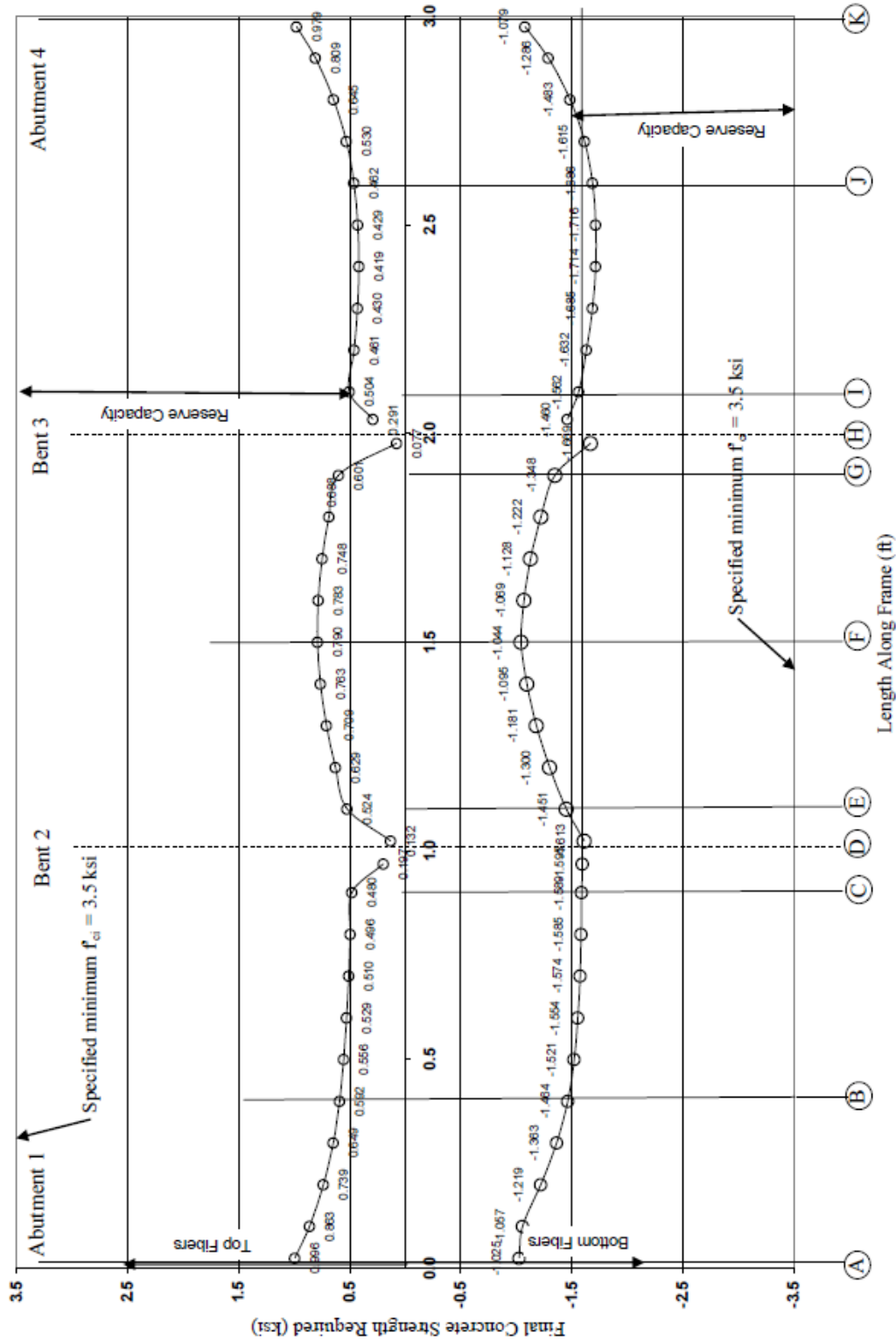Using Figure 5.2.12.12-4, the minimum f'_{ci} = 3.5 ksi controls.

Figure 5.2.12.12-4 Determining Initial Concrete Strength Required

5.2.12.13 Flexural Design

5.2.12.13.1   General

For rectangular or flanged sections subjected to flexure about one axis where the approximate stress distribution as specified in Article 5.6.2.2 (AASHTO, 2017) is used and for which f_{pe} is not less than 0.5 f_{pu}, the average stress in the prestressing steel, f_{ps}, may be taken as:

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$

(AASHTO 5.6.3.1.1-1)

$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right)$$

(AASHTO 5.6.3.1.1-2)

Alternatively, the stress in the prestressing steel may be determined by strain compatibility (see Article 5.6.3.2.5).

For rectangular section behavior, the distance between the neutral axis and the compressive face can be represented as:

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{\alpha_1 f'_c \, \beta_1 b + kA_{ps}\dfrac{f_{pu}}{d_p}}$$

(AASHTO 5.6.3.1.1-4)

$$\alpha_1 = 0.85$$

(AASHTO 5.6.2.2)

The factored resistance M_r shall be taken as:

$$M_r = \phi \, M_n$$

(AASHTO 5.6.3.2.1-1)

For flanged sections subjected to flexure about one axis and for biaxial flexure with the axial load as specified in Article 5.6.4.5, (AASHTO, 2017), where the approximate stress distribution specified in Article 5.6.2.2 (AASHTO, 2017) is used and the tendons are bonded, and where the compression flange depth is less than $\alpha = \beta_1$, as determined in accordance with Equation 5.6.3.2.2-1, the nominal flexural resistance may be taken as:

$$M_n = \underbrace{A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right)}_{\substack{\text{Presstressed} \\ \text{Steel Only}}} + \underbrace{A_s f_s\left(d_s - \frac{a}{2}\right)}_{\substack{\text{Additional} \\ \text{Flexural Steel}}} - \underbrace{A'_s f'_s\left(d'_s - \frac{a}{2}\right)}_{\substack{\text{Compression} \\ \text{Steel}}} + \underbrace{\alpha_1 f'_c\left(b - b_w\right)h_f\left(\frac{a}{2} - \frac{h_f}{2}\right)}_{\substack{\text{Flanged Section Component} \\ \alpha = \beta_1 c \le h_f}}$$

$$\frac{f_{ps}}{f_{pu}} = 0.9$$

(AASHTO Table C5.6.3.1.1-1)

$$k = 0.28 \qquad\qquad\qquad \text{(AASHTO Table C5.6.3.1.1-1)}$$

$$A_{ps} = \frac{P_j}{0.75 f_{pu}}$$

where:

A_{ps}	=	area of prestressing steel (in.^{2})
A_s	=	area of non-prestressed tension reinforcement (in.^{2})
b	=	width of the compression face of a member (in.)
c	=	distance from extreme compression fiber to the neutral axis (in.)
d_p	=	distance from extreme compression fiber to the centroid of the prestressing tendons (in.)
f'_c	=	specified compressive strength of concrete used in design (ksi)
f_{ps}	=	average stress in prestressing steel at the time for which the nominal resistance is required (ksi)
f_{pu}	=	specified tensile strength of prestressing steel (ksi)
M_r	=	factored flexural resistance of a section in bending (kip-in.)
M_n	=	nominal flexure resistance (kip-in.)
β_1	=	ratio of the depth of the equivalent uniformly stressed compression zone assumed in the strength Limit States to the depth of the actual compression zone
ϕ	=	resistance factor

The factored ultimate moment, M_u , shall be taken as the greater of the following two Strength I and II Limit States as defined in California Amendments Article 3.4.1 and Table 3.4.1-1 (Caltrans, 2019).

Strength I: $M_{u(HL93)}$ = 1.25 (M_{DC}) + 1.50 (M_{DW}) + 1.75 (M_{HL93}) + 1.00 $(M_{P/S\,s})$

Strength II: $M_{u(P\text{-}15)}$ = 1.25 (M_{DC}) + 1.50 (M_{DW}) + 1.35 $(M_{P\text{-}15})$ + 1.00 $(M_{P/S\,s})$

The largest value of M_u indicated the governing Limit States at a given location.

It is possible to have different Limit States at different locations.

Unless otherwise specified, at any section of a flexural component, the amount of prestressed and nonprestressed tensile reinforcement shall be adequate to develop a factored flexural resistance, $M_{r\,(min)}$, at least equal to the lesser of:

1.33 M_u as defined in Article 5.6.3.3 (AASHTO, 2017)

$$M_{cr} = \gamma_3 \left[\left(\gamma_1 f_r + \gamma_2 f_{cpe} \right) S_c - M_{dnc} \left(\frac{S_c}{S_{nc}} - 1 \right) \right] \qquad \text{(AASHTO 5.6.3.3-1)}$$

$$f_r = 0.24\lambda\sqrt{f'_c} \hspace{4cm} \text{(AASHTO 5.4.2.6)}$$

The second part of AASHTO Equation 5.6.3.3-1 will not be needed for this example because the superstructure is not a composite section.

Article 5.6.3.3 defines:

γ_1 = 1.6 for superstructures that are not precast segmental

γ_2 = 1.1 for bonded tensions

γ_3 = 0.75 if additional mild reinforcement is A 706, grade 60 reinforcement

where:

f_r = modulus of rupture of concrete (ksi)

f_{cpe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at the extreme fiber of section where tensile stress is caused by externally applied loads (ksi)

M_{cr} = cracking moment (kip-in.)

S_c = section modulus for the extreme fiber of the composite sections where tensile stress is caused by externally applied loads (in^3)

γ_1 = flexural cracking variability factor

γ_2 = prestress variability factor

γ_3 = ratio of specified minimum yield strength to ultimate tensile strength of reinforcement

λ = the concrete density modification factor

The prestressing steel present in the section by itself may be enough to resist the applied factored moment. However, additional flexural steel may have to be added to provide adequate moment resistance for the Strength I and II Limit States. Flexural steel provided for seismic resistance can be relied upon for Strength Limit States.

AASHTO Article 5.7.3.3.1 (removed from the 6th edition), which defined a limit on tension steel to prevent over-reinforced sections, was eliminated in the 2006 interims. The current approach involves reducing the flexural resistance factor when the tensile strain in the reinforcement falls below 0.005. In other words, over-reinforced sections are allowed by the code, but a more conservative resistance factor is applied. Conventional designs will likely result in tensile strains greater than 0.005. The tensile strain can be determined using the *c/d_e* ratio. From a simple plane strain diagram assuming a concrete strain of 0.003, a c/de ratio of 0.375 corresponds to a tensile strain of 0.005. If the *c/d_e* ratio exceeds 0.375, then a reduced ϕ must be used as defined in Article 5.5.4.2 (Caltrans, 2019).

Figure 5.2.12.13-1 shows unfactored moments due to dead loads and live loads for this
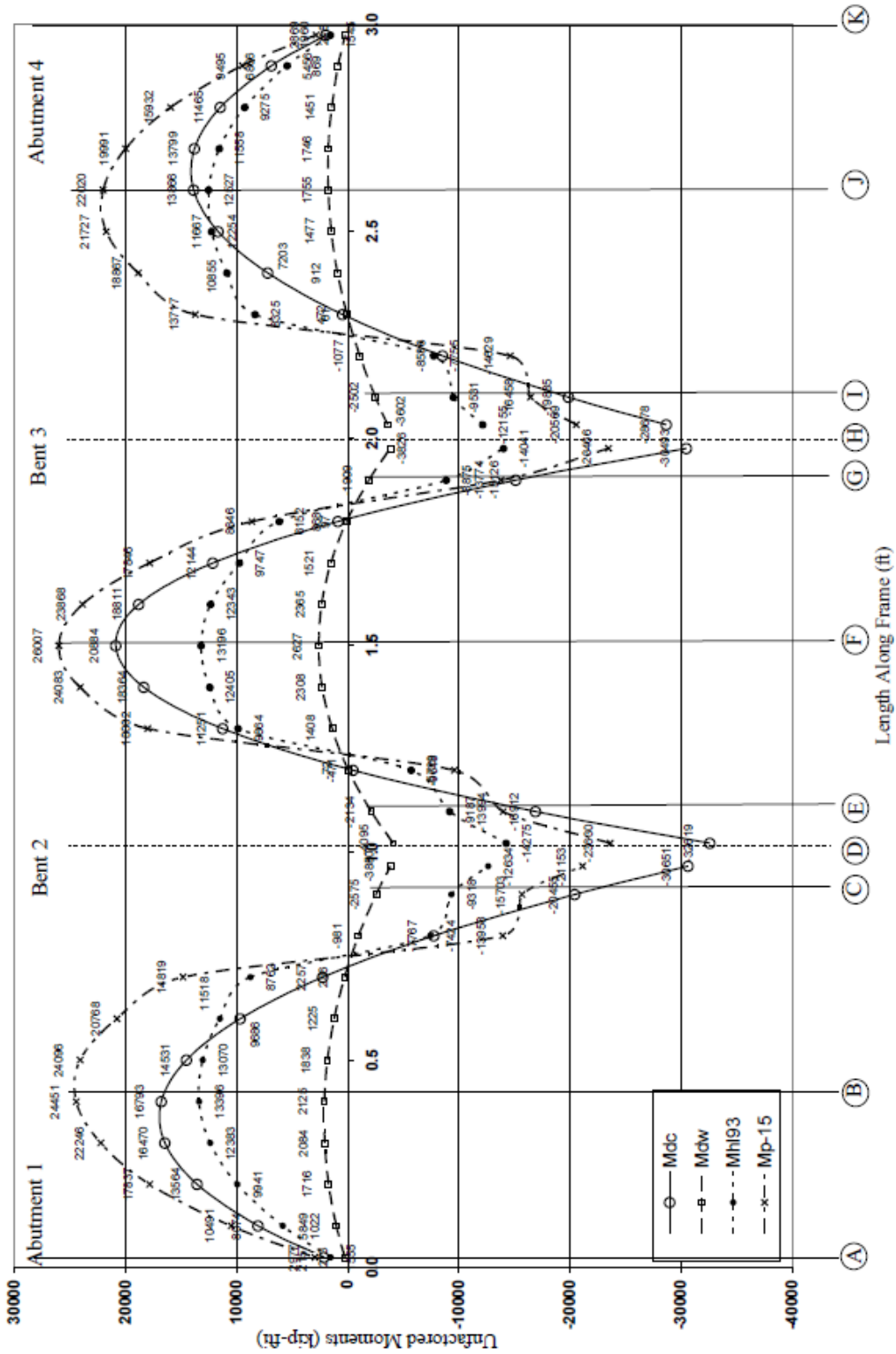
example.

Figure 5.2.12.13-1 Gravity Load Moment Envelopes for Design of Flexural Resistance

5.2.12.13.2   Check Section at Right Face of Cap at Bent 2

Find the flexural resistance of the section at the right face of the cap at Bent 2 considering the Area of *P/S* steel only. If required, find the amount of additional flexural steel needed to resist the factored nominal resistance ϕM_n.

$$M_{P/S\ s} = P_j \left(MC_s \right) \tag{5.2.12.13-2}$$

where:

   $M_{P/S}$ =   moment due to the secondary effects of prestressing (k-ft)

Step 1:   Determine the controlling Strength Limit State used to determine the factored ultimate moment, M_u:

   Strength I:

   $(MC_s) = 0.856$ ft (Figure 5.2.12.9-1)

   $M_{P/S\ s} = (9{,}120)(0.856) = 7{,}810$ kip-ft

   $M_{u(HL93)} = 1.25 \left(M_{DC} \right) + 1.50 \left(M_{DW} \right) + 1.75 \left(M_{HL93} \right) + 1.00 \left(M_{P/S\ s} \right)$

   $M_{u(HL93)} = 1.25 \left(-32{,}619 \right) + 1.50 \left(-4{,}095 \right) + 1.75 \left(-14{,}275 \right) + 1.00 \left(7{,}810 \right)$
   $= -64{,}090$ kip-ft

   Strength II:

   $M_{u(P-15)} = 1.25 \left(M_{DC} \right) + 1.50 \left(M_{DW} \right) + 1.75 \left(M_{P-15} \right) + 1.00 \left(M_{P/S\ s} \right)$

   $M_{u(P-15)} = 1.25 \left(-32{,}619 \right) + 1.50 \left(-4{,}095 \right) + 1.35 \left(-23{,}630 \right) + 1.00 \left(7{,}810 \right)$
   $= -71{,}010$ kip-ft

   The Strength II Limit State controls, M_u = -71,010 kip-ft

Step 2: Compute M_{cr} to determine which criteria governs the design of the factored resistance, M_r (1.33M_u or using AASHTO Equation 5.6.3.3-1).

$$S_c = \frac{I_{bent\ face}}{y_{bent\ face}} = \frac{\left(824\ ft^4 \right) 12^4}{(3.25\ ft) 12} = 438{,}115\ in.^3$$

$$f_r = 0.24 \lambda \sqrt{f'_c} = 0.24 (1) \sqrt{4} = 0.48\ ksi$$

f_{cpe} = 1.025 ksi (from Figure 5.2.12.12-2)

$$M_{cr} = \gamma_3 \left[\left(\gamma_1 f_r + \gamma_2 f_{cpe} \right) S_c - M_{dnc} \left(\frac{S_c}{S_{nc}} - 1 \right) \right]$$

$$= 0.75 \left[\left(1.6(0.48) + 1.1(1.025) \right) 438,115 - 0 \right]$$

$$= 622,835 \text{ kip-in.} = 51,903 \text{ kip-ft}$$

$$M_{r(\text{min})} = \text{the lesser of} \begin{bmatrix} M_{cr} = 51,903 \text{ kip-ft} \\ 1.33 M_u = 1.33(71,010) = 94,443 \text{ kip-ft} \end{bmatrix}$$

$$= 51,903 \text{ kip-ft}$$

Step 3: Compute the nominal moment resistance of the section based on the effects of the prestressing steel using AASHTO 5.6.3.1.1-4 only and substituting out A_{ps}:

$$c = \frac{A_{ps} f_{pu} + A_s f_s - A'_s f'_s}{\alpha_1 f'_c \beta_1 b + k A_{ps} \dfrac{f_{pu}}{d_p}} = \frac{\dfrac{P_j}{0.75 f_{pu}} f_{pu} + A_s f_s - A'_s f'_s}{\alpha_1 f'_c \beta_1 b + k \dfrac{P_j}{0.75 f_{pu}} \dfrac{f_{pu}}{d_p}}$$

Assuming no compression or tension resisting mild steel, A_s and A'_s both equal zero.

b = the soffit width = overall width – overhang width (including slope)

$b = 59.5(12) - 2 \left[5(12) + 5.75/2 \right] = 525$ in. (Figure 5.2.12.3-1)

d_p = structure depth – prestressing force distance to the deck (interpolated between points D and E)

$d_p = 81 - 17.0 = 64.0$ in.

$$c = \frac{\dfrac{9,120}{0.75} + 0 + 0}{0.85(4)(0.85)(525) + 0.28 \left(\dfrac{9,120}{0.75(64)} \right)} = 7.74 \text{ in.}$$

7.74 in. < h_{soffit} = 12 in.; therefore, the rectangular section assumption is satisfied

Using AASHTO Equation 5.6.3.1.1-1:

$$f_{ps} = f_{pu} \left(1 - k \frac{c}{d_p} \right) = 270 \left(1 - 0.28 \frac{7.74}{64.0} \right) = 260.9 \text{ ksi}$$

Modifying AASHTO Equation 5.6.3.2.2-1 for rectangular sections produces Equation 5.2.12.13-3:

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$$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right)$$

(5.2.12.13-3)

From Article 5.6.2.2 (AASHTO, 2017):

$$a = \beta_1 c$$

(5.2.12.13-4)

$$a = \beta_1 c = 0.85(7.74) = 6.58 \text{ in.}$$

$$A_{ps} = \frac{P_j}{0.75f_{pu}} = \frac{9,120}{0.75(270)} = 45.0 \text{ in.}^2$$

$$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) = 45.0(260.9)\left(64.0 - \frac{6.58}{2}\right)$$

$$= 712,766 \text{ kip-in.} = 59,397 \text{ kip-ft}$$

Assume $\phi = 0.95$ and obtain $\phi M_n = 0.95 (59,397) = 56,427$ kip-ft $< M_u = 71,010$ kip-ft. It shows that additional mild steel reinforcement is required. For illustrative purposes, let's determine A_s based on $M_r = 77,300$ kip-ft.

Step 4: Compute the area of mild steel required to have the factored resistance, $M_r = 77,300$ kip-ft.

Rearranging AASHTO 5.6.3.1.1-4 and Equation 5.2.12.13-4 and substituting out the values for f_{ps} and A_{ps}, results in Equation 5.2.12.13-5.

$$a = \frac{\dfrac{P_j}{0.75f_{pu}}f_{pu} + A_s f_s - A_s' f_s'}{\alpha_1 f_c' b + k\dfrac{kP_j f_{pu}}{\beta_1\left(0.75f_{pu}\right)d_p}}$$

(5.2.12.13-5)

$$\frac{a}{2} = \frac{\dfrac{9,120}{0.75} + (60)A_s - 0}{2\left(0.85(4)(525) + \dfrac{0.28(9,120)}{(0.85)(0.75)(64.0)}\right)} = 3.29 + 0.0162A_s$$

Note that we will assume $f_s = f_y$. This assumption is valid if the reinforcement at the extreme steel tension fiber fails. We can check this by measuring ε_t at the end of the calculation.

Modifying AASHTO Equation 5.6.3.2.2-1 for rectangular sections produces Equation 5.2.12.13-6.

$$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_s\left(d_s - \frac{a}{2}\right)$$

(5.2.12.13-6)

where:

d_s =   distance from extreme compression fiber to the centroid of the nonprestressed tensile reinforcement (in.)

$d_s = d - 0.5(h_{deck}) = 81 - 0.5(9) = 76.5$ in.

$$\left(\frac{77,300(12)}{0.95}\right) = (45.0)(260.9)\left(64.0 - (3.29 + 0.0162A_s)\right)$$

$$+ \ 60A_s\left(76.5 - (3.29 + 0.0162A_s)\right)$$

$$0.97A_s^2 - 4,202A_s + 263,655 = 0$$

Solving the quadratic equation: $A_s = 63.68$ in.^{2} (65 # 9 bars $A_s = 65$ in.^{2})

Step 5: Verify Assumptions – Two assumptions were made in the determination of A_s. The first was that the mild steel would yield and we could use f_y for f_s. The validity of this assumption can be checked by calculating the ε_t. According to the California Amendments (Caltrans, 2019) and Figure 5.2.12.13-2 if $(\varepsilon_t - \varepsilon_{cl}) / (\varepsilon_{tl} - \varepsilon_{cl})$ is > 0.005 then the section is tension controlled and $\phi = 0.95$ we used initially. These values can be easily obtained with a simple strain diagram setting the concrete strain to 0.003.
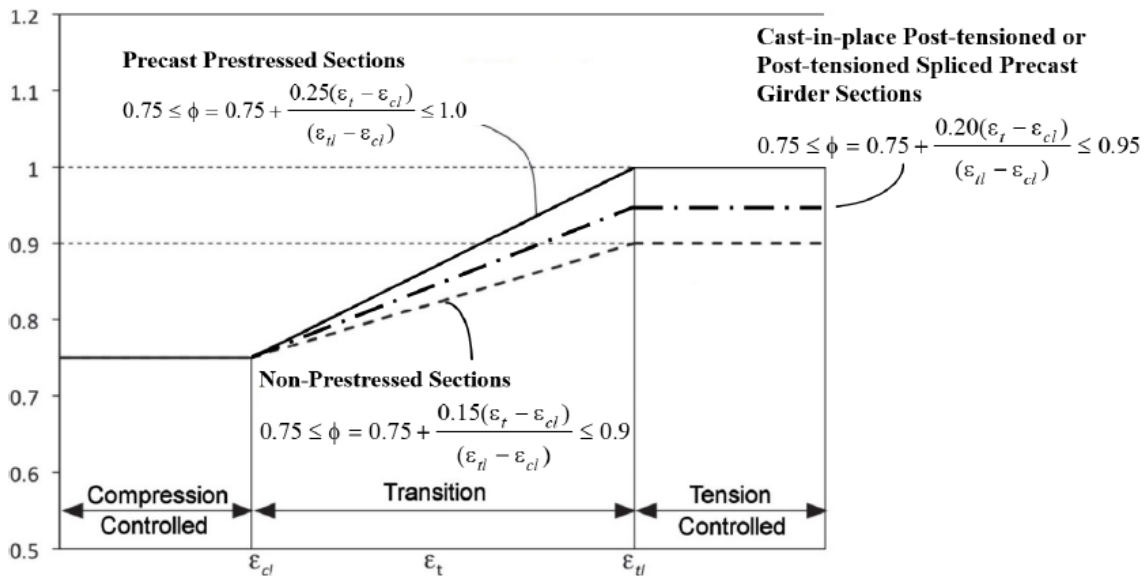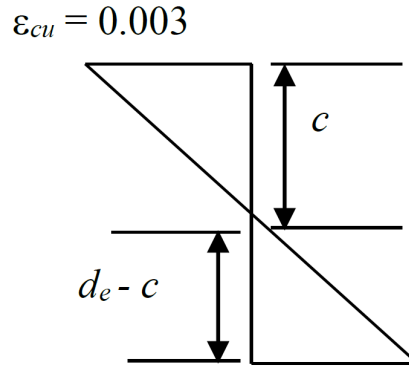

Figure 5.2.12.13-2 Variation of ϕ with Net Tensile Strain ε_t for Nonprestressed and Prestressed Members (California Amendments Figure C5.5.4.2-1, 2019)

$$\varepsilon_{cu} = 0.003$$

Figure 5.2.12.13-3 Strain Diagram

where:

c = distance from extreme compression fiber to the neutral axis (in.)

d_e = effective depth from extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (in.)

ε_{cu} = failure strain of concrete in compression (in./in.)

ε_t = net tensile strain in extreme tension steel at nominal resistance (in./in.)

Having just calculated A_s it is possible to calculate both c and d_e using AASHTO 5.6.3.1.1-4 and substituting out f_{ps} and A_{ps}:

$$c = \frac{\dfrac{P_j}{0.75} + A_s f_s - A'_s f'_s}{\alpha_1 f'_c \beta_1 b + k \dfrac{P_j}{0.75}\dfrac{1}{d_p}}$$

$$= \frac{\dfrac{9{,}120}{0.75} + (65.0)(60) - 0}{\left(0.85(4)(0.85)(525) + (0.28)\dfrac{9{,}120}{0.75}\left(\dfrac{1}{64.0}\right)\right)} = 10.11 \text{ in.}$$

Therefore c = 10is .11 in. < 12 in., the rectangular section assumption satisfied.

$$d_e = \frac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y}$$

$$== \frac{(45.0)(260.9)(64.0) + (65.0)(60)(76.50)}{(45.0)(260.9) + (65.0)(60)} = 67.12 \text{ in}$$

By similar triangles:

$$\frac{\varepsilon_c}{c} = \frac{\varepsilon_t}{d_e - c} :$$

$$\varepsilon_t = \frac{\varepsilon_c}{c}(d_e - c) = \frac{0.003}{10.11}(67.12 - 10.11) = 0.017 > 0.005$$

Therefore, the assumptions of yielding of the mild steel and $\phi = 0.95$ are valid.

5.2.12.13.3   Check Section at Midspan of Span 2

Find the flexural resistance of the section at midspan of Span 2 considering the area of *P/S* steel only. If required, find the amount of additional flexural steel needed to resist the factored nominal resistance, ϕM_n :

Step 1:   Determine the controlling Strength Limit State used to determine the factored ultimate moment, M_u:

Strength I:

$(MC_s) = 0.854$ ft (Figure 5.2.12.9-1)

$M_{P/S\ s} = (9{,}120)(0.854) = 7{,}789$ kip-ft

$M_{u(HL93)} = 1.25(M_{DC}) + 1.50(M_{DW}) + 1.75(M_{HL93}) + 1.00(M_{P/S\ s})$

$M_{u(HL93)} = 1.25(20{,}884) + 1.50(2{,}627) + 1.75(13{,}196) + 1.00(7{,}789)$
$\qquad\qquad = 60{,}928$ kip-ft

Strength II:

$M_{u(P-15)} = 1.25(M_{DC}) + 1.50(M_{DW}) + 1.35(M_{P-15}) + 1.00(M_{P/S\ s})$

$M_{u(P-15)} = 1.25(20{,}884) + 1.50(2{,}627) + 1.35(26{,}007) + 1.00(7{,}789)$
$\qquad\qquad = 72{,}944$ kip-ft

The Strength II Limit State controls, $M_u = 72{,}944$ kip-ft

Step 2: Compute M_{cr} to determine which criteria governs the design of the factored resistance, M_r (1.33M_u or using AASHTO 5.6.3.2.2-1).

From Section 5.2.12.4, $I_{mid} = 72{,}9$ ft^4 and $y_{b\text{-}mid} = 3.8$ ft.

$$S_c = \frac{I_{mid}}{y_{b-mid}} = \frac{(729)}{3.8} \text{ ft}^3 = 331{,}503 \text{ in.}^3$$

$$f_r = 0.24\lambda\sqrt{f'_c} = 0.24(1)\sqrt{4} = 0.48 \text{ ksi}$$

f_{cpe} = 0.852 ksi (from Figure 5.2.12.12-2)

$$M_{cr} = \gamma_3\left[\left(\gamma_1 f_r + \gamma_2 f_{cpe}\right)S_c - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right)\right]$$

$$= 0.75\left[\left(1.6(0.48) + 1.1(0.852)\right)331{,}503 - 0\right]$$

$$= 423{,}959 \text{ kip-in.} = 35{,}330 \text{ kip-ft}$$

$$M_{r(\min)} = \text{the lesser of}\begin{bmatrix}M_{cr} = 35{,}330 \text{ kip-ft} \\ 1.33M_u = 1.33(72{,}940) = 97{,}010 \text{ kip-ft}\end{bmatrix}$$

$$= 35{,}330 \text{ kip-ft}$$

Step 3: Compute the nominal moment resistance of the section based on the effects of the prestressing steel using AASHTO 5.6.3.1.1-4 only and substituting out f_{ps} and A_{ps}:

$$c = \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{\alpha_1 f'_c \beta_1 b + kA_{ps}\dfrac{f_{pu}}{d_p}} = \frac{\dfrac{P_j}{0.75f_{pu}}f_{pu} + A_s f_s - A'_s f'_s}{\alpha_1 f'_c \beta_1 b + k\dfrac{P_j}{0.75f_{pu}}\dfrac{f_{pu}}{d_p}}$$

Assuming no compression or tension resisting mild steel, A_s and A'_s both equal zero.

b = compression (top) flange width = 59.5 ft = 714 in.

d_p = structure depth – prestressing force distance to soffit

d_p = 81 – 12.5 = 68.5 in.

$$c = \frac{\dfrac{9{,}120}{0.75} + 0 - 0}{\left(0.85(4)(0.85)(714) + (0.28)\left(\dfrac{9{,}120}{0.75}\right)\dfrac{1}{68.5}\right)} = 5.51 \text{ in.}$$

5.51 in. < h_{deck} = 9.0 in., therefore, rectangular section assumption satisfied.

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right) = 270\left(1 - 0.28\frac{5.51}{68.5}\right) = 263.9 \text{ ksi}$$

Using Equations 5.2.12.13-3 and 5.2.12.13-4:

$$a = \beta_1 c = 0.85(5.51) = 4.68 \text{ in.}$$

$$M_n = A_{ps} f_{ps} \left(d_p - \frac{a}{2} \right) = 45.0(263.9)\left(68.5 - \frac{4.68}{2} \right)$$

$$= 783{,}385 \text{ kip-in.} = 65{,}449 \text{ kip-ft}$$

Assume $\phi = 0.95$ and obtain $\phi M_n = 0.95\,(65{,}449) = 62{,}177$ kip-ft $< 72{,}944$ kip-ft. It shows that additional mild steel reinforcement is required. For a conservative design, let's determine A_s based on $M_r = 75{,}000$ kip-ft.

Step 4: Compute the area of mild steel required to increase ϕM_n to resist the full factored, M_r .

Using Equation 5.2.12.13-5:

$$a = \frac{\dfrac{P_j}{0.75 f_{pu}} f_{pu} + A_s f_s - A_s' f_s'}{\alpha_1 f_c' b + \dfrac{k}{\beta_1} \dfrac{P_j}{0.75 f_{pu}} \dfrac{f_{pu}}{d_p}}$$

$$\frac{a}{2} = \frac{\dfrac{9{,}120}{0.75} + (60) A_s - 0}{2\left(0.85(4)(714) + \dfrac{0.28}{0.85} \left(\dfrac{9{,}120}{0.75} \right) \dfrac{1}{68.5} \right)} = 2.418 + 0.012 A_s$$

Again, assume $f_s = f_y$. This assumption is valid if the reinforcement at the extreme steel tension fiber fails. We can check this by measuring ε_t at the end of the calculation.

Using Equation 5.2.12.13-6:

$$M_n = A_{ps} f_{ps} \left(d_p - \frac{a}{2} \right) + A_s f_s \left(d_s - \frac{a}{2} \right)$$

$$d_s = d - 0.5\left(h_{soffit} \right) = 81 - 0.5(8.25) = 76.88 \text{ in.}$$

$$\left(\frac{75{,}000(12)}{0.95} \right) = (45.0)(263.9)\big(68.5 - (2.418 + 0.012 A_s) \big) + 60 A_s \big(76.88 - (2.418 + 0.012 A_s) \big)$$

$$0.72 A_s^2 - 4{,}470 A_s + 162{,}612 = 0$$

Solving the quadratic equation: $A_s = 36.59$ in.^{2} (37 # 9 bars $A_s = 37.00$ in.^{2})

Step 5: Verify the two assumptions that were made in the determination of A_s. The first was that the mild steel would yield and we could use f_y for f_s. The validity of this

assumption can be checked by calculating the ε_t. According to the California Amendments (Caltrans, 2019) and Figure 5.2.12.13-2 if $\varepsilon_t > 0.005$ then the section is tension controlled and $\phi = 0.95$. These values can be easily obtained with a simple strain diagram setting the concrete strain to 0.003.

$$\varepsilon_{cu} = 0.003$$
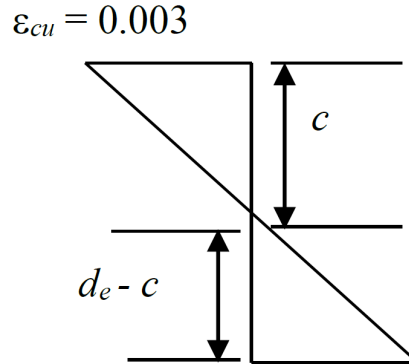
Figure 5.2.12.13-4 Strain Diagram

Having just calculated A_s., it is possible to calculate both c and d_e using AASHTO 5.6.3.1.1-4 and substituting out f_{ps} and A_{ps}:

$$c = \frac{\dfrac{P_j}{0.75} + A_s f_s - A'_s f'_s}{\alpha_1 f'_c \beta_1 b + k \dfrac{P_j}{0.75}\dfrac{1}{d_p}}$$

$$= \frac{\dfrac{9{,}120}{0.75} + (37.0)(60) - 0}{\left(0.85(4)(0.85)(714) + (0.28)\dfrac{9{,}120}{0.75}\left(\dfrac{1}{68.5}\right)\right)} = 6.80 \text{ in.}$$

t_s = 8.25 in.

c = 6.80 in. < 8.25 in., therefore, the rectangular section assumption is satisfied.

$$d_e = \frac{A_{ps}f_{ps}d_p + A_s f_y d_p}{A_{ps}f_{ps} + A_s f_y}$$

$$= \frac{(45.0)(263.9)(68.5) + (37.00)(60)(76.88)}{(45.0)(263.9) + (37.00)(60)} = 69.82 \text{ in.}$$

By similar triangles:

$$\frac{\varepsilon_c}{c} = \frac{\varepsilon_t}{d_e - c} :$$

$$\varepsilon_t = \frac{\varepsilon_c}{c}(d_e - c) = \frac{0.003}{6.8}(69.82 - 6.8) = 0.028 > 0.005$$

Therefore, the assumptions of the mild steel yielding and $\phi = 0.95$ are valid.

A similar procedure can be used to check flexural resistances for all other locations.

5.2.12.14   Shear Design

5.2.12.14.1   General

Where it is reasonable to assume that plane sections remain plane after loading, regions of components shall be designed for shear using either the sectional method as specified in Article 5.7.3 (AASHTO, 2017), or the strut-and-tie method as specified in Article 5.8.2 (AASHTO, 2017). When designing for nominal shear resistance in box-girders, it is appropriate to use the sectional method.

In the sectional design approach, the component is investigated by comparing the factored shear force and the factored shear resistance at a number of sections along its length. Usually, this check is made at the tenth point of the span and locations near the supports.

Where the reaction force in the direction of the applied shear introduces compression into the end region of a member, the location of the critical section for shear shall be taken as d_v from the internal face of support.

Figure CB5.2-5 illustrates the shear design process using a flow chart. This Figure is based on the simplified assumption that $0.5 \cot \theta = 1.0$.

Figure 5.2.12.14-1 shows unfactored shears due to dead loads and live loads for this example.

Start

Determine d_v,
Calculate V_p,
Check that b_v satisfies
Eqn. 5.7.3.3-2

Calculate shear stress
ratio v_u/f'_c using
Eqn. 5.7.2.8-1

If section is within the
transfer length of any
strands, then calculate
the effective value of f_{pe},
else assume $f_{pe} = 0.7f_{pu}$

Calculate ε_x using Eqn.
B5.2-3, 4 or 5

Choose values of θ and β
corresponding to next
larger ε_x in Table B5.2-1

Determine transverse
reinforcement, V_s, to ensure:
$V_u \le \phi(V_c + V_s + V_p)$
(Eqns. 5.7.2.1-1, 5.7.3.3-1)

Can longitudinal
reinforcement resist required
tension?
(T in Eqn. 5.7.3.5-1)

Yes

End

No

Yes

Can transverse
reinforcement, V_s, be
increased thereby reducing
T, i.e., the longitudinal steel
reinforcement in Eqn.
5.7.3.5-1?

No

Provide additional
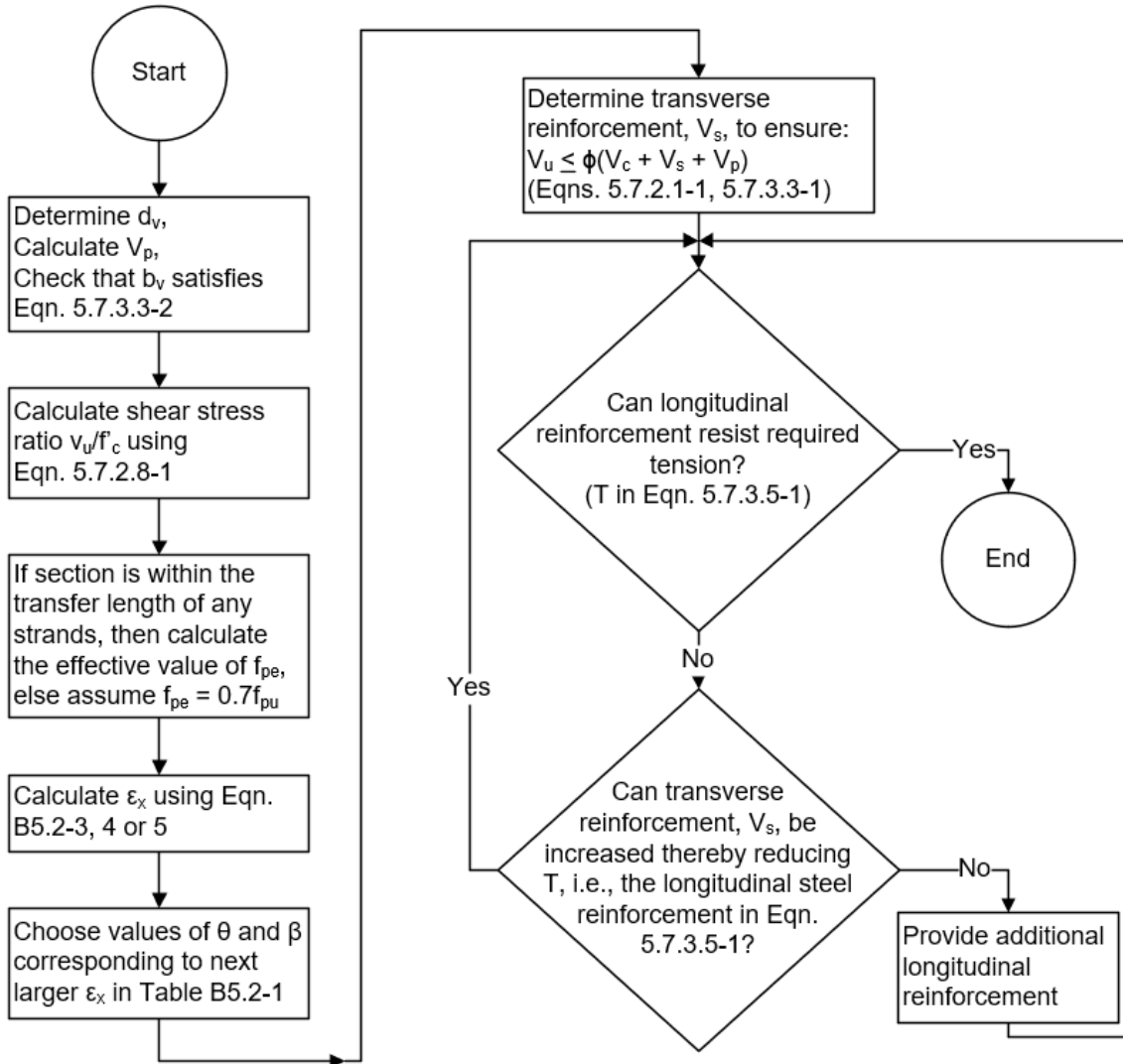longitudinal
reinforcement

**Figure CB5.2-5 Flow Chart for Shear Design Containing at Least Minimum
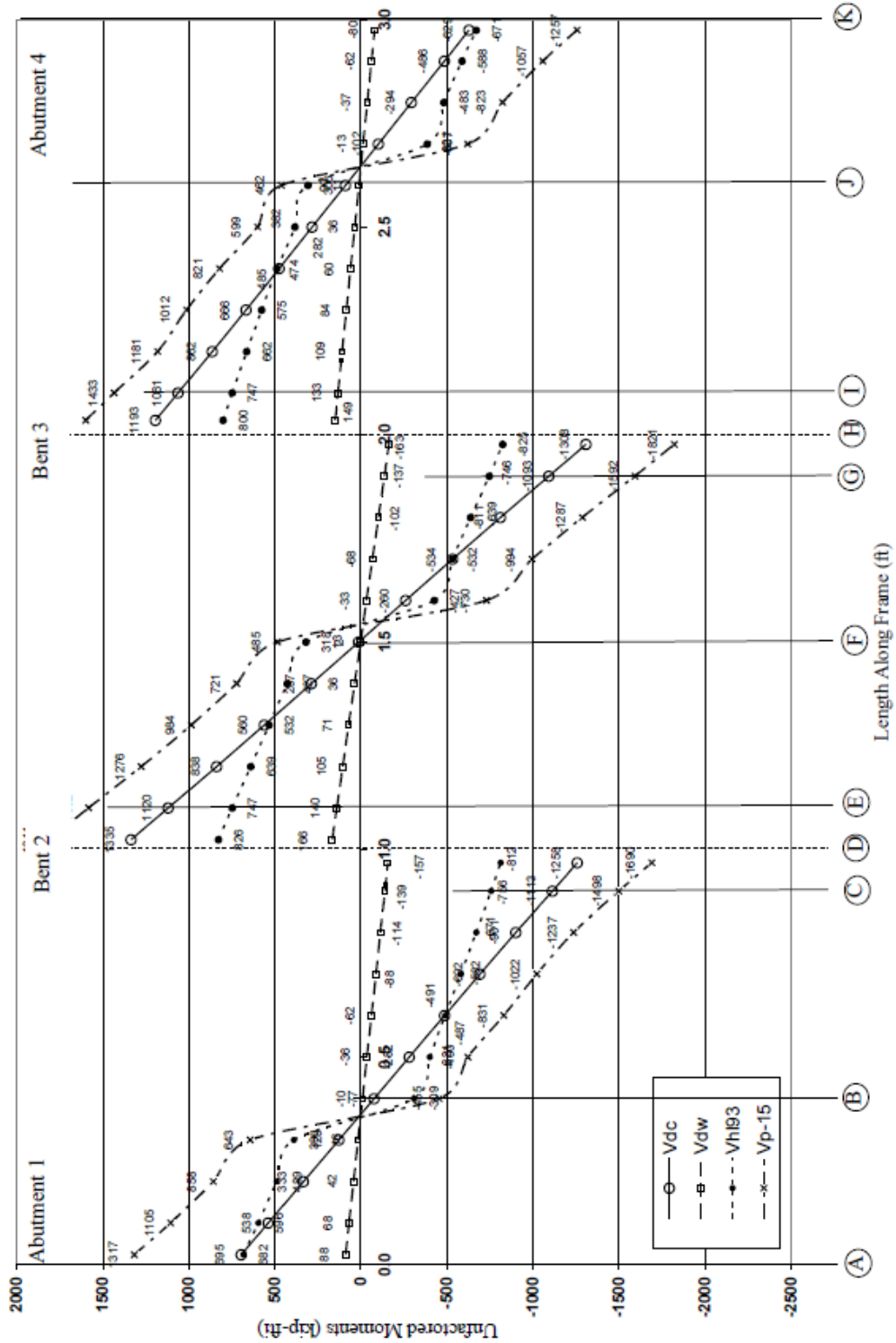Transverse Reinforcement (AASHTO, 2017)**

Figure 5.2.12.14-1 Gravity Load Unfactored Shear Envelopes for Design of Shear Resistance

5.2.12.14.2   Check Interior Girders at Right Cap Face at Bent 2

Design the interior girders at the right cap face at Bent 2 to resist the factored shear. Use Figure 5.2.12.14-1 as a guide and take advantage of the reduction in shears by using the critical section for shear as d_v from the internal face of support.

Step 1: Determine d_v, calculate V_p. Check that b_v satisfies AASHTO Equation 5.7.3.3-2. (AASHTO, 2017)

$$d_v = \frac{M_n}{A_s f_y + A_{ps} f_{ps}} \qquad (5.2.12.14\text{-}1)$$

where:

A_{ps} = area of prestressing steel (in.^{2})

A_s = area of non-prestressed tension reinforcement (in.^{2})

d_v = the effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure (in.)

f_{ps} = average stress in prestressing steel at the time for which the nominal resistance is required (ksi)

f_y = specified minimum yield strength of reinforcement (ksi)

M_n = nominal flexure resistance (kip-in.)

At the right cap face of Bent 2, $M_n = M_r/\phi = 77{,}300/0.95 = 81{,}368$ kip-ft, d_v becomes:

$$d_v = \frac{M_n}{A_s f_y + A_{ps} f_{ps}} = \frac{81{,}368}{65.00(60) + 45.0(260.9)} = 5.20 \text{ ft} = 62.4 \text{ in.}$$

Article 5.7.2.8 (AASHTO, 2017) states that d_v should not be less than the greater of $0.9d_e$ or $0.72h$.

d_e = effective depth from extreme compression fiber to the centroid of the tensile force in the tensile reinforcement = 67.12 in.

$$d_v = \frac{M_n}{A_s f_y + A_{ps} f_{ps}} = 62.4 \text{ in.} > \begin{cases} 0.9d_e = 0.9(67.12) = 60.41 \text{ in.} \\ 0.72h = 0.72(81) = 58.32 \text{ in.} \end{cases} \text{OK}$$

Finding V_p after establishing Equation 5.2.12.14-2:

$$V_p = P_j(\alpha) \text{ kips} \qquad (5.2.12.14\text{-}2)$$

where:

P_j =   force in prestress strands before losses (kip)

V_p =   the component in the direction of the applied shear of the effective prestressing force; positive if resisting the applied shear (kip).

α   =   total angular change of prestressing steel path from jacking end to a point under investigation (rad)

Since the angle that is formed between the tangent of the parabola and the horizontal changes as the location on the parabola changes, we will take two α's at each point. One looking back to the previous point, and the other looking forward to the next point.

Given such a short distance along the larger parabolas, a triangle can be used to approximate the angle change of the much smaller parabola segments:
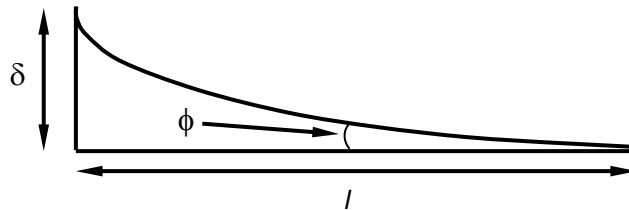
Figure 5.2.12.14-2 Components of Alpha

The general equation of a parabola: $y = ax^2$

For any parabola with side l and δ: $a = δ / l^2$

The angle change at any given point on the parabola is its first derivative:

$$\phi(\text{rads}) = \frac{dy}{dx} = \frac{2δ}{l^2} x \qquad\qquad (5.2.12.14\text{-}3)$$

ϕ at a distance d_v from the face of the cap at Bent 2

$$\phi(\text{rads}) = \frac{2\left[(2.12+3.5)-(0.90+3.8)\right]}{\left(0.1(168)\right)^2}\left(4+\frac{63.0}{12}\right) = 0.06$$

$V_p = P_j(α)$ kips $= 9,120$ (0.06 rad) $= 547$ kips

V_n is the lesser of the following:

$$V_n = V_c + V_s + V_p \qquad\qquad \text{(AASHTO 5.7.3.3-1)}$$

$$V_n = 0.25f'_c b_v d_v + V_p \qquad\qquad \text{(AASHTO 5.7.3.3-2)}$$

where:

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where:

b_v = effective web width taken as the minimum web width, measured parallel to the neutral axis, between resultants of the tensile and compressive forces due to flexure. This value lies within the depth d_v (in.)

d_v = the effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure (in.)

For now, we will use AASHTO 5.7.3.3-2, and return to AASHTO 5.7.3.3-1 later.

$$b_v\left(req'd\right) = \frac{V_n - V_p}{0.25f_c'd_v}$$ (5.2.12.14-4)

where:

V_n = the nominal shear resistance of the section considered (kip)

V_p = the component in the direction of the applied shear of the effective prestressing force; positive if resisting the applied shear (kip)

$$V_n = \frac{V_u}{\phi}$$ (5.2.12.14-5)

Using Figure 5.2.12.14-1 to compare Strength I and Strength II results of V_u at the Bent 2 cap face:

$$V_n@face\left(Str\ I\right) = \frac{V_u}{\phi} = \frac{1.25\left(1,335\right) + 1.50\left(166\right) + 1.75\left(826\right)}{0.90} = 3,737 \text{ kips}$$

$$V_n@pt\ E\left(Str\ I\right) = \frac{V_u}{\phi} = \frac{1.25\left(1,120\right) + 1.50\left(140\right) + 1.75\left(747\right)}{0.90} = 3,241 \text{ kips}$$

$$V_n@face\left(Str\ II\right) = \frac{V_u}{\phi} = \frac{1.25\left(1,335\right) + 1.50\left(166\right) + 1.35\left(1,810\right)}{0.90} = 4,846 \text{ kips}$$

$$V_n@pt\ E\left(Str\ II\right) = \frac{V_u}{\phi} = \frac{1.25\left(1,120\right) + 1.50\left(140\right) + 1.35\left(1,582\right)}{0.90} = 4,162 \text{ kips}$$

Strength II controls so now interpolate to find V_n a distance d_v from the face of Bent 2:

$$V_n@\ d_v \text{ from face} = \frac{4,162 - 4,846}{0.1\left(168\right) - 4}\left(\frac{63.0}{12} + 4 - 4\right) + 4,846 = 4,565 \text{ kips}$$

From above f_c' = 4 ksi and now using Equation 5.2.14.12-4 to find b_v:

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$$b_v\left(req'd\right) = \frac{V_n - V_p}{0.25 f_c' d_v} = \frac{4{,}565 - 547}{0.25(4)62.4} = 64.4 \text{ in.}$$

This results in $b_v \approx 12.88$ in. per girder. Flare the interior girders at bent faces to 13 in. for added capacity in future calculations.

b_v (13 in. flare) = (5 girders)(13 in.) = 65 in.

Step 2: Calculate shear stress ratio v_u / f_c' as follows:

$$v_u = \frac{\left|V_u - \phi V_p\right|}{\phi b_v d_v} \qquad\qquad \text{(AASHTO 5.7.2.8-1)}$$

where:

v_u = shear stress (ksi) (AASHTO 5.7.2.6)

V_u @ d_v from face = 0.90 (4565) = 4,108 kips

ϕV_p = 0.9(547) = 492 kips

$$v_u = \frac{\left|V_u - \phi V_p\right|}{\phi b_v d_v} = \frac{\left|4{,}108 - 492\right|}{0.9(65)62.4} = 0.991 \text{ ksi}$$

$$\frac{v_u}{f_c'} = \frac{0.991}{4} = 0.248$$

Step 3: If the section is within the transfer length of any strands, then calculate the effective value of f_{po}, else assume $f_{po} = 0.7 f_{pu}$. This step is necessary for members without anchorages.

$f_{po} = 0.7 f_{pu} = 189$ ksi

Step 4: Calculate ε_x using Equations B5.2-3, B5.2-4 or B5.2-5 (AASHTO, 2017)

Assuming the section meets the requirements specified in Article 5.7.3.4.2 (AASHTO, 2017)

$$\varepsilon_x = \frac{\dfrac{\left|M_u\right|}{d_v} + 0.5 N_u + 0.5\left|V_u - V_p\right|\cot\theta - A_{ps} f_{po}}{2\left(E_s A_s + E_p A_{ps}\right)} \qquad\qquad \text{(AASHTO B5.2-3)}$$

where:

f_{po} = a parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete (ksi)

N_u = applied factored axial force taken as positive if tensile (kip)

ε_x = longitudinal strain in the web reinforcement on the flexural tension side of the member (in/in)

θ = angle of inclination of diagonal compressive stresses (degrees)

From Figure 5.2.12.13-1, we obtain:

$$M_u @ \ face \ = \ 1.25(-32,619)+1.50(-4,095)+1.35(-23,630)+0.856(9,120)$$
$$= -71,010 \text{ kip-ft}$$

$$M_u @ \ pt \ E \ = \ 1.25(-16,912)+1.50(-2,134)+1.35(-13994)+0.856(9,120)$$
$$= \ -35,426 \text{ kip-ft}$$

By interpolation, find M_u a distance d_v from the face of Bent 2:

$$M_u @ \ d_v \ \text{from face} = \frac{-35,426+71,010}{0.1(168)-4}\left(\frac{62.4}{12}+4-4\right)-71,010$$

$$= \text{-56,415 kip-ft} = 676,980 \text{ kip-in}$$

Now using AASHTO B5.2-3 with $N_u = 0$, begin with cot θ = 1, and values calculated earlier

$$\varepsilon_x = \frac{\dfrac{|-676,980|}{62.4}+0.5(0)+0.5|4,108-547|(1)-45.0(189)}{2[29,000(65)+28,500(45.0)]} = 0.000651$$

Step 5: Choose values of θ and β corresponding to the next-larger ε_x from

AASHTO Table B5.2-1 (AASHTO, 2017).

Based on calculated values of $v_u / f'_c = 0.248$; ε_x = 0.000651, we obtain:

θ = 34.3 and β = 1.58.

Chapter 5.2 – Post-Tensioned Concrete Girders

Table B5.2-1 Values of θ and β for Sections with Transverse Reinforcement (AASHTO, 2017)

$\dfrac{v_u}{f'_c}$	$\varepsilon_x \times 1,000$								
	≤−0.20	≤−0.10	≤−0.05	≤0	≤0.125	≤0.25	≤0.50	≤0.75	≤1.00
≤0.075	22.3 6.32	20.4 4.75	21.0 4.10	21.8 3.75	24.3 3.24	26.6 2.94	30.5 2.59	33.7 2.38	36.4 2.23
≤0.100	18.1 3.79	20.4 3.38	21.4 3.24	22.5 3.14	24.9 2.91	27.1 2.75	30.8 2.50	34.0 2.32	36.7 2.18
≤0.125	19.9 3.18	21.9 2.99	22.8 2.94	23.7 2.87	25.9 2.74	27.9 2.62	31.4 2.42	34.4 2.26	37.0 2.13
≤0.150	21.6 2.88	23.3 2.79	24.2 2.78	25.0 2.72	26.9 2.60	28.8 2.52	32.1 2.36	34.9 2.21	37.3 2.08
≤0.175	23.2 2.73	24.7 2.66	25.5 2.65	26.2 2.60	28.0 2.52	29.7 2.44	32.7 2.28	35.2 2.14	36.8 1.96
≤0.200	24.7 2.63	26.1 2.59	26.7 2.52	27.4 2.51	29.0 2.43	30.6 2.37	32.8 2.14	34.5 1.94	36.1 1.79
≤0.225	26.1 2.53	27.3 2.45	27.9 2.42	28.5 2.40	30.0 2.34	30.8 2.14	32.3 1.86	34.0 1.73	35.7 1.64
≤0.250	27.5 2.39	28.6 2.39	29.1 2.33	29.7 2.33	30.6 2.12	31.3 1.93	32.8 1.70	34.3 1.58	35.8 1.50

Step 6: Determine transverse reinforcement, V_s, to ensure: $V_u \le (V_c + V_s + V_p)$ Equations 5.7.2.1-1, 5.7.3.3-1 (AASHTO, 2017). First, we should introduce the concrete component of shear resistance:

$$v_c = 0.0316\beta\lambda\sqrt{f'_c}\,b_v d_v$$

(AASHTO 5.7.3.3-3)

where:

V_c = nominal shear resistance provided by the tensile stresses in the concrete (kip)

β = factor relating effect of longitudinal strain on the shear capacity of concrete as indicated by the ability of diagonally cracked concrete to transmit tension

λ = concrete density modification factor

$$v_c = 0.0316\beta\lambda\sqrt{f'_c}\,b_v d_v = 0.0316(1.58)(1)\sqrt{4}(65)(63.0) = 409 \text{ kips}$$

V_p = 547 kips

V_u = 4,108 kips

Combining AASHTO Equations 5.7.2.1-1 and 5.7.3.3-1 (AASHTO, 2017) results in design equation as follows:

$$V_u \le \phi\,(V_c + V_s + V_p)$$

Rearranging and solving for V_s:

$$V_s \geq \frac{V_u}{\phi} - V_c - V_p = \frac{4,108}{0.90} - 409 - 547 = 3,608 \text{ kips}$$

$$V_s = \frac{A_v f_y d_v \left(\cot\theta + \cot\alpha \right) \sin\alpha}{s} \qquad \text{(AASHTO 5.7.3.3-4)}$$

where:

A_v = area of transverse reinforcement within distance s (in.^{2}.)

α = angle of inclination of transverse reinforcement to the longitudinal axis (°)

s = spacing of reinforcing bars (in.)

V_s = shear resistance provided by the transverse reinforcement at the section under investigation as given by AASHTO 5.7.3.3-4, except V_s shall not be taken greater than V_u/ϕ (kip)

Then $\alpha = 90°$ since stirrups in this bridge are perpendicular to the deck. Thus Equation 5.7.3.3-4 reduces to:

$$V_s = \frac{A_v f_y d_v \left(\cot\theta \right)}{s} \qquad \text{(AASHTO C5.7.3.3-1)}$$

Rearranging terms to solve for s

$$s \leq \frac{A_v f_y d_v \left(\cot\theta \right)}{V_s} \qquad \text{(5.2.12.14-6)}$$

Assume 2 - #5 legs per stirrup ×5 girders, A_v = 2(5)(0.31) = 3.10 in.^{2}

$$s \leq \frac{A_v f_y d_v \left(\cot\theta \right)}{V_s} = \frac{3.10(60)(62.4)\left(\cot 34.3° \right)}{3,608} = 4.72 \text{ in.}$$

Use # 5 stirrups, s = 4 in. spacing

$$V_s = A_v f_y d_v \left(\cot\theta \right) \frac{A_v f_y d_v \left(\cot\theta \right)}{s} = \frac{3.10(60)(62.4)\left(\cot 34.3° \right)}{4} = 4,254 \text{ kips}$$

Step 7: Use AASHTO Equation 5.7.3.5-1 (AASTHO, 2017) to check if the longitudinal reinforcement can resist the required tension.

$$A_{ps}f_{ps} + A_s f_y \geq \frac{|M_u|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left| \frac{V_u}{\phi_v} - V_p \right| - 0.5V_s \right) \cot \theta \qquad \text{(AASHTO 5.7.3.5-1)}$$

Breaking into parts and solving both sides of Equation 5.7.3.5-1 results in:

$$45.0(260.9) + 65.0(60) = 15{,}641 \text{ kips}$$

$$\left| \frac{-56{,}415(12)}{62.4(0.95)} \right| + 0 + \left(|4{,}564 - 547| - 0.5(4{,}254) \right) \cot(34.3°) = 14{,}191 \text{ kips}$$

Since 15,641 kips is greater than 14,191 kips therefore the shear design is complete. Had the left side being smaller than the right side we would use the following procedure to determine A_s. If the conditions of AASHTO Equation 5.7.3.5-1 (AASHTO, 2017) were met then the shear design process is complete.

Step 8: If the right side of AASHTO 5.7.3.5-1 was greater than the left side, we would need to solve AASHTO Equation 5.7.3.5-1 to increase A_s to meet the minimum requirements of 5.7.3.5-1.

$$A_s \geq \frac{\dfrac{|M_u|}{d_v \phi_f} + 0.5\dfrac{N_u}{\phi_c} + \left(\left| \dfrac{V_u}{\phi_v} - V_p \right| - 0.5V_s \right) \cot \theta - A_{ps}f_{ps}}{f_y}$$

5.2.12.14.3   Check Exterior Girders at Right Cap Face at Bent 2

Design the exterior girders at the right cap face at Bent 2 to resist the factored shear. Use Figure 5.12.14-1 as a guide and take advantage of the reduction in shears by using the critical section for shear as d_v from the internal face of support. In this example, all values will be per girder, since only the exterior girder is affected by this analysis. Use the equations found in Table 4.6.2.2.3c-1 (CA Amendments) to amplify the values of V_u.

Step 1: Determine d_v, calculate V_p. Check that b_v satisfies AASHTO Equation 5.7.3.3-2. (AASHTO, 2017)

From Section 5.2.12.14.2:

d_v = 62.4 in.

V_p = 547 kips or 110 kips/girder

V_n is the lesser of the following:

$$V_n = V_c + V_s + V_p \qquad \text{(AASHTO 5.7.3.3-1)}$$

$$V_n = 0.25 f_c' b_v d_v + V_p \qquad \text{(AASHTO 5.7.3.3-2)}$$

For now, we will use AASHTO 5.7.3.3-2, and return to AASHTO 5.7.3.3-1 later.

$$b_v\left(req'd\right) = \frac{V_n - V_p}{0.25f_c'd_v} \qquad\qquad (5.2.12.14\text{-}4)$$

$$V_n = \frac{V_u}{\phi} \qquad\qquad (5.2.12.14\text{-}5)$$

From Section 5.2.12.14.2, Strength II controls:

V_n @ d_v from face = 4,565 kips

We can now use the equation found in Table 4.6.2.2.3c-1 (CA Amendments) to amplify the obtuse exterior girders shear.

For an exterior girder with a 20º skew,

$$1.0 + \frac{\theta}{50} = 1.0 + \frac{20}{50} = 1.4 < 1.6$$

Now $V_{n(mod)}$ = V_n (ext) = 1.4 (4,565 / 5) = 1278 kips

From above f_c' = 4 ksi and now using Equation 5.2.12.14-4 to find b_v:

$$b_v\left(req'd\right) = \frac{V_n - V_p}{0.25f_c'd_v} = \frac{1,278 - 110}{0.25(4)(62.4)} = 18.72 \text{ in.}$$

This results in b_v = 18.72 in. per girder. We will flare exterior girders at bent faces to 19 in. for added capacity in future calculations.

Step 2: Calculate shear stress ratio v_u / f_c' using AASHTO Equation 5.7.2.8-1. (AASHTO, 2017)

$$v_u = \frac{\left|V_u - \phi V_p\right|}{\phi b_v d_v}$$

V_u @ d_v from face = 0.9 (1,278) = 1,150 kips

$$v_u = \frac{\left|V_u - \phi V_p\right|}{\phi b_v d_v} = \frac{\left|1,150 - (0.9)110\right|}{0.9(19)(62.4)} = 0.985 \text{ ksi}$$

$$\frac{v_u}{f_c'} = \frac{0.985}{4} = 0.246$$

Step 3: If the section is within the transfer length of any strands, then calculate the effective value of f_{po}, else assume $f_{po} = 0.7f_{pu}$. This step is necessary for members with no anchorage devices.

$f_{po} = 0.7f_{pu} = 189$ ksi

Step 4: Calculate ε_x using Equations Appendix B5.2-3, B5.2-4 or B5.2-5 (AASHTO, 2017)

Assuming the section meets the requirements specified in Article 5.7.2.5 (AASHTO, 2017)

$$\varepsilon_x = \frac{\dfrac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}}{2(E_s A_s + E_p A_{ps})} \qquad \text{(AASHTO B5.2-3)}$$

From Section 5.2.12.14.2 and in terms of a single girder:

M_u @ d_v from face = -676,980 kip-in / 5 = -135,396 kip – in

Now using AASHTO B5.2-3 with $N_u = 0$, begin with $\cot\theta = 1$, and values calculated earlier,

$$\varepsilon_x = \frac{\dfrac{|-135,396|}{62.4} + 0.5(0) + 0.5|1,150 - 110|(1) - \dfrac{45.0}{5}(189)}{2\left[29,000\left(\dfrac{65}{5}\right) + 28,500\left(\dfrac{45.0}{5}\right)\right]} = 0.000651$$

Step 5: Choose values of θ and β corresponding to next-larger ε_x from AASHTO Table B5.2-1 (AASHTO, 2017).

Based on calculated values of $v_u / f_c' = 0.246$; $\varepsilon_x = 0.000651 < 0.001$, we obtain:

θ = 35.8 and β = 1.50

Table B5.2-1 Values of θ and β for Sections with Transverse Reinforcement (AASHTO, 2017)

$\frac{v_u}{f_c'}$	$\varepsilon_x \times 1,000$								
	≤−0.20	≤−0.10	≤−0.05	≤0	≤0.125	≤0.25	≤0.50	≤0.75	≤1.00
≤0.075	22.3	20.4	21.0	21.8	24.3	26.6	30.5	33.7	36.4
	6.32	4.75	4.10	3.75	3.24	2.94	2.59	2.38	2.23
≤0.100	18.1	20.4	21.4	22.5	24.9	27.1	30.8	34.0	36.7
	3.79	3.38	3.24	3.14	2.91	2.75	2.50	2.32	2.18
≤0.125	19.9	21.9	22.8	23.7	25.9	27.9	31.4	34.4	37.0
	3.18	2.99	2.94	2.87	2.74	2.62	2.42	2.26	2.13
≤0.150	21.6	23.3	24.2	25.0	26.9	28.8	32.1	34.9	37.3
	2.88	2.79	2.78	2.72	2.60	2.52	2.36	2.21	2.08
≤0.175	23.2	24.7	25.5	26.2	28.0	29.7	32.7	35.2	36.8
	2.73	2.66	2.65	2.60	2.52	2.44	2.28	2.14	1.96
≤0.200	24.7	26.1	26.7	27.4	29.0	30.6	32.8	34.5	36.1
	2.63	2.59	2.52	2.51	2.43	2.37	2.14	1.94	1.79
≤0.225	26.1	27.3	27.9	28.5	30.0	30.8	32.3	34.0	35.7
	2.53	2.45	2.42	2.40	2.34	2.14	1.86	1.73	1.64
≤0.250	27.5	28.6	29.1	29.7	30.6	31.3	32.8	34.3	35.8
	2.39	2.39	2.33	2.33	2.12	1.93	1.70	1.58	1.50

Step 6: Determine transverse reinforcement, V_s, to ensure: $V_u \le (V_c + V_s + V_p)$ Equations 5.7.2.1-1, 5.7.3.3-1 (AASHTO, 2017). First use AASHTO 5.7.3.3-3 to introduce the concrete component of shear resistance.

$$v_c = 0.0316\beta\lambda\sqrt{f'_c}\,b_v d_v$$

$$v_c = 0.0316\beta\lambda\sqrt{f'_c}\,b_v d_v = 0.0316(1.50)(1)\sqrt{4}\,(19)(62.4) = 112.4 \text{ kips}$$

V_p = 110 kips

V_u = 1,150 kips

Combining Equations 5.7.2.1the -1 and 5.7.3.3-1 (AASHTO, 2017) results in design equation as follows:

$$V_u \le \phi\,(V_c + V_s + V_p)$$

Rearranging and solving for V_s:

$$V_s \ge \frac{V_u}{\phi} - V_c - V_p = \frac{1,150}{0.90} - 112.4 - 110 = 1,055 \text{ kips}$$

Then using Eq. 5.2.12.14-6

$$s \le \frac{A_v f_y d_v (\cot\theta)}{V_s}$$

Assume 2 - #5 legs per stirrup ×1 girder, A_v = 2(1)(0.31) = 0.62 in.^{2}

$$s \leq \frac{A_v f_y d_v (\cot \theta)}{V_s} = \frac{0.62(60)(62.4)(\cot 35.8^\circ)}{1,055} = 3.05 \text{ in.}$$

Use # 5 stirrups, s = 3 in. spacing

Step 7: Use Modified Equation 5.7.3.5-1 (AASTHO, 2017) to check if the longitudinal reinforcement per girder can resist the required tension.

$$A_{ps} f_{ps} + A_s f_y \geq \frac{|M_u|}{d_v \phi_f} + 0.5 \frac{N_u}{\phi_c} + \left(\left| \frac{V_u}{\phi_v} - V_p \right| - 0.5 V_s \right) \cot \theta$$

Breaking into parts and solving both sides of AASHTO Equation 5.7.3.5-1 results in:

45.0(260.9) / 5+ 65.0(60) / 5 = 3,128 kips

$$\left| \frac{-135,396}{62.4(0.9)} \right| + 0 + \left(\left| \frac{1,150}{0.9} - 110 \right| - 0.5(1,055) \right) \cot 35.8 = 3,299 \text{ kips}$$

3,128 is not greater than 3,299. Use the following procedure to determine A_s.

Step 8: Solve AASHTO Equation 5.7.3.5-1 to increase A_s to meet the minimum requirements:

$$A_s \geq \frac{\dfrac{|M_u|}{d_v \phi_f} + 0.5 \dfrac{N_u}{\phi_c} + \left(\left| \dfrac{V_u}{\phi_v} - V_p \right| - 0.5 V_s \right) \cot \theta - A_{ps} f_{ps}}{f_y}$$

$$A_s = \frac{3,299 - \dfrac{45.0(260.9)}{5}}{60} = 15.84 \text{ in.}^2$$

The right side of AASHTO Equation 5.7.3.5-1 is equal to 3,299, the exterior 1/2 bay reinforcement should be increased from A_s = 65 / 5 = 13.0 in.^{2} to A_s = 16 in.^{2}

5.2.12.15   Calculate the Prestressing Elongation

Tendon elongation calculations are necessary to help ensure the proper jacking force is delivered to the superstructure. Elongation calculations are one way for construction field personnel to check the actual P_j force applied to tendons.

Since the structure has been designed for two-end stressing, both first and second end elongations need to be computed.

Based on the location and magnitude of f_{pF} (stress with friction losses) shown on the contract plans, the post-tensioning fabricator develops a simplified diagram, like the one shown in Figure 5.2.12.15-1.
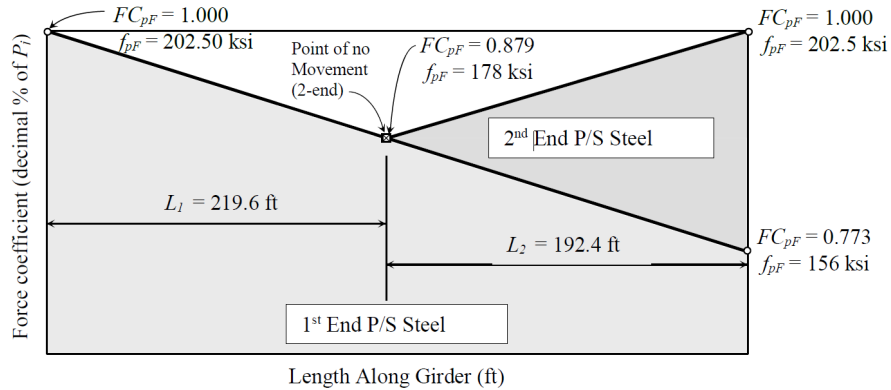

Figure showing Force coefficient (decimal % of P_i) versus Length Along Girder (ft):

- $FC_{pF} = 1.000$, $f_{pF} = 202.50$ ksi (top left)
- Point of no Movement (2-end)
- $FC_{pF} = 0.879$, $f_{pF} = 178$ ksi
- $FC_{pF} = 1.000$, $f_{pF} = 202.5$ ksi (top right)
- 2nd End P/S Steel
- $L_1 = 219.6$ ft
- $L_2 = 192.4$ ft
- $FC_{pF} = 0.773$, $f_{pF} = 156$ ksi
- 1st End P/S Steel

Figure 5.2.12.15-1 Simplified Diagram Example

The 1st end elongation calculation (from the 1st end of the jacking side to the anchorage side, the entire length of the span):

The prestressing elongation is based on the stress-strain relationship and results in the following:

$$\Delta_E = \frac{f_{avg}L_x}{E_p} \qquad (5.2.12.15\text{-}1)$$

where

$\quad E_p \quad = \quad$ modulus of elasticity of prestressing tendon (ksi)

$\quad f_{avg} \quad = \quad$ average stress in the strand from jacking end to point of no movement (ksi)

$\quad \Delta_E \quad = \quad$ change in length of prestressing tendons due to jacking (in.)

For one-end stressing: $\quad \Delta_E = \dfrac{T_o\left(1+\otimes\right)\left(L+3.5'\right)}{2E_p} \qquad (5.2.12.15\text{-}2)$

In Equation 5.2.12.15-2, the 3.5' term is the expected length of jack.

For two end stressing:

$$\Delta_{1st} = \frac{T_o}{2E_p}\left[\left(1+\otimes\right)L_1 + \left(3\otimes -1\right)L_2\right] \qquad (5.2.12.15\text{-}3)$$

$$\Delta_{2nd} = \frac{T_o(1+\otimes)L_2}{E_p}$$

(5.2.12-15-4)

where:

Δ_{1st} = elongation after stressing the first end (in)

Δ_{2nd} = elongation after stressing the second end (in)

T_o = steel stress at the jacking end before seating (generally 202.5 ksi) (ksi)

$\otimes$ = initial force coefficient at the point of no movement

L = Length of tendon (ft)

L_1 = Length of tendon from the first stressing end to the point of no movement (ft)

L_2 = Length of the tendon from the point of no movement to the second stressing end (ft)

For our bridge, let's use the values shown on the graph above and applying the 3 ft length of the jack:

$$\Delta_{1st} = \frac{202.5}{(2)28,500}\left[(1+0.879)(219.6)+((3x0.879)-1)(192.40)\right]$$

$$= 2.585 \text{ ft} = 31.02 \text{ in.}$$

When applying these measurements in the field, it is necessary to determine the "measurable" elongation. The measurable elongation includes the length of tendon in the contractor's jack (3 ft). It has been found that by applying 20% of P_j before monitoring elongations, the tendon is allowed to shift from its resting place to its final position. Therefore, only 80% of the elongation calculated above is measurable.

Δ_{1st} = (0.8)(31.02) = 24.81 in.

The 2nd end elongation calculation (from the 2nd end of jacking side to the point of no

movement): $$\Delta_{2nd} = \frac{202.5(1-0.879)(192.4)}{28,500} = 0.165 \text{ ft} = 1.98 \text{ in.}$$

Chapter 5.2 – Post-Tensioned Concrete Girders

NOTATION

A_g	=	gross area of section (in.^{2})
A_{ps}	=	area of prestressing steel (in.^{2})
A_s	=	area of non-prestressed tension reinforcement (in.^{2})
A'_s	=	area of compression reinforcement (in.^{2})
A_v	=	area of transverse reinforcement within distance s (in.^{2})
b	=	width of the compression face of a member (in.)
b_v	=	effective web width taken as the minimum web width, measured parallel to the neutral axis, between resultants of the tensile and compressive forces due to flexure. b_v lies within the depth d_v (in.)
b_w	=	web width (in.)
c	=	distance from the extreme compression fiber to the neutral axis (in.)
clr_{int}	=	clearance from the interior face of the bay to the first mat of steel in the soffit or deck (Usually taken as 1 in.) (in.)
D	=	distance between C.G. of the prestressing steel and the bottom of the soffit or top of the deck plus the cover clearance from interior face (in.)
D_c	=	column diameter (ft)
d	=	depth of member (in.)
d_e	=	defective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (in.)
d_i	=	distance between C.G. of the ith duct and the ith duct LOL (See Figure 5.2.12.6-5) (in.)
d_p	=	distance from the extreme compression fiber to the centroid of the prestressing tendons (in.)
d_s	=	distance from the extreme compression fiber to the centroid of the nonprestressed tensile reinforcement (in.)
d_v	=	the effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure (in.)
E_c	=	modulus of elasticity of concrete (ksi)
E_{ci}	=	modulus of elasticity of concrete at transfer (ksi)
E_{ct}	=	modulus of elasticity of concrete at transfer or time of load application (ksi)
E_p	=	modulus of elasticity of prestressing tendons (ksi)
FC_{pA}	=	force coefficient for loss from anchor set
FC_{pES}	=	force coefficient for loss from elastic shortening

FC_{pF} = force coefficient for loss from friction

FC_{pT} = total force coefficient for loss

FC_{pLT} = force coefficient for long-term loss

e = eccentricity of resultant of prestressing with respect to the centroid of the cross section, always taken as a positive (ft); base of natural logarithms

e_x = eccentricity as a function of x along parabolic segment (ft)

f_{avg} = average stress in the strand from the jacking end to point of no movement (ksi)

f'_c = specified compressive strength of concrete used in design (ksi)

f'_{ci} = specified compressive strength of concrete at the time of initial loading or prestressing (ksi); nominal concrete strength at the time of application of tendon force (ksi)

f_{pe} = effective stress in the prestressing steel after losses (ksi)

f_{pi} = initial stress in the prestressing steel after losses, considering only the effects of friction loss. No other P/S losses have occurred (ksi)

f'_s = stress in the nonprestressed compression reinforcement at nominal flexural resistance (ksi)

f_{cgp} = concrete stress at the center of gravity of prestressing tendons, that results from the prestressing force at either transfer or jacking and the self-weight of the member at maximum moment sections (ksi)

f_{cpe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at the extreme fiber of the section where tensile stress is caused by externally applied loads (ksi)

f_{DC+DW} = stress in concrete from DC and DW load cases (ksi)

$f_{DCw/o\ b}$ = stress in concrete due to the Dead Load of the structural section only (ksi)

f_g = stress in the member from dead load (ksi)

f_{HL93} = stress in concrete from $HL93$ load cases (ksi)

f_{pe} = effective stress in prestressing steel after losses (ksi)

f_{pi} = stress in prestressing steel immediately prior to transfer (ksi)

f_{pj} = stress in the prestressing steel at jacking (ksi)

f_{po} = a parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete (ksi)

f_{ps} = average stress in prestressing steel at the time for which the nominal resistance is required (ksi)

f_{pu}	=	specified tensile strength of prestressing steel (ksi)
f_{py}	=	yield strength of prestressing steel (ksi)
f_r	=	modulus of rupture of concrete (ksi)
f_s	=	stress in the nonprestressed tension reinforcement at nominal flexural resistance (ksi)
f_u	=	specified tensile strength of reinforcement (ksi)
f_y	=	specified minimum yield strength of reinforcement (ksi)
H	=	average annual ambient mean relative humidity (percent)
h_f	=	compression flange depth (in.)
I_{cr}	=	moment of inertia of the cracked section, transformed to concrete (in.^{4})
I_e	=	effective moment of inertia (in.^{4})
I_g	=	moment of inertia of the gross concrete section about the centroidal axis, neglecting reinforcement (in.^{4})
K	=	distance to the closest duct to the bottom of the soffit or top of the deck (in.)
k	=	wobble friction coefficient (per ft of tendon)
L	=	distance to a point of known stress loss (ft), individual span length (ft) ($60 \leq L \leq 240$)
Lin_{frame}	=	length of frame to be post-tensioned (ft)
M_a	=	maximum moment in a member at the stage which the deformation is computed (kip-in.)
M_{cr}	=	cracking moment (kip-in.)
M_n	=	nominal flexure resistance (kip-in.)
M_r	=	factored flexural resistance of a section in bending (kip-in.)
M_{DL}	=	dead load moment of structure (kip-in.)
$M_{P/S}$	=	moment due to the secondary effects of prestressing (k-ft)
MC_p	=	primary moment force coefficient for loss (ft)
MC_s	=	secondary moment force coefficient for loss (ft)
MC_{PT}	=	total moment force coefficient for loss (ft)
N	=	number of identical prestressing tendons
N_c	=	number of cells in a concrete box girder (Nc $\geq$ 3)
N_u	=	applied factored axial force taken as positive if tensile (kip)
P_j	=	force in prestress strands before losses (kip)
l_{ij}	=	length of individual parabola (in.)
n_i	=	number of strands in the ith duct

S	=	slab span length (ft)
S_c	=	section modulus for the extreme fiber of the composite sections where tensile stress is caused by externally applied loads (in.^{3})
s	=	spacing of reinforcing bars (in.)
t	=	thickness of soffit or deck (in.)
t_d	=	thickness of deck (in.)
t_s	=	thickness of soffit (in.)
V_c	=	nominal shear resistance provided by tensile stresses in the concrete (kip)
V_n	=	the nominal shear resistance of the section considered (kip)
V_p	=	the component in the direction of the applied shear of the effective prestressing force; positive if resisting the applied shear (kip)
V_s	=	shear resistance provided by the transverse reinforcement at the section under investigation as given by AASHTO 5.7.3.3-4, except V_s shall not be taken greater than V_u / ϕ (kip)
v_u	=	average factored shear stress on the concrete (ksi)
W	=	weight of prestressing steel obtained by using Figure 5.2.12-6-1 (lb)
w_c	=	unit weight of concrete (kcf)
x	=	length of a prestressing tendon from the jacking end to any point under consideration (ft); location along parabolic segment where eccentricity is calculated (% span L)
x_{ij}	=	length of parabolic segment under consideration (ft)
x_{pA}	=	influence length of anchor set (ft)
y	=	general distance from the neutral axis to a point on member cross-section (in.)
y_{ij}	=	height of individual parabola (in.)
y_t	=	distance from the neutral axis to the extreme tension fiber (in.)
Z	=	C.G. tendon shift within the duct (in.)
α	=	angle of inclination of transverse reinforcement to the longitudinal axis (°); sum of the absolute values of angular change of prestressing steel path from jacking end, or from the nearest jacking end if tensioning is done equally at both ends, to the point under investigation (rad.)
β	=	factor relating effect of longitudinal strain on the shear capacity of concrete, as indicated by the ability of diagonally cracked concrete to transmit tension
β_1	=	ratio of the depth of the equivalent uniformly stressed compression zone assumed in the strength limit state to the depth of the actual compression zone

δ_{hp}	=	offset from deck to centroid of the duct (in.)
δ_{lp}	=	offset from soffit to centroid of the duct (in.)
Δ_{Aset}	=	anchor set length (in.)
ΔE	=	change in length of prestressing tendons due to jacking (in.)
Δf_i	=	change in force in prestressing tendon due to an individual loss (ksi)
Δf_L	=	friction loss at the point of known stress loss (ksi)
Δf_{pA}	=	loss due to the anchorage set (ksi)
Δf_{pCR}	=	prestress loss due to creep of girder concrete between the transfer and the deck placement (ksi)
Δf_{pES}	=	sum of all losses or gains due to elastic shortening or extension at the time of application of prestress and/or external loads (ksi)
Δf_{pF}	=	loss due to friction (ksi)
Δf_{pSR}	=	prestress loss due to shrinkage of girder concrete between transfer and deck placement (ksi)
Δf_{pR}	=	an estimate of relaxation loss taken as 2.4 ksi for low relaxation strand and in accordance with manufacturers recommendation for other types of strand (ksi)
ε_{cu}	=	failure strain of concrete in compression (in./in.)
ε_s	=	net longitudinal tensile strain in section at the centroid of the tension reinforcement (in./in.)
ε_t	=	net tensile strain in extreme tension steel at nominal resistance (in./in.)
ε_{cl}	=	compression-controlled strain limit in the extreme tension steel (in./in.)
ε_{tl}	=	tension-controlled strain limit in the extreme tension steel (in./in.)
λ	=	concrete density modification factor
θ	=	angle of inclination of diagonal compressive stresses (degrees)
ϕ	=	resistance factor
μ	=	friction factor
γ_1	=	flexural cracking variability factor
γ_2	=	prestress variability factor
γ_3	=	ratio of specified minimum yield strength to ultimate tensile strength of reinforcement

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